4feb
26

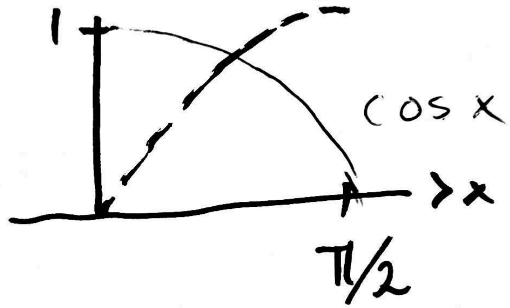
11F

$(\sin x)' = \cos x$

$(\cos x)' = -\sin x$

vinkelenhet er radianer

sin x voksende



cos x avtagende

$$(\tan x)' = \left(\frac{\sin x}{\cos x}\right)'$$

$$= \frac{(\sin x)' \cdot \cos x - \sin x \cdot (\cos x)'}{\cos^2 x}$$

$$= \frac{\cos^2 x + \sin^2 x}{\cos^2 x}$$

$$= \frac{1}{\cos^2 x} \quad \text{(ved Pytagoras)}$$

$$(\tan x)' = 1 + \tan^2 x$$

Harmonisk svingning (sinus funksjon)

Periode $\frac{2\pi}{c}$

$$f(x) = A\sin(cx+\varphi) + d$$

$$f'(x) = A(\sin(cx+\varphi))' = A\cos(cx+\varphi)(cx+\varphi)'$$

$$= Ac\cos(cx+\varphi)$$

(benytter $\cos x = \sin\left(\frac{\pi}{2}+x\right)$)

$$= \underline{Ac\sin\left(cx+\left(\varphi+\frac{\pi}{2}\right)\right)}$$

Den deriverte er også en harmonisk svingning.

$$(\sin x)'' = (\cos x)' = -\sin x$$

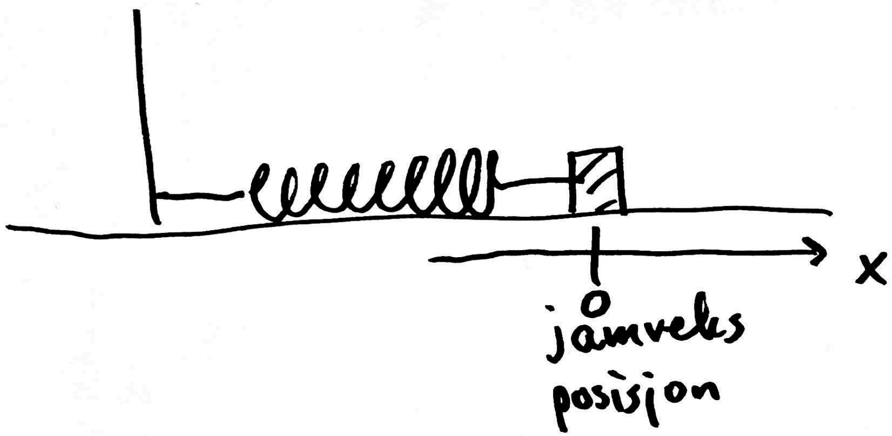
$$(a\sin kx)'' = (ak\cos(kx))' = -ak^2\sin(kx)$$

$$= -k^2(a\sin kx).$$

Tilsvarende for $b\cos kx$.

$f(x) = a\sin kx + b\cos kx$ har egenskapen

$$\underline{f'' + k^2 f = 0}$$



Hookes lov

kraften $F = -kx$

$F = m \cdot x''$

Newtons lov

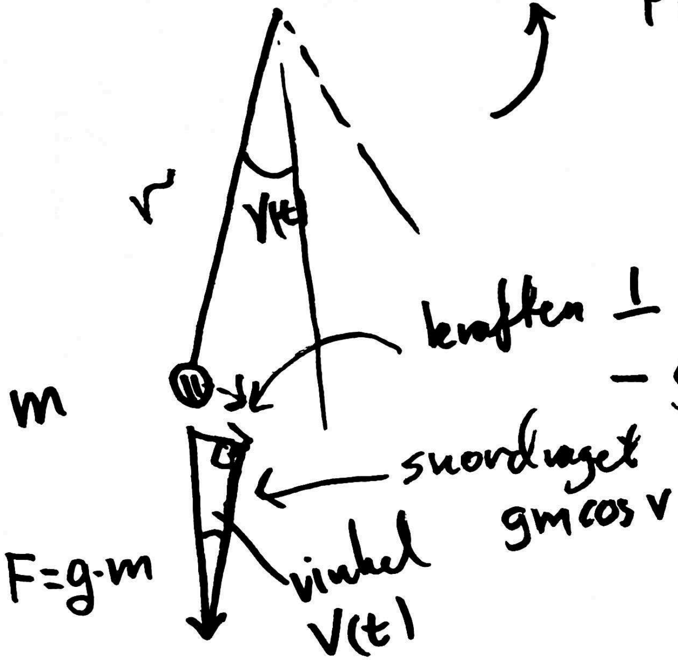
$m \cdot x'' = -k \cdot x$

(Differensial likning) $x'' = -\frac{k}{m} x$

Resultat: Løsningene er på formen $x(t) = a \sin\left(\sqrt{\frac{k}{m}}\, t\right) + b \cos\left(\sqrt{\frac{k}{m}}\, t\right)$

Perioden er $\frac{2\pi}{\sqrt{k/m}} = 2\pi\sqrt{\frac{m}{k}}$.

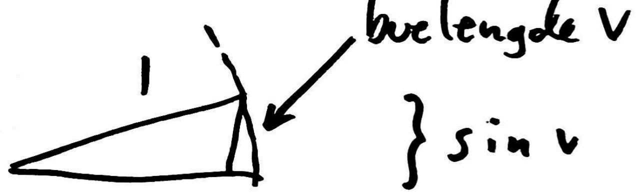
Pendel som svinger fritt.



kraften ⊥ på snoren

$$\underbrace{-gm\sin v}_{\text{kraften} \perp \text{på snoren}} = m \cdot r \cdot \underbrace{v''(t)}_{\text{akselerasjon}}$$

$$\underline{v''(t) = -\frac{g}{r}\sin v}$$



$$\lim_{v \to 0} \frac{\sin v}{v} = 1$$

så $\sin v \sim v$ når v er liten.

$$v'' \cong -\frac{g}{r} v$$

(tilnærmet) harmonisk svingning!

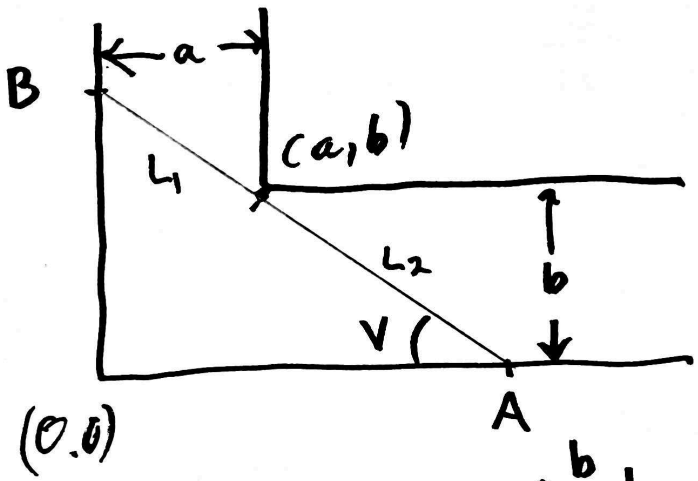
Pendel med små utslag.

$$v(t) = a\sin\left(\sqrt{\frac{g}{r}}\,t\right) + b\cos\left(\sqrt{\frac{g}{r}}\,t\right)$$

(a, b små ...!)

Perioden er $\underline{2\pi\sqrt{\frac{r}{g}}}$ (tilnærming. Perioden avhenger av utslagene.)

2dimensjonalt.



Stokk rundt hjørne.

Hvor lang stokk får vi rundt hjørnet?

$a = b$

$\underline{L = 2\sqrt{2}\,b}$

Resultat: Lengste stokk er

$$\left(a^{2/3} + b^{2/3}\right)^{3/2}$$

Vi viser dette resultatet i de neste 2 sidene.

stemmer for $a = b$ ✓

$a << 1$ veldig liten $\sim b$ ✓

$L = AB$ lengden til stokken

$\tan V = \frac{B}{A}$.

$L_2 \cdot \sin V = b$ så $L_2 = \frac{b}{\sin V}$

$L_1 \cdot \cos V = a$ så $L_1 = \frac{a}{\cos V}$.

$$L = L_1 + L_2 = \frac{a}{\cos V} + \frac{b}{\sin V}.$$

Finner V slik at $L'(V) = 0$.

$$L'(V) = \left(a(\cos V)^{-1} + b(\sin V)^{-1}\right)'$$

$$= -a(\cos V)^{-2}(\cos V)' - b(\sin V)^{-2}(\sin V)'$$

$$= -\frac{a(-\sin V)}{(\cos V)^2} - \frac{b\cos V}{(\sin V)^2}.$$

felles nevner

$$L'(V) = \frac{a\sin^3 V - b\cos^3 V}{\cos^2 V \sin^2 V}$$

$L'(v) = 0$ när $a\sin^3 v = b\cos^3 v$

$$\tan^3 v = \frac{\sin^3 v}{\cos^3 v} = \frac{b}{a}$$

$$\underline{\tan v = \sqrt[3]{\frac{b}{a}}}$$

$$\frac{1}{\cos^2 v} = 1 + \tan^2 v = 1 + \left(\frac{b}{a}\right)^{2/3}.$$

så

$$\frac{1}{\cos v} = \left(1 + \left(\frac{b}{a}\right)^{2/3}\right)^{1/2}$$

$$L = \frac{a}{\cos v} + \frac{b}{\sin v}$$

$$= \frac{1}{\cos v}\left(a + \frac{b}{\sin v/\cos v}\right)$$

$$L = \left(1 + \left(\frac{b}{a}\right)^{2/3}\right)^{1/2}\underbrace{\left(a + \frac{b}{(b/a)^{1/3}}\right)}_{\left(a + \frac{b}{b^{1/3}}a^{1/3}\right)}$$

$$\left(\left(\frac{a}{a}\right)^{2/3} + \left(\frac{b}{a}\right)^{2/3}\right)^{1/2}\left(a + \frac{b}{b^{1/3}}a^{1/3}\right)$$

$$\left(\frac{a^{2/3} + b^{2/3}}{a^{2/3}}\right)^{1/2}\underbrace{\left(a + b^{2/3}a^{1/3}\right)}_{a^{1/3}\left(a^{2/3} + b^{2/3}\right)} = \frac{\left(a^{2/3} + b^{2/3}\right)^{1/2}}{a^{1/3}}a^{1/3}\left(a^{2/3} + b^{2/3}\right)$$

$$\underline{= \left(a^{2/3} + b^{2/3}\right)^{3/2}}$$