26.01.26

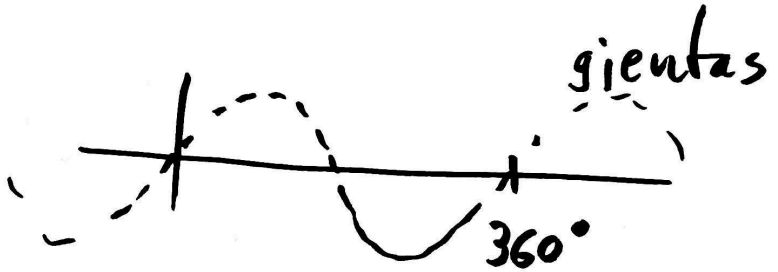
11 Trigonometrinės lygtys

A $\sin x = \frac{1}{2}$ $\quad x \in [-90°, 500°\rangle$

B $2\sin(3x-1) = \cos(3x-1)$

C $\sin x \geq \frac{1}{2}$ $\quad x \in [0, 180°]$

D $f(x) = \sin x$



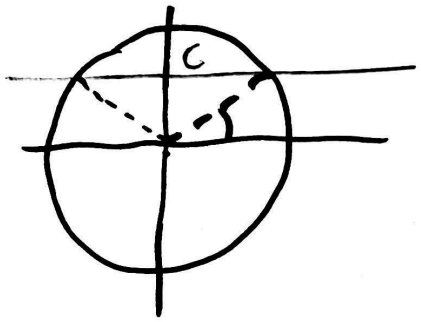
E $(\sin x)' = \cos x$

$(\cos x)' = -\sin x$

$\lim\limits_{x \to 0} \frac{\sin x}{x} = 1$

11A Trigonometriske grunnlikninger

$\sin(x) = c$



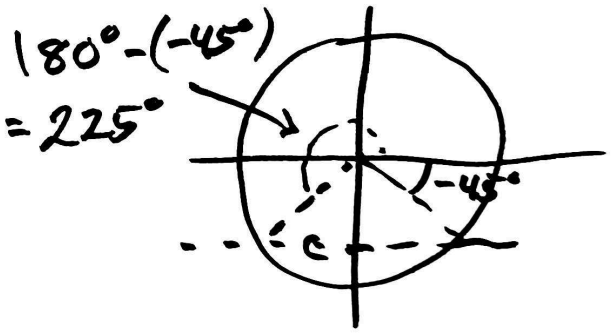
Løsningene er

$x = \arcsin(c) + 360° \cdot k$

$x = 180° - \arcsin(c) + 360° \cdot k$

eks: $\sin x = -\sqrt{2}/2 = -1/\sqrt{2}$

$\arcsin x = -45°$



Løsningene:

$x = -45° + 360° \cdot k$ $\quad k \in \mathbb{Z}$

$x = 225° + 360° \cdot k$

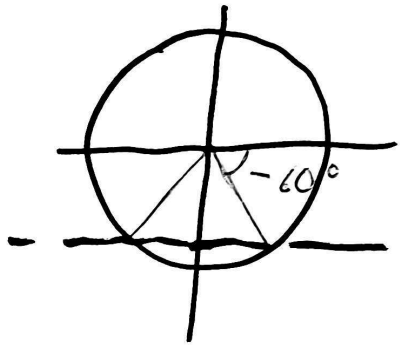
(alternativt: $-45° + 360° \cdot k$

$= 315° + 360°(k-1)$)

$$2\sin x = -\sqrt{3} \qquad x \in [0, 360°\rangle$$

$$\Updownarrow$$

$$\sin x = -\sqrt{3}/2 \sim -0.866$$



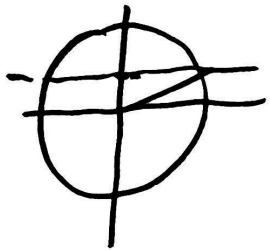
$$x = \arcsin(-\sqrt{3}/2) = -60° + 360°\cdot k$$

og $$x = 180° - \arcsin(-\sqrt{3}/2) + 360°\,k$$

$$= 240° + 360°\cdot k$$

Løsningene i $[0, 360°\rangle$ er $x = 240°, 300°$

$\sin x = 0.4$



$\arcsin(0.4) = 23.58°$

$180° - \quad \text{-''-} \quad = 156.42°$

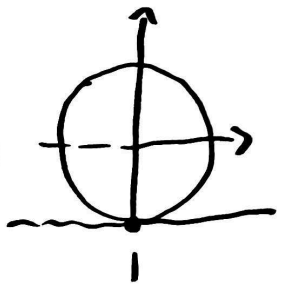
$x \cong 23.58° + 360° \cdot k$

og $\quad x \cong 156.42° + 360° k$.

Løs: a) $\sin x = -1$

opg b) $\sin x = 2$

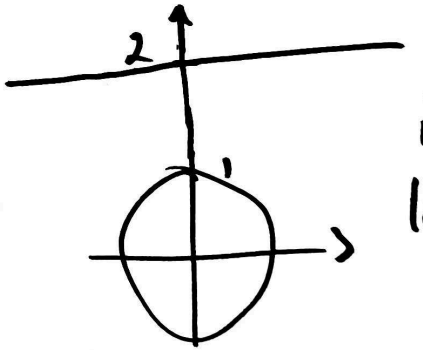
a)



$\arcsin -1 = -90°$.

$x = -90° + 360° k$

b)

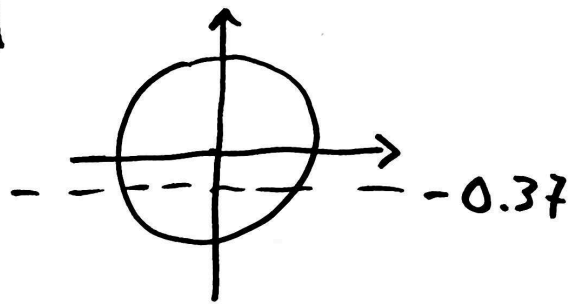


ingen løsning (i $\mathbb{R}$)

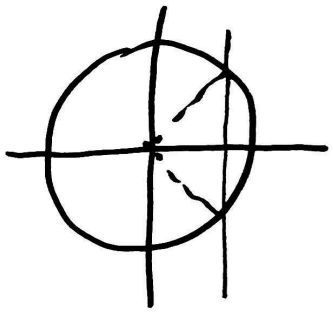
$\sin x = -0.37$ $\quad x \in [0, 180°]$

$\sin x \geq 0$ för $x \in [0°, 180°]$,

så $\sin x = -0.37$ ingen lösning i $[0, 180°]$.



$\cos x = \sqrt{3}/2$



$x = \arccos(\sqrt{3}/2) + 360° \cdot k$

$= 30° + 360° \cdot k$

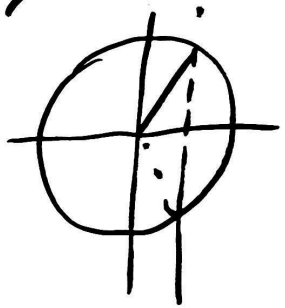
Samt $\quad x = -30° + 360°k = 330° + 360°(k-1)$

Generelt: Løsningene til $\cos x = c$

er $x = \arccos(c) + 360° \cdot k$

og $x = -\arccos(c) + 360° k$.

eks $\cos x = 1/2$ $\quad x \in [0, 720°\rangle$

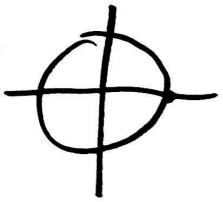


$x = \pm \arccos(1/2) + 360° k$

$x = \pm 60° + 360° k$

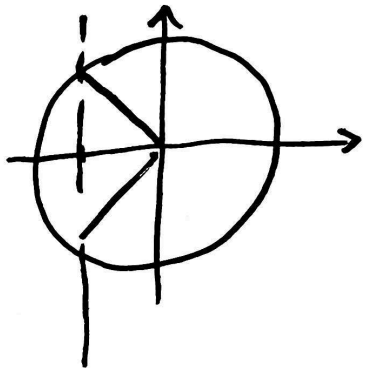
Løsningsmengden: $\{ 60°, 420°, 300°, 660° \}$

eks $\cos x = 1$ $\quad x = 0° + 360° \cdot k$



I intervallet $[0, 360°]$ er løsningene 0° og 360°

oppg $2\cos x = -\sqrt{2}$ $\quad x \in [0, 540°\rangle$



$$\cos x = \frac{-\sqrt{2}}{2} = \frac{-1}{\sqrt{2}} \sim -0.707$$

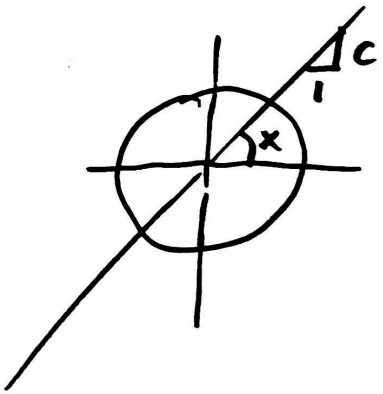
$\arccos(-\sqrt{2}/2) = 135°$.

$x = 135° + 360° \cdot k$

$x = -135° + 360° \cdot k$

Løsningene er $\underline{\{135°, 225°, 495°\}}$

$\tan x = c \quad (c \in \mathbb{R})$

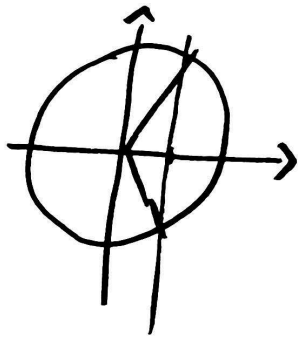


$x = \arctan(c) + 180°k$

eks: $\tan x = 1$, $x = \arctan(1) + 180°k$

$= 45° + 180°k$

$$\cos(\underbrace{3x}_{V}) = 1/2 \qquad x \in [0, 360°]$$



I $\quad \cos(V) = 1/2$

II $\quad 3x = V, \quad x = V/3$

I $\quad V = \pm \arccos(1/2) + 360°k$

$$V = \pm 60° + 360°k$$

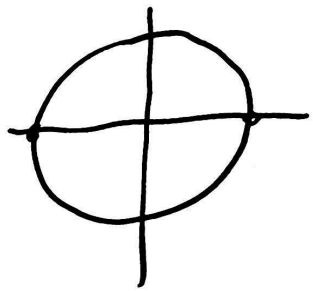
II $\quad x = V/3 = \pm 20° + 120°k$

I $[0, 360°]$:

$$20°, \ 140°, \ 260°$$
$$100°, \ 220°, \ 340°$$

Løsningene er $\quad x \in \{20°, 100°, 140°, 220°, 260°, 340°\}$

$\sin x \cdot \cos x = 0$ $\quad x \in [0, 360°]$



$\Leftrightarrow \quad \sin x = 0$ eller $\cos x = 0$

$\sin x = 0 \quad x \in \{0°, 180°, 360°\}$

$\cos x = 0 \quad x \in \{90°, 270°\}$

Løsningene til $\sin x \cos x = 0$ er $\{0°, 90°, 180°, 270°, 360°\}$

Alternativt, hvor vi benytter dobling av vinkel formel

$$\sin(2x) = 2\sin x \cos x$$

$\sin x \cos x = 0 \quad \Leftrightarrow \quad \sin(2x) = 0$

$2x = 0 + 360° \cdot k$ eller $2x = 180° + 360° k$

$\Leftrightarrow \quad 2x = 0 + 180° \cdot n$

$\Leftrightarrow \quad x = 0 + \frac{180°}{2} \cdot n = 0 + 90° \cdot n$ som gir

$x \in \{0°, 90°, 180°, 270°, 360°\}$.

$x \in [0, 360°\rangle$

$$2\sin x \cos x = \sin x$$

$$2\sin x \cos x - \sin x = 0$$

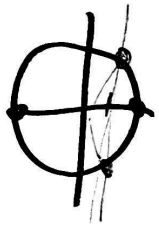
$$\sin x (2\cos x - 1) = 0$$

$\Leftrightarrow \sin x = 0$ eller $2\cos x - 1 = 0$

$\Updownarrow$

$\cos x = 1/2$.

$\sin x = 0$



$x \in \{0, 180°\}$

$\cos x = 1/2$

$x \in \{60°, 300°\}$

Løsningene er $\underline{\{0°, 60°, 180°, 300°\}}$

Alternativt:

$$2\sin x \cos x = \sin(2x)$$

Så

$$2\sin x \cos x = \sin x$$

$$\Leftrightarrow \quad \sin(2x) = \sin(x)$$