Prøveeksamen 18 april 2024

1 $\frac{1}{\sin x} = \frac{1}{\sin 2x}$ $x \in [0, 2\pi]$

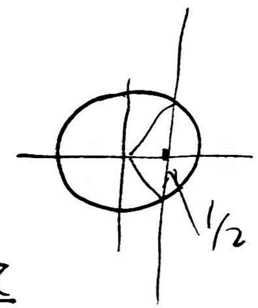
Ikke definert når $\sin(x)$ eller $\cos(x)$ er lik 0.

$\sin(2x) = 2\sin x \cos x$

For $\sin x, \cos x \neq 0$ er likningen ekvivalent til

$\sin(x) = \sin(2x) \Leftrightarrow \sin x = 2\sin x \cos x$

$\Leftrightarrow \frac{1}{2} = \cos x$




Løsningene er $\underline{x = \frac{\pi}{3}}$ og $x = 2\pi - \frac{\pi}{3} = \underline{\frac{5\pi}{3}}$

2 $\frac{1}{x-2} \geq \frac{3}{x-3} \Leftrightarrow \frac{1}{x-2} - \frac{3}{x-3} \geq 0$

$\Leftrightarrow \frac{(x-3) - 3(x-2)}{(x-2)(x-3)} \geq 0 \Leftrightarrow \frac{-2x+3}{(x-2)(x-3)} \geq 0$

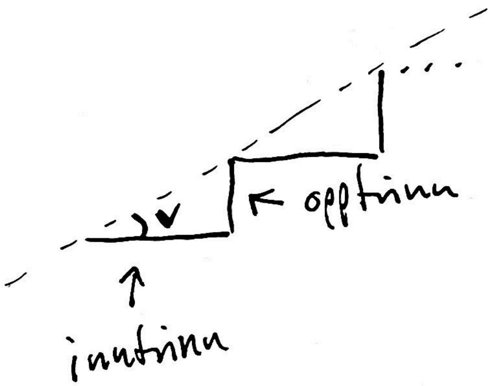
Fortegnsskjema

| | | $\frac{3}{2}$ | | $2$ | | $3$ | |
|---|---|---|---|---|---|---|---|
| $\frac{1}{x-2}$ | - - - | - - | - - | x | —— | —— | —— |
| $\frac{1}{x-3}$ | - - - | - - | - - | - - | - - | x | —— |
| $3-2x$ | —— | 0 | - - | - - | - - | - - | - - |
| $\frac{3-2x}{(x-2)(x-3)}$ | —— | 0 | - - | x | —— | x | - - - |

Løsningsmengden er

$x \in \langle -\infty, \frac{3}{2}] \cup \langle 2, 3 \rangle$

3




$v$ stigningsvinkelen

$V = 30^\circ$ Finn inntrinnet $i$ og opptrinnet $o$.

$$i + 2o = 62$$

$$\frac{1/2}{\sqrt{3}/2} = \tan(30^\circ) = \frac{o}{i} \quad \text{så} \quad \frac{1}{\sqrt{3}} = \frac{o}{i}.$$

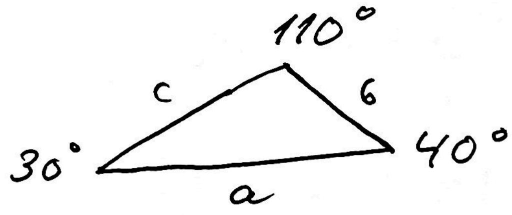
$$\Leftrightarrow \quad i = \sqrt{3}\, o$$

Likningssystem
$$i + 2o = 62$$
$$i = \sqrt{3}\, o$$

$$\text{så} \quad (\sqrt{3} + 2)o = 62 \quad \text{så} \quad o = \frac{62}{2+\sqrt{3}} = \underline{16.6\text{cm}}$$

$$i = 62\text{cm} - 2(16.6\text{cm}) = \underline{28.8\text{cm}}$$

4




Siste vinkel: 110° $= 180° - 30° - 40°$

side a, motstående den største vinkelen, er lengst.

$$A = \frac{a \cdot c}{2} \underbrace{\sin 30°}_{1/2} = 10 \qquad \text{arealsetningen}$$

$$\text{Så} \quad a \cdot c = \underline{40}$$

Sinussetningen :

$$\frac{a}{\sin 110°} = \frac{c}{\sin 40°}$$

$$\text{Så} \quad c = a \cdot \sin(40°) / \sin(110°)$$

$$a\left(a \frac{\sin(40°)}{\sin(110°)}\right) = 40$$

$$a^2 = 40 \cdot \frac{\sin(110°)}{\sin(40°)} \qquad a > 0$$

$$a = 2\sqrt{10}\sqrt{\frac{\sin(110°)}{\sin(40°)}}$$

$$\sim \underline{7.647} \text{ cm}$$

5 $\quad v(t) = t e^{-t/4} \qquad t \in [0, 10] = I$

produkt regelen

a) $a(t) = v'(t) = (t \cdot e^{-t/4})'$

$= \underbrace{(t)'}_{1} e^{-t/4} + t \underbrace{(e^{-t/4})'}_{e^{-t/4}(-t/4)'}$

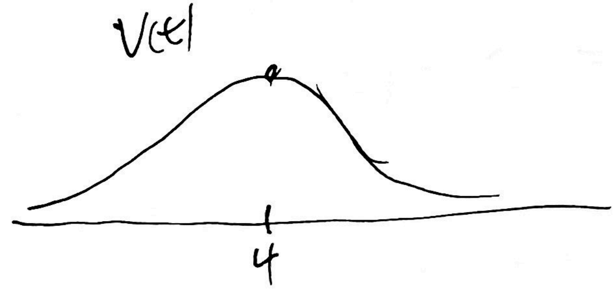
$a(t) = e^{-t/4} - \frac{1}{4} t e^{-t/4}$

$= \underline{(1 - \frac{t}{4}) e^{-t/4}}$

$a(t) > 0, \quad 0 \leq t < 4$

$a(t) < 0 \quad 4 < t < 10.$

$a(t) = 0$ når $t = 4$




$|v(t)|$ er størst for $t = 4$

største fart er $|v(4)| = |4 e^{-4/4}|$

$= \underline{\underline{\frac{4}{e}}}$

b)

$s(0) = 2$

$s'(t) = v(t) = t e^{-t/4}$

$s(t)$ er en antiderivert til $v(t)$.

delvis integras

$$\int \underbrace{t}_{v} \underbrace{e^{-t/4}}_{u'} \, dt \qquad u' = e^{-t/4} \qquad u = -4e^{-t/4}$$

$$= t(-4e^{-t/4}) - \int \underbrace{1}_{v'} (-4e^{-t/4}) \, dt$$

$$= -4te^{-t/4} - ((-4)(-4e^{-t/4})) + c$$

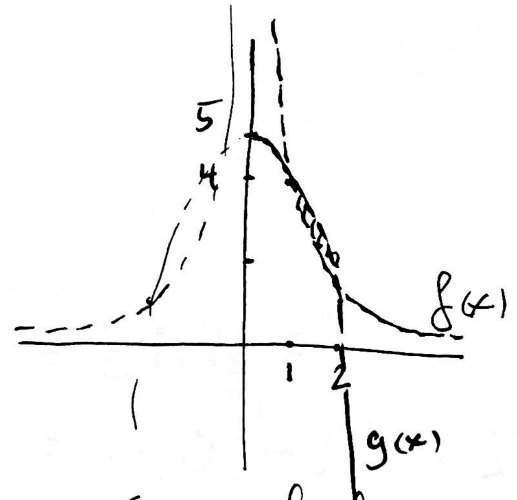
$$= s(t) = -4te^{-t/4} - 16e^{-t/4} + c$$

$$s(0) = 2 = 0 - 16e^{0} + c = c - 16$$

$$c = 2 + 16 = 18$$

$$s(t) = (-4te^{-t/4} - 16e^{-t/4} + 18)\ \mathrm{m}$$

6




symmetrisk om y-akse.

$f(x) = g(x)$

$\frac{4}{x^2} = 5 - x^2$

$(x^2)^2 - 5x^2 + 4 = 0$

$(x^2 - 4)(x^2 - 1) = 0$

$x = \pm 1$

$x = \pm 2$

$$A = \underset{\substack{\uparrow \\ \text{to like} \\ \text{områder}}}{2} \int_1^2 \underbrace{5 - x^2 - \frac{4}{x^2}}_{\geq 0} dx$$

$$= 2 \left[ 5x - \frac{x^3}{3} + \frac{4}{x} \right]_1^2$$

$$= 2 \left[ 5(2-1) - \frac{1}{3}(2^3 - 1^3) + 4\left(\frac{1}{2} - \frac{1}{1}\right) \right]$$

$$= 2 \left[ 5 - \frac{7}{3} + 4\left(\frac{-1}{2}\right) \right] = 2\left[5 - 2 - \frac{7}{3}\right]$$

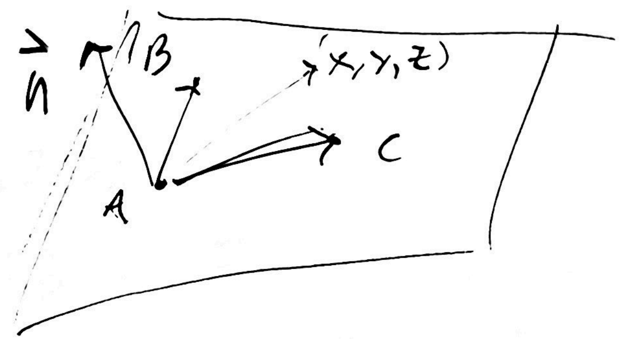
$$= 2 \left[ \frac{3 \cdot 3 - 7}{3} \right] = \underline{\underline{\frac{4}{3}}}$$

Arealet til de to områdene begrenset av f og g er tilsammen $\underline{\underline{4/3}}$

7. $A(1,3,0)$

$B(1,1,-2)$

$C(3,1,1)$




$\vec{AB} = \vec{OB} - \vec{OA} = [1,1,-2] - [1,3,0]$

$= [0,-2,-2] = -2[0,1,1]$

$\vec{AC} = \vec{OC} - \vec{OA} = [3,1,1] - [1,3,0] = [2,-2,1]$

$\vec{AB} \times \vec{AC} = -2 \begin{vmatrix} i & j & k \\ 0 & 1 & 1 \\ 2 & -2 & 1 \end{vmatrix}$

$= -2\left[ \begin{vmatrix} 1 & 1 \\ -2 & 1 \end{vmatrix}, - \begin{vmatrix} 0 & 1 \\ 2 & 1 \end{vmatrix}, \begin{vmatrix} 0 & 1 \\ 2 & -2 \end{vmatrix} \right]$

$= -2[3, 2, -2]$

$\vec{n} = [3,2,-2]$.

Planet $[3,2,-2] \cdot [x,y,z] = [3,2,-2] \cdot \underbrace{[1,3,0]}_{\vec{OA}}$

$\underline{3x + 2y - 2z = 3 + 2 \cdot 3 = 9}$

Linje gjennom $P(2,0,3)$

retningsvektor: $\vec{r} = [1,2,3]$

parametrisering: $[x,y,z] = \vec{OP} + t\vec{r}$

$= [2,0,3] + t[1,2,3]$

punkt på linjen ligger i planet når de oppfyller $3x+2y-2z = 9$

$$3(\underbrace{2+t}_{x}) + 2(\underbrace{2t}_{y}) - 2(3+3t) = 9$$

$$6+3t + 4t - 2\cdot 3 - 2\cdot 3t = 9$$

$$6-6 + 3t+4t-6t = 9$$

$$\underline{t = 9}$$

$$[x,y,z] = [2,0,3] + \underbrace{9[1,2,3]}_{[9,18,27]}$$

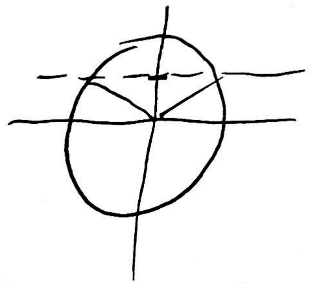
$$= [11,18,30]$$

Snittpunktet er $\underline{\underline{(11,18,30)}}$

8 $f(x) = \cos(2x) + x$ $D_f = [0, 2]$ (radianer)

a) $f'(x) = -\sin(2x) \cdot (2x)' + 1$

$= -2\sin(2x) + 1.$



$f'(x) = 0$ : $\sin(2x) = \frac{1}{2}$

$u = 2x$ $\sin(u) = \frac{1}{2}$

$2x = u = \frac{\pi}{6} + 2\pi \cdot n$

$2x = u = \frac{5\pi}{6} + 2\pi \cdot n$

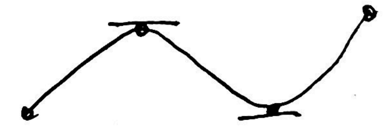
$x = \frac{\pi}{12} + \pi \cdot n$ $n = 0$

så $x = \frac{5\pi}{12} + \pi \cdot n$ $n = 0$

$f'(x) = 0$ når $\underline{x = \frac{\pi}{12} \text{ og } \frac{5\pi}{12}}.$

$0 \quad \pi/12 \quad \frac{5\pi}{12} \quad 2$

$f' = 1 - 2\sin 2x$ ——o- - - - -o——



$f(x)$ vokser i $[0, \pi/12]$
og i $[\frac{5\pi}{12}, 2]$

$f(x)$ avtar i $[\frac{\pi}{12}, \frac{5\pi}{12}]$.

$f(x)$ kontinuerlig funksjon på et lukket begrenset intervall.

Ekstremalverdisetningen gir da eksistens av største og minste verdi:

$f(0) = \cos(0) - 0 = 1$ endepunkt

$f(2) = \cos(4) + 2 \approx 1.346$ —||—

$f(\pi/12) = \cos(\frac{\pi}{6}) + \frac{\pi}{12} = \frac{\sqrt{3}}{2} + \frac{\pi}{12} \sim 1.128$

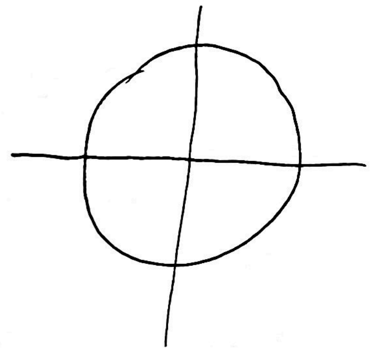
$f(\frac{5\pi}{12}) = \cos(\frac{5\pi}{6}) + \frac{5\pi}{12} = -\frac{\sqrt{3}}{2} + \frac{5\pi}{12} \sim 0.443$

Toppunkt $(2, 1.346)$ globalt, $(\pi/12, 1.128)$ | Bunnpunkt $(0,1)$ og $(\frac{5\pi}{12}, 0.443)$ globalt

b) $f''(x) = \left(-2\sin(2x)+1\right)'$

$= -2\cos(2x)\cdot(2x)' + 0$

$= -4\cos(2x).$



$u = 2x$

$0 \leq u \leq 4$

$\cos u > 0 \qquad u \in [0, \pi/2\rangle$

$\cos u = 0 \qquad u = \pi/2$

$\cos u < 0 \qquad u \in \langle \pi/2, 4]$

$2x = u = \frac{\pi}{2}$

$\underline{x = \pi/4}$

$f''(x) < 0 \qquad x \in [0, \pi/4\rangle$ konkav ned

$f''(x) = 0 \qquad x = \pi/4$

$f''(x) > 0 \qquad x \in \langle \pi/4, 2]$ konkav opp

Vendepunkt $\left(\frac{\pi}{4}, f\left(\frac{\pi}{4}\right)\right) = \left(\frac{\pi}{4}, \overbrace{\cos\left(2\cdot\frac{\pi}{4}\right)}^{0} + \frac{\pi}{4}\right)$

$= \underline{\left(\frac{\pi}{4}, \frac{\pi}{4}\right)}$

$f'\left(\frac{\pi}{4}\right) = \left(-2\sin\left(2\cdot\frac{\pi}{4}\right)+1\right) = -2\cdot 1 + 1 = \underline{\underline{-1}}$

Vendetangenten er $y = -\left(x - \frac{\pi}{4}\right) + \frac{\pi}{4}$

$\underline{y = -x + \frac{\pi}{2}}$

9 $\ln(5) = \int_1^5 \frac{1}{x}\,dx$

Simpsons formel med 2 doble intervaller

$\underbrace{\Delta x = 2}$

$2 \quad 4$

$1 \quad 3 \quad 5$

Vekting: $\frac{1}{6}\ \frac{4}{6}\ \frac{2}{6}\ \frac{4}{6}\ \frac{1}{6}$

(2 er bredden)

$$\int_1^5 \frac{1}{x}dx \sim S_2 = \frac{1}{6}\left[1\cdot\frac{1}{1} + 4\cdot\frac{1}{2} + 2\cdot\frac{1}{3} + 4\cdot\frac{1}{4} + 1\cdot\frac{1}{5}\right]\cdot 2$$

$$= \frac{2}{6}\left[1 + 2 + \frac{2}{3} + 1 + \frac{1}{5}\right] = \frac{1}{3}\left[\frac{4\cdot 15 + 2\cdot 5 + 1\cdot 3}{3\cdot 5}\right]$$

$$= \frac{73}{3\cdot 15} = \frac{73}{45} = \underline{1.62}$$

$(\ln 5 \sim 1.609437\ldots)$

10 $$\sum_{i=0}^{99} \left(\frac{1}{i+3}\right)^2 = \sum_{n=1}^{100} \frac{1}{(n+2)^2}$$

- ---- print... blir da en del av for-løkken skriver da ut alle delsummene.

- $$\sum_{n=1}^{300} \frac{1}{n^2+1}$$

```
N = 300
:
for i in range(300)    eller N
    sum = sum + 1/((i+1)**2
:
```