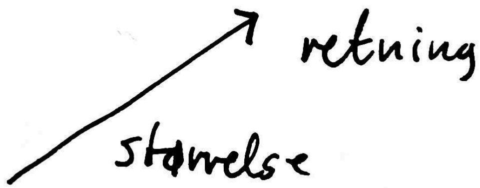
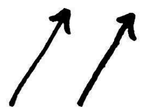
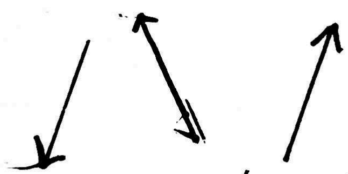
9 feb
26

12 Vektorer

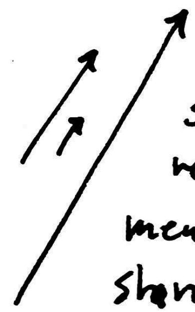




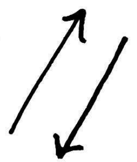
to like vektorer.



samme størrelse,
men ulike retninger



samme
retning,
men ulike
størrelser



<u>motsattvektorer</u>
samme størrelse,
motsatte retninger

$\vec{0}$ nullvektoren

$\cdot$

ingen retning
(alle retninger)

vektorer navngis med bokstaver med pil over

$\vec{u}$ $\vec{v}$ eller ved en tykk bokstav $\mathbb{V}$.

Skalering. $k \in \mathbb{R}$

$k > 0$

$\vec{v}$ skaleres med $k > 0$

samme retning
k ganger lengre.

$\vec{u}$

$3\vec{u}$

$1 \cdot \vec{u} = \vec{u}$

$\frac{1}{2}\vec{u}$

$\vec{w}$

$(-1)\vec{w} = -\vec{w}$ er motsattvektoren til $\vec{w}$.

$k < 0$

Snu retningen og skalere lengden med $|k|$

$(-1)\vec{w} = -\vec{w}$
snur retningen.

$k = 0$

$0 \cdot \vec{v} = \vec{0}$

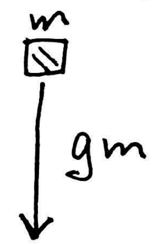
$\vec{z}$

$-3\vec{z}$

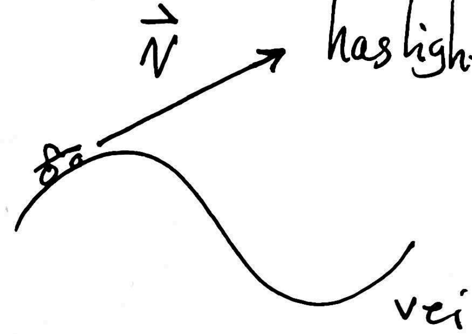
$\left(\frac{1}{3}\right)\vec{z}$

$-\frac{1}{3}\vec{z}$

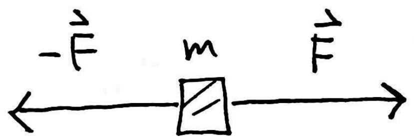
Hvorfor vektorer?



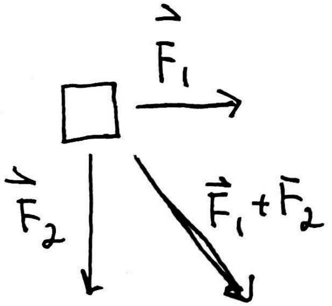
krefter er vektorer



hastighetsvektor)

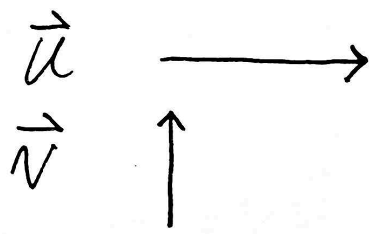


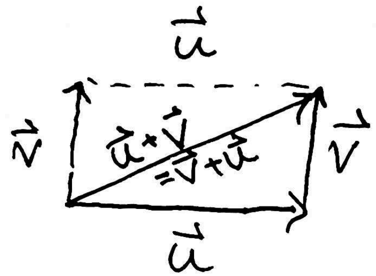
summen av kreftene er $\vec{0}$
(vektorene

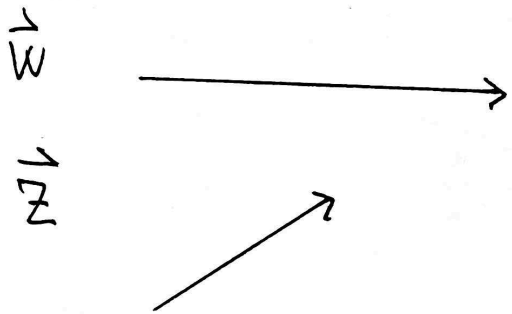


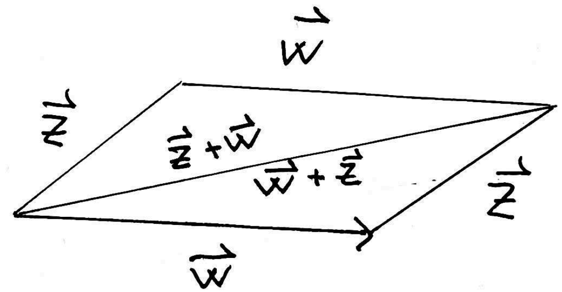
resultant kraften (summen av kreftene F_1 og F_2)

Sum av vektorer









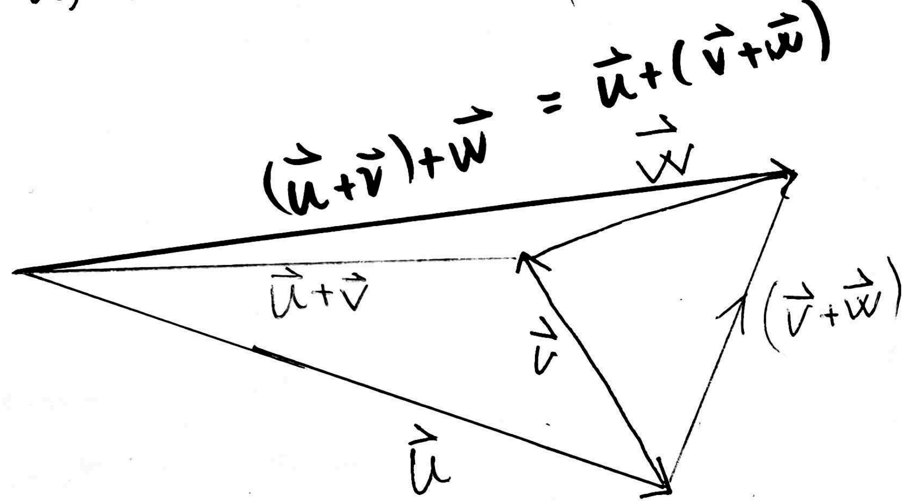
$\vec{u}+\vec{v}$ er vektoren fra starten av $\vec{u}$ til slutten av $\vec{v}$, når $\vec{v}$ etterfølger $\vec{u}$.

$$\vec{u}+\vec{v} = \vec{v}+\vec{u}$$

(diagonal i et parallellogram)

Tre vektore
$\vec{u}, \vec{v}, \vec{w}$

$$(\vec{u}+\vec{v})+\vec{w} = \vec{u}+(\vec{v}+\vec{w})$$



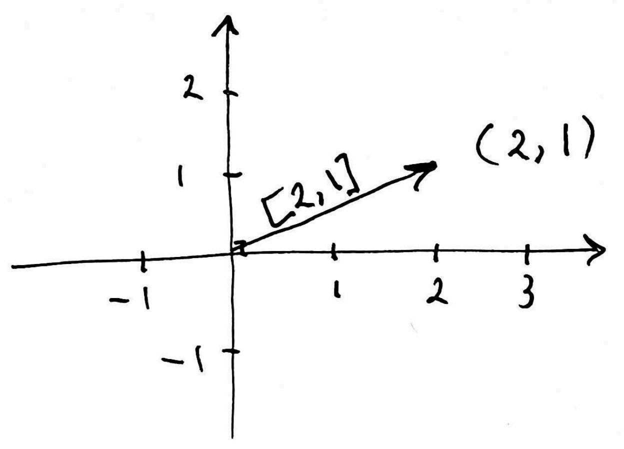
Pause.

$$k_1\vec{v} + k_2\vec{v} = (k_1+k_2)\vec{v}$$

$$2\vec{v} + 3\vec{v} = (2+3)\vec{v} = 5\vec{v}$$

$$2\vec{v} \quad -3\vec{v} \qquad (2-3)\vec{v} = (-1)\vec{v} = -\vec{v}$$

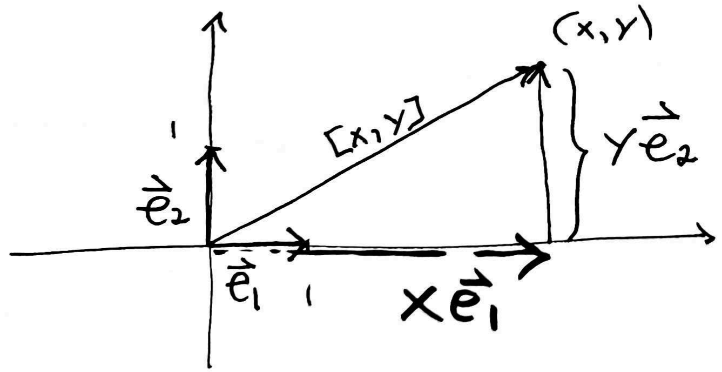
$$k_2(k_1\vec{v}) = (k_2 \cdot k_1)\vec{v}$$



$\vec{v}$

parallellforskyver $\vec{v}$ slik at den starter i origo $(0,0)$. Vektoren vil da ende i et punkt $P(x,y)$.

Motsatt, gitt et punkt P i planet så kan vi lage (tilordne) en vektor, ved å la den starte i origo og slutte i P.



punkt i planet $\longleftrightarrow$ Vektorer i planet

$(x,y) \longleftrightarrow [x,y]$

firkantparenteser

$$[x,y] = x\vec{e}_1 + y\vec{e}_2$$

($\vec{e}_1, \vec{e}_2$ er en basis for vektorene i planet)

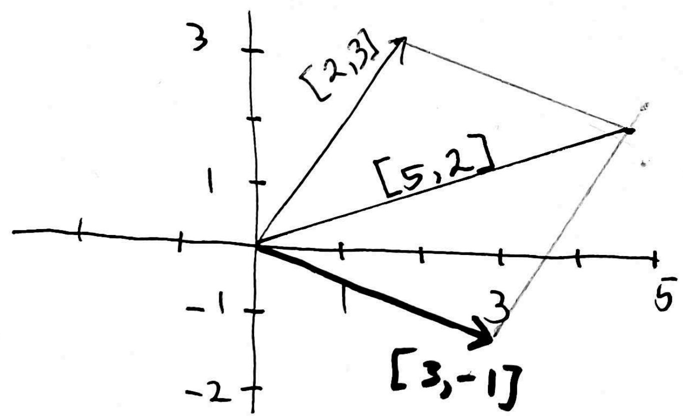
$$k[x,y] = [kx, ky]$$

$$\left(k(x\vec{e}_1 + y\vec{e}_2) = k(x\vec{e}_1) + k(y\vec{e}_2) = (kx)\vec{e}_1 + (ky)\vec{e}_2 = [kx, ky] \right)$$

$$[x_1, y_1] + [x_2, y_2] = [x_1 + x_2, y_1 + y_2]$$

$$(x_1\vec{e}_1 + y_1\vec{e}_2) + (x_2\vec{e}_1 + y_2\vec{e}_2) = (x_1\vec{e}_1 + x_2\vec{e}_1) + (y_1\vec{e}_2 + y_2\vec{e}_2)$$

$$= (x_1+x_2)\vec{e}_1 + (y_1+y_2)\vec{e}_2 = [x_1+x_2, y_1+y_2].$$

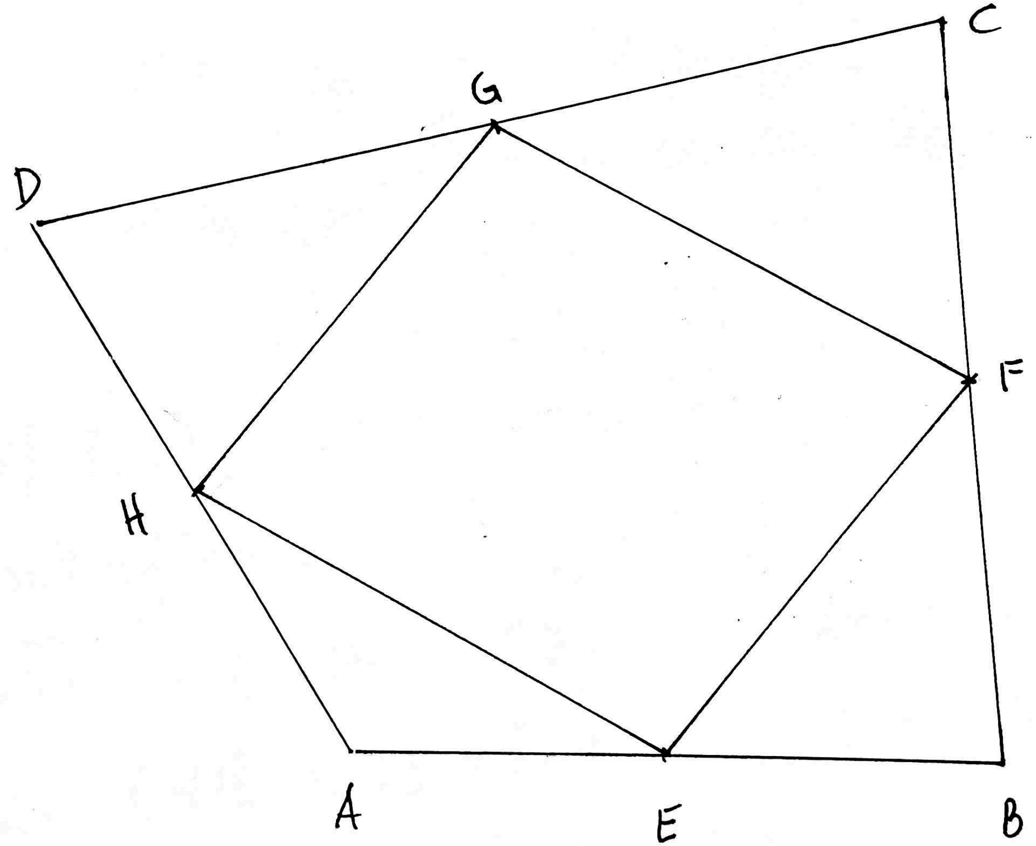


$$[2,3] + [3,-1]$$

$$= [2+3, 3-1] = [5,2]$$

oppg $(-3)[1,-2] = [(-3)\cdot 1, (-3)(-2)] = \underline{[-3, 6]}$

$[1,8] + [0,-5] = [1+0, 8+(-5)] = \underline{[1, 3]}$



Virkårlig firkant
ABCD

<u>Resultat</u>

Firkanten
EFGH
er et parallellogram.

$\vec{AB} + \vec{BC} = \vec{AC}$

$\vec{EF} = \vec{EB} + \vec{BF} = \frac{1}{2}\vec{AB} + \frac{1}{2}\vec{BC} = \frac{1}{2}(\vec{AB} + \vec{BC}) = \frac{1}{2}(\vec{AC})$

Tilsvarende er $\vec{HG} = (\vec{HD} + \vec{DG}) = \frac{1}{2}(\vec{AD} + \vec{DC}) = \frac{1}{2}(\vec{AC})$.

$\vec{EF} = \vec{HG}$ og linjestykkene EF og HG er parallelle og like lange.

Tilsvarende HE og GF.

Oppg $\vec{u} = [-1, 3]$

Finn motsatt vektoren.

Finn $5\vec{u}$, $-8\vec{u}$ og $3\vec{u} + 2\vec{u} - 7\vec{u} + 5\vec{u}$

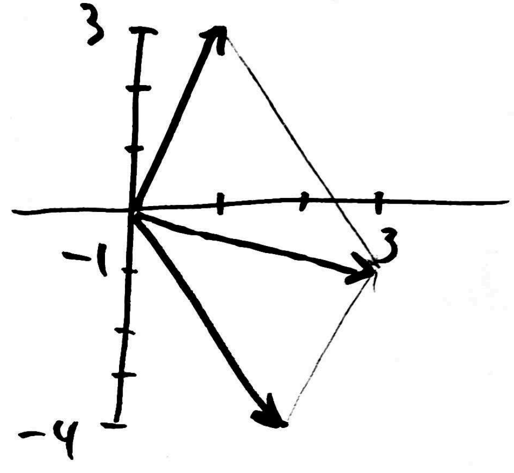
Motsattvektoren: $(-1)[-1, 3] = [(-1)^2, -1 \cdot 3] = \underline{[1, -3]}$

$5\vec{u} = 5[-1, 3] = [5(-1), 5 \cdot 3] = \underline{[-5, 15]}$

$-8\vec{u} = (-8)[-1, 3] = [(-8)(-1), (-8)3] = [8, -24]$

$3\vec{u} + 2\vec{u} - 7\vec{u} + 5\vec{u} = \underbrace{(3+2-7+5)}_{3} \cdot \vec{u} = 3\vec{u} = 3[-1, 3] = \underline{[-3, 9]}$

* $[1,3] + [2,-4] = [1+2,\ 3-4] = \underline{[3,-1]}$



* $[-2,5] + [5,-3] + 2[1,-2]$

$= [-2,5] + [5,-3] + [2,-4]$

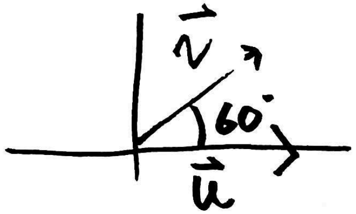
(bytter plass, slik at utregningen blir enklere)

$= \underbrace{[-2,5] + [2,-4]}_{[0,1]} + [5,-3]$

$= \underline{[5,-2]}$

oppg. a) $[1,-7] + 3[1,2] + [2,2]$

b)



$\vec{u}$ lengde 2 i positiv x-retning

$\vec{v}$ lengde 2, men gjør en vinkel 60° med x-aksen.

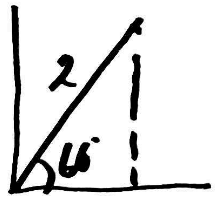
Hva er $\vec{u}+\vec{v}$?

---

a) $\underbrace{[1,-7] + [3,6]}_{[4,-1]} + [2,2]$

$[4,-1] + [2,2] = \underline{[6,1]}$

b)



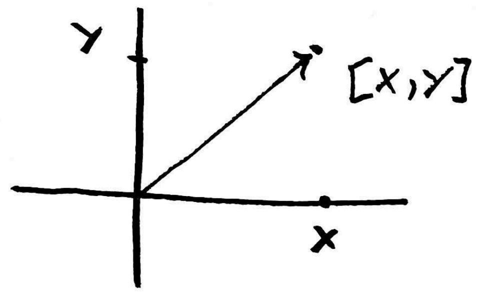
$\vec{v} = [2\overbrace{\cos 60°}^{1/2}, 2\overbrace{\sin 60°}^{\sqrt{3}/2}]$

$= [1, \sqrt{3}]$

$\vec{u}+\vec{v} = [2,0] + [1,\sqrt{3}] = \underline{[3,\sqrt{3}]} = \sqrt{3}[\sqrt{3},1]$

(Dette er vektoren med lengde $\sqrt{3}\cdot 2$ og 30° til x-aksen)

12.9



Lengden er gitt ved Pytagoras

$|[x,y]| = \sqrt{x^2+y^2}$. lengden til $[x,y]$

$|[1,-3]| = \sqrt{1^2+(-3)^2} = \sqrt{10}$.

$|[x,y,z]| = \sqrt{x^2+y^2+z^2}$.

12.9d) $|[3t, e^t, -1]| = \sqrt{(3t)^2+(e^t)^2+(-1)^2}$

$$= \underline{\sqrt{9t^2+e^{2t}+1}}$$

12.10 a) $\vec{u} = [3t, 4t]$

När har $\vec{u}$ lengde 20?

$|\vec{u}| = \sqrt{(3t)^2+(4t)^2} = \sqrt{(9+16)t^2}$

$= \sqrt{25}\sqrt{t^2} = \underline{5|t|}$

$|\vec{u}| = 20$ så $|t| = \frac{20}{5} = 4$

$5|t| = 20$ Løsningene er

$t = \underline{-4, 4}$