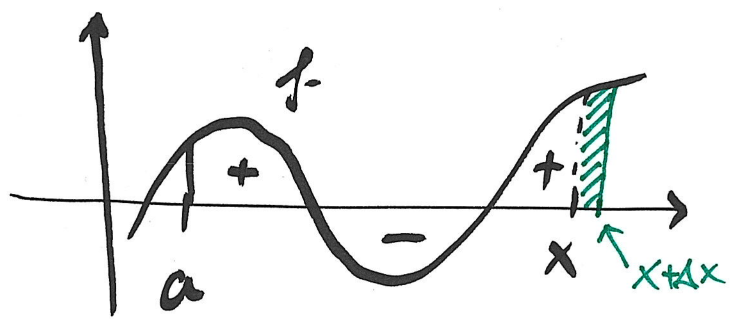
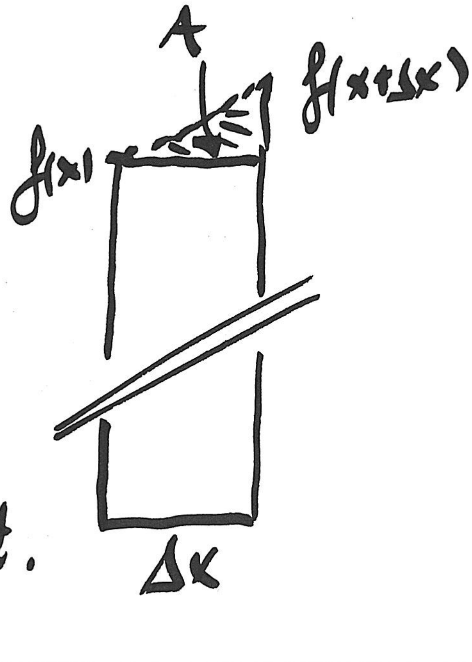
14 april 26 15D Fundamentalteoremet i kalkulus.

Resultat Hvis $f(x)$ er <u>kontinuerlig</u> i $[a,b]$, da er $\int_a^x f(t)\,dt$ en antiderivert til $f(x)$ i $[a,b]$.



$$G(x) = \int_a^x f(t)\,dt$$

$$\begin{aligned}\Delta G(x) &= G(x+\Delta x) - G(x) \\ &= \int_a^{x+\Delta x} f(t)\,dt - \int_a^x f(t)\,dt \\ &= \int_x^{x+\Delta x} f(t)\,dt\end{aligned}$$



$$\begin{aligned}\frac{\Delta G}{\Delta x} &= \frac{\Delta x \cdot f(x)}{\Delta x} + \frac{A}{\Delta x} \\ &= f(x) + \frac{A}{\Delta x}\end{aligned}$$

$\lim\limits_{\Delta x \to 0} \frac{A}{\Delta x} = 0$ siden $f(x)$ er kont.

$$\lim_{\Delta x \to 0} \frac{\Delta G}{\Delta x} = f(x).$$

$$G'(x) = f(x)$$

så $G(x)$ er en antiderivert til $f(x)$.

Resultat. Anta at $f(x)$ er kontinuerlig i $[a,b]$, da er

$$\int_a^b f(t)\,dt = G(b) - G(a) = G(x)\Big|_a^b$$

for en antiderivert $G(x)$ av $f(x)$.

Hvis $G(x)$ er en antiderivert til $f(x)$

så $$G(x) = \int_a^x f(t)\,dt + \underset{\uparrow\,\text{konstant}}{C}$$

antideriverte til f.

$x=a$: $$G(a) = \underbrace{\int_a^a f(t)\,dt}_{0} + C$$ så $C = G(a)$

$$\int_a^x f(t)\,dt = G(x) - G(a)$$

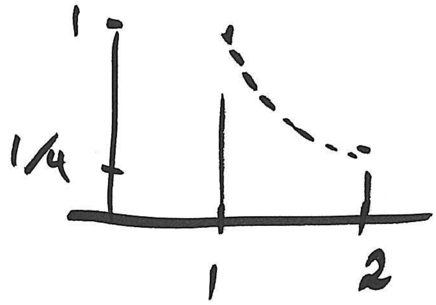
$x=b$ gir resultatet

Hvis $F(x)$ er en annen antiderivert til $f(x)$: $F(x) = G(x) + k$ ← konstant.

$$F(x)\Big|_a^b = F(b) - F(a)$$
$$= (G(b)+k) - (G(a)+k)$$
$$= G(b) - G(a) + k - k$$
$$= G(x)\Big|_a^b .$$

Eksempel

$$\int_1^2 \frac{1}{x^2}\,dx$$



$$\frac{1}{x^2} = x^{-2}$$

$$(x^{-1})' = (-1)x^{-2}$$
$$(-x^{-1})' = x^{-2}.$$

$$\int \frac{1}{x^2}\,dx = \frac{-1}{x} + C$$

Fundamentalteoremet: $\int_1^2 \frac{1}{x^2}\,dx = \frac{-1}{x}\Big|_1^2$

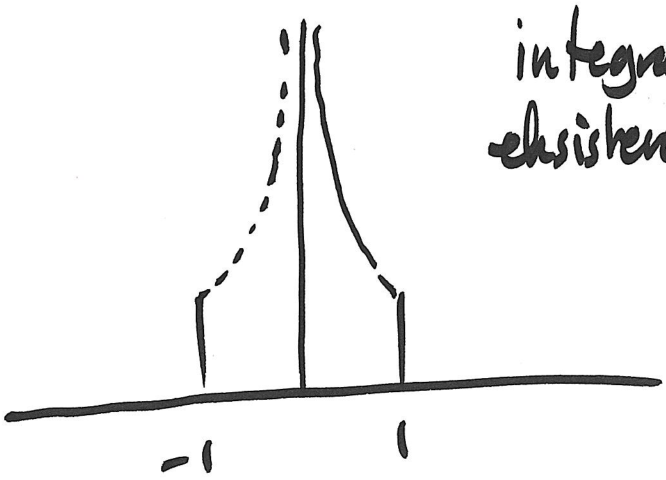
$$= \frac{-1}{2} - \left(\frac{-1}{1}\right) = 1 - \frac{1}{2} = \underline{\underline{\frac{1}{2}}}$$

$$\int_{-1}^{1} \frac{1}{x^2}\,dx = \frac{-1}{x}\Big|_{-1}^{1}$$

$$= \frac{-1}{1} - \left(\frac{-1}{-1}\right)$$

$$= -1 - 1 = \underline{-2}$$

GALT!



integralet eksisterer ikke!

$\Big($ $\int_a^1 \frac{1}{x^2}\,dx$ også la $a \to 0^+$

$\frac{1}{a} - 1 \to \infty$ når $a \to 0^+$ $\Big)$

inneholder et rektangel fra $x=0$ til $x=a>0$ med høyde $1/a^2$.
Arealet til rektangelet er $\frac{1}{a}$ som går mot ∞ når $a \to 0^+$

—

Ubestemte integral

$$\int f(x)\,dx = \text{samlingen av alle antideriverte til } f(x)$$

$$= F(x) + C$$

(på hver komponent til definisjonsmengden til $f(x)$)

$$\int x^r\,dx = \begin{cases} \ln|x| + C & r = -1 \\ \dfrac{x^{r+1}}{r+1} & k \neq -1 \end{cases}$$

$$\int xy^2\,dx = y^2 \int x\,dx = y^2 \cdot \frac{x^2}{2} + C$$

$$\int xy^2\,dy = x \int y^2\,dy = x\frac{y^3}{3} + C$$

Egenskaper: Det ubestemte integralet er "lineært".

$$\int f+g\,dx = \int f\,dx + \int g\,dx$$

$$\int k \cdot f(x)\,dx = k \int f(x)\,dx \quad (+C)$$

konstant

$$\int 3x^5 - \frac{4}{\sqrt[3]{x}}\, dx$$

$$= 3\int x^5 dx - 4\int (x)^{-1/3} dx$$

$$= 3\,\frac{x^6}{6} - 4\,\frac{x^{2/3}}{2/3} + c$$

$$= \underline{\frac{x^6}{2} - 6x^{2/3} + c}$$

FORK1120 - Matematikk forkurs Oslomet
Test
Tirsdag 14. april 2026

Oppgave 1. Finn de bestemte integralene

a) $\displaystyle\int_0^3 2x - 1 \,\mathrm{d}x$

b) $\displaystyle\int_{-1}^2 |x| \,\mathrm{d}x$

c) $\displaystyle\int_1^2 \frac{3-x}{x} \,\mathrm{d}x$

Oppgave 2. Finn de ubestemte integralene

a) $\displaystyle\int \sqrt{z} \,\mathrm{d}z$

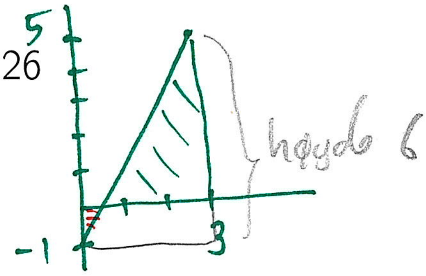
b) $\displaystyle\int 3\sin(x) + 2\cos(x) \,\mathrm{d}x$

c) $\displaystyle\int \frac{1}{|x|} \,\mathrm{d}x$

FORK1120 - Matematikk forkurs Oslomet

Test

Tirsdag 14. april 2026

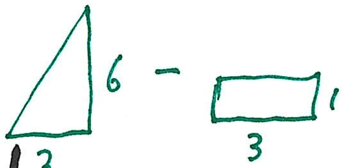


Oppgave 1. Finn de bestemte integralene

a) $\int_0^3 2x - 1\,\mathrm{d}x$

$= x^2 - x \Big|_0^3 = 3^2 - 3 - (0)$

$= 9 - 3 = \underline{6}$



$= \frac{6 \cdot 3}{2} - 3$

$= 9 - 3 = \underline{6}$

$|x| = \begin{cases} x & x \geq 0 \\ -x & x < 0 \end{cases}$

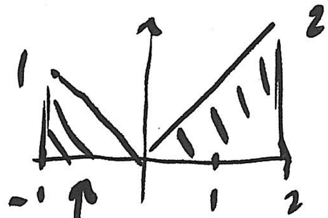
kont.

b) $\int_{-1}^2 |x|\,\mathrm{d}x$ $= \frac{x^2}{2}\Big|^2 - \left(\frac{-x^2}{2}\right)\Big|_{-1}$

en antiderivert $\begin{cases} x^2/2 & x \geq 0 \\ -x^2/2 & x < 0 \end{cases}$

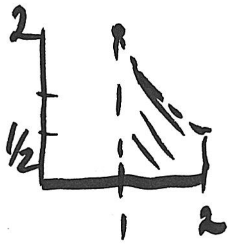
$= \frac{2^2}{2} - -\frac{(-1)^2}{2}$

$= 2 + 1/2 = \underline{\frac{5}{2}}$



$\frac{1 \cdot 1}{2} + \frac{2 \cdot 2}{2} = \underline{2.5}$

c) $\int_1^2 \frac{3 - x}{x}\,\mathrm{d}x$



$= \int_1^2 \frac{3}{x} - \frac{x}{x}\,dx = \int_1^2 3 \cdot x^{-1} - 1\,dx$

$= 3\ln|x| - x\Big|_1^2 = 3(\ln 2 - \ln 1) - (2 - 1)$ $= \underline{3\ln(2) - 1}$ (~ 1.1)

Oppgave 2. Finn de ubestemte integralene

a) $\int \sqrt{z}\,\mathrm{d}z$ $= \int z^{1/2}\,dz$

$= \frac{z^{1/2+1}}{3/2} = \underline{\frac{2}{3} z^{3/2} + C}$ $\left(= \frac{2}{3} z\sqrt{z} + C\right)$

b) $\int 3\sin(x) + 2\cos(x)\,\mathrm{d}x$

$= 3\underbrace{\int \sin x\,dx}_{-\cos x} + 2\underbrace{\int \cos x\,dx}_{\sin x} = \underline{-3\cos x + 2\sin x + C}$

c) $\int \frac{1}{|x|}\,\mathrm{d}x$

$\frac{1}{|x|} = \begin{cases} \frac{1}{x} & x > 0 \\ \frac{-1}{x} & x < 0 \end{cases}$

$\int \frac{1}{|x|}\,dx = \begin{cases} \ln(x) + C_1 & x > 0 \\ -\ln(-x) + C_2 & x < 0 \end{cases}$