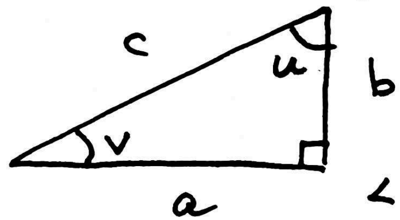
19.01
25

10D Sinussetningen

$u = \angle A$



$v = \angle B$

$\angle C = 90^\circ$

Rettvinklet trekant $(\sin \angle C = 1)$

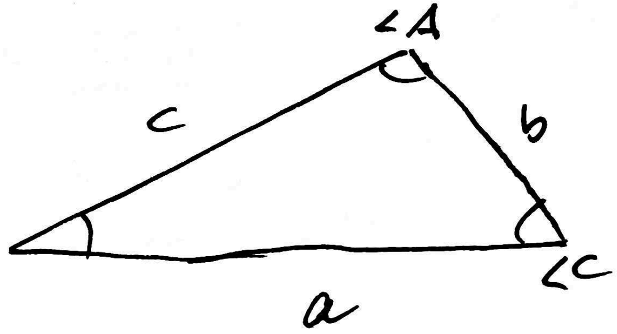
$\sin(\angle A) = \frac{a}{c}$ $(\cos \angle B)$

$\sin(\angle B) = \frac{b}{c}$ $(= \cos \angle A)$

$c = \frac{a}{\sin \angle A} = \frac{b}{\sin \angle B}$.

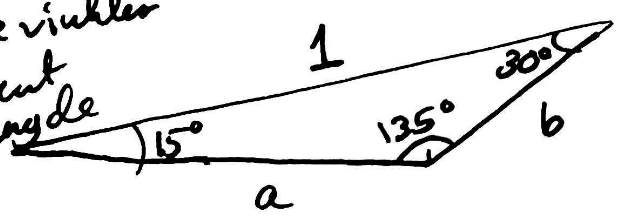
Sinussetningen

Generell trekant



$$\frac{a}{\sin \angle A} = \frac{b}{\sin \angle B} = \frac{c}{\sin \angle C}$$

Eksempel
Kjente vinkler
én kjent sidelengde



$$\frac{1}{\sin(135^\circ)} = \frac{a}{\sin(30^\circ)} = \frac{b}{\sin(15^\circ)}$$

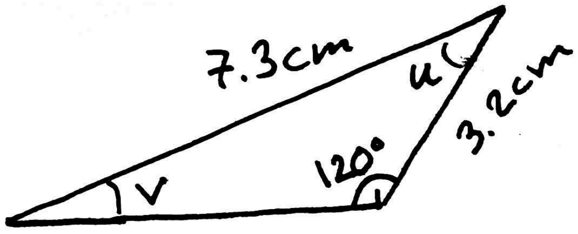
$$\frac{1}{1/\sqrt{2}} = \frac{a}{1/2}$$ så $a = \frac{1}{2} \cdot \frac{1}{1/\sqrt{2}} = \frac{1}{\sqrt{2}}$

$a \sim \underline{0.707}$

$\sqrt{2} = \frac{b}{\sin(15^\circ)}$ så $b = \sqrt{2}\sin(15^\circ) \sim \underline{0.366}$

Eksempel

To kjente sidelengder og én motstående vinkel til en av dem

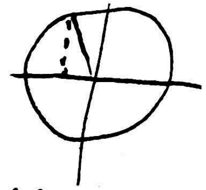


$$\frac{7.3}{\sin 120^\circ} = \frac{3.2}{\sin V} = \frac{a}{\sin u}$$

$u = 180° - 120° - V.$

$$\sin V = 3.2 \cdot \frac{\sin 120^\circ}{7.3} \approx 0.379627\ldots$$

$\sin(120°) = \sin(180° - 120°) = \sin(60°) \approx 0.866$



$\underline{V = 22.3106\ldots}^\circ \approx \underline{22.31^\circ}$

(og $180° - 22.3106°\ldots$ ikke mulig siden sum vinklene $= 180°$)

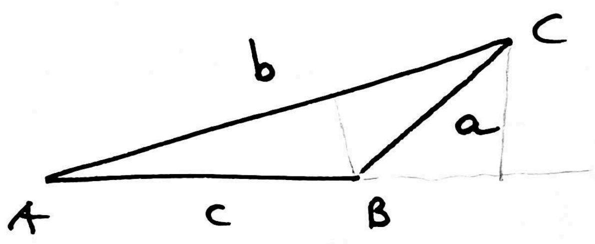
$u = 180° - 120° - 22.31° = 180° - 142.31°$

$\underline{u = 37.69°}$

$$\underline{a} = \sin u \cdot \frac{7.3}{\sin(120^\circ)} \underline{= 5.15}$$

(stemmer med figuren

Arealsetningen gir oss sinussetningen



$$A = \frac{1}{2} bc \sin(\angle A) = \frac{1}{2} ab \sin(\angle C) = \frac{1}{2} ac \sin(\angle B)$$

$$\frac{2A}{abc} = \frac{bc\sin(\angle A)}{abc} = \frac{ab\sin(\angle C)}{abc} = \frac{ac\sin(\angle B)}{abc}$$

så $$\frac{2A}{abc} = \frac{\sin(\angle A)}{a} = \frac{\sin(\angle C)}{c} = \frac{\sin(\angle B)}{b}$$

inversverdiene: $$\frac{abc}{2a} = \frac{a}{\sin(\angle A)} = \frac{c}{\sin(\angle C)} = \frac{b}{\sin(\angle B)}$$

Trekant: $\angle A = 30^\circ$ $\quad a = 4$

$c = 10$

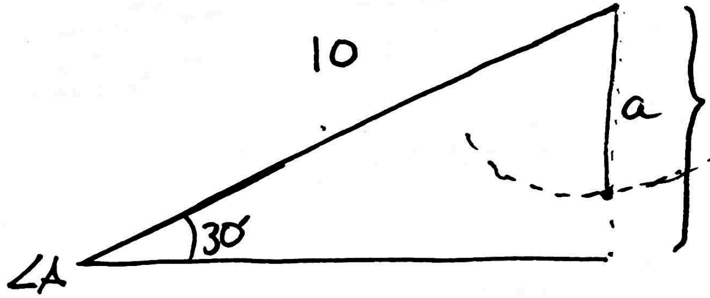
I

Ikke mulig å finne en slik trekant.

$$\frac{a}{\sin\angle A} = \frac{4}{\sin 30^\circ} = \frac{4}{1/2} = 8 = \frac{c}{\sin(\angle C)} = \frac{10}{\sin\angle C}$$

så $\sin(\angle C) = \frac{10}{8} = \frac{5}{4} = 1.25$

ikke mulig (siden $|\sin v| \leq 1$).



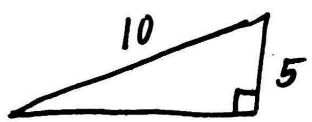
$5 = 10\sin 30^\circ$

Vi rekker ikke ned!

II $a = 5$ $\angle A = 30^\circ$, $c = 10$

$$\frac{a}{\sin\angle A} = \frac{5}{1/2} = 10 = \frac{c}{\sin\angle C} = \frac{10}{\sin\angle C}$$

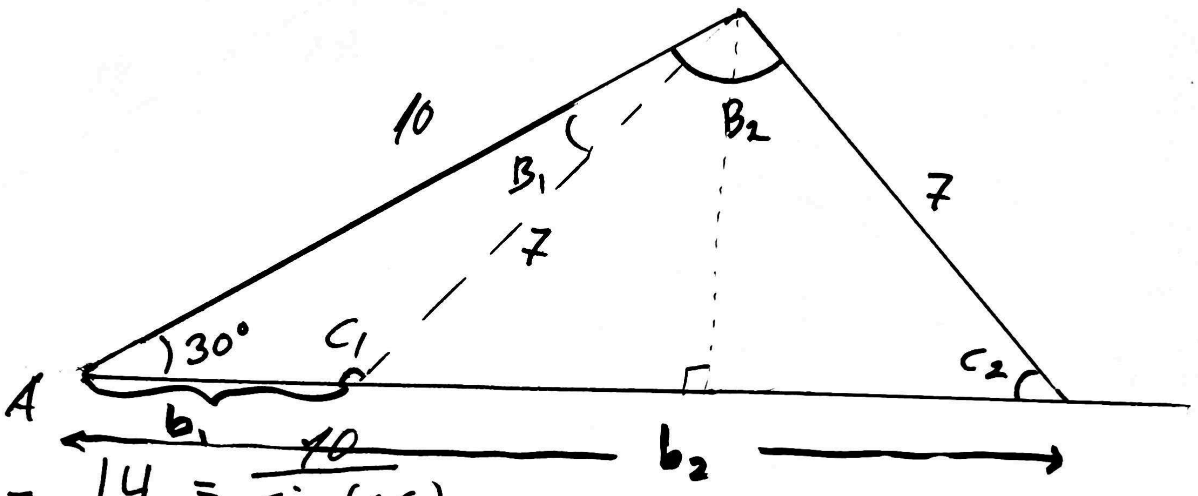
så $\sin\angle C = 1$ og $\underline{\angle C = 90^\circ}$



III $\angle A = 30°$ $a = 7$

$c = 10$

To mulige løsninger :



$$\frac{a}{\sin\angle A} = \frac{7}{\sin 30°} = \frac{7}{1/2} = 14 = \frac{10}{\sin(\angle C)}$$

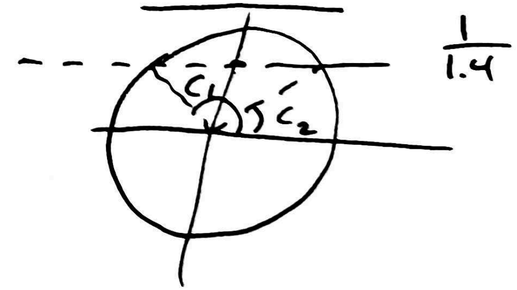
$$\sin(\angle C) = \frac{10}{14} = \frac{1}{1.4}$$

$\angle C_2 \cong 45.58°$ $= \arcsin\left(\frac{1}{1.4}\right)$

men det er én løsning til :

$\angle C_1 = 180° - \angle C_2$

$= \underline{134.42°}$



$$\angle B_1 = \underbrace{180° - \angle C_1}_{\angle C_2} - 30° \cong 15.58°$$

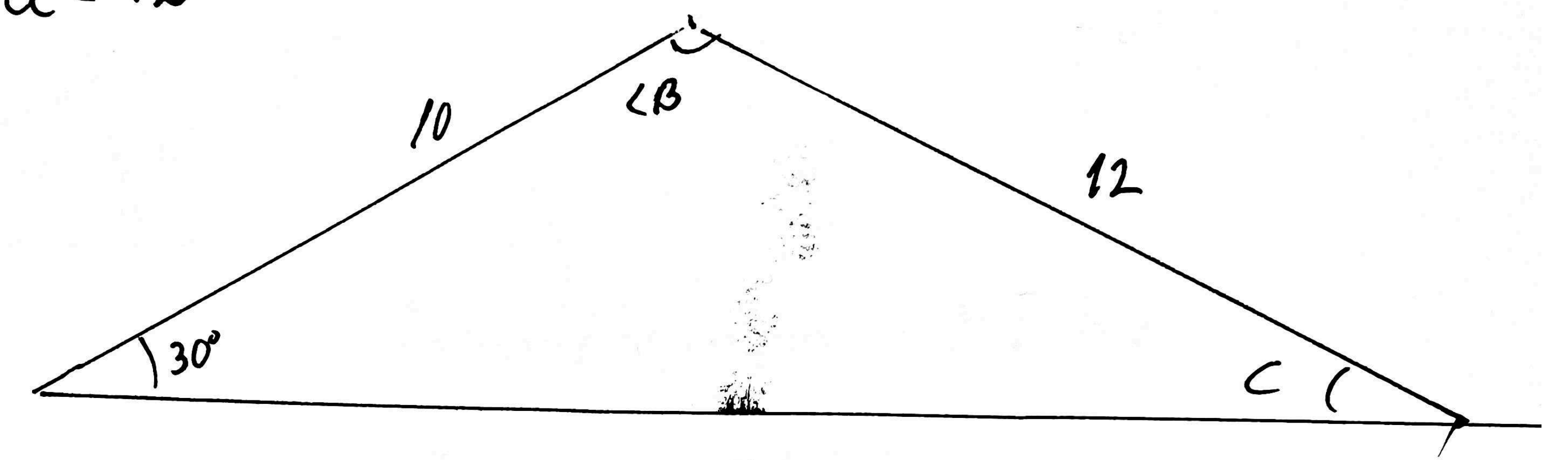
$b_1 = \sin(\angle B_1) \cdot 14 \sim \underline{3.76}$

$$\angle B_2 = \underbrace{180° - \angle C_2}_{\angle C_1} - 30° \simeq 104.42°$$

$b_2 = \sin(\angle B_2) \cdot 14 \sim \underline{13.56}$

IV $\angle A = 30^\circ$ $a = 12$

$c = 10$



Sinussetningen: $\dfrac{a}{\sin(\angle A)} = \dfrac{12}{\sin(30^\circ)} = \dfrac{12}{1/2} = 24 = \dfrac{10}{\sin C}$

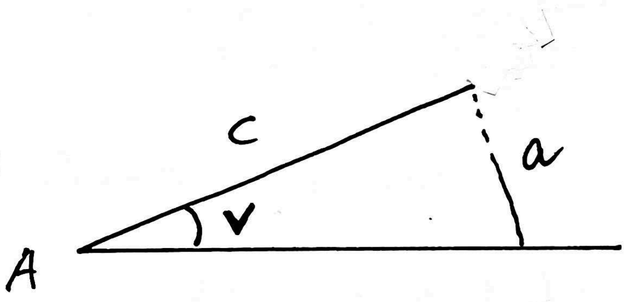
$\sin C = \dfrac{10}{24} = \dfrac{1}{2.4}$ $\angle C = \arcsin\left(\dfrac{1}{2.4}\right) = 24.624^\circ$

(andre løsning $180^\circ - 24.624^\circ$.)
ikke gyldig

$\angle B = 180^\circ - 30 - 24.624^\circ \cong 150^\circ - 24.624^\circ = \underline{125.376^\circ}$

$\dfrac{b}{\sin\angle B} = 24$ så $b = 24\sin(125.376^\circ) \sim \underline{19.57}$

($V = \angle A$)



1 $a < c \cdot \sin V$: ingen løsning (ingen mulig Δ)

2 $a = c \sin V$: én rettvingla Δ, $\angle C = 90°$

3 $c \sin V < a < c$: to mulige Δ

4 $a \geq c$: én løsning. ($\angle C < 90°$)

tilfeller

Øving

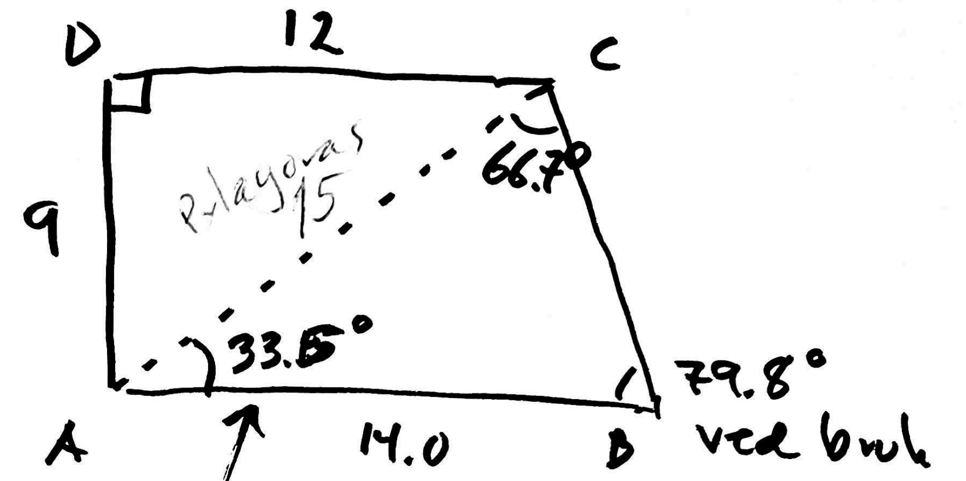
oppg

10.67 c) Areal □ABCD

$= \text{areal } \triangle ACD + \triangle ABC$

$= \dfrac{9 \cdot 12}{2} + \underbrace{\dfrac{1}{2} 15 \cdot 14.0 \cdot \sin(33.5^\circ)}_{\text{arealsetningen}}$

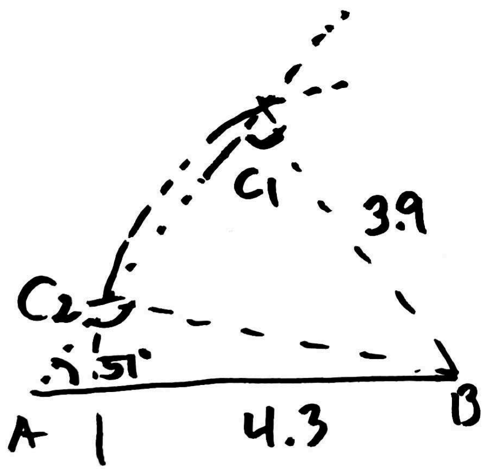
$= \underline{112}$



summen av vinklene i Δ er 180°

ved bruk av sinussetning

10.69



$\dfrac{\sin A}{a} = \dfrac{\sin 51^\circ}{3.9} = \dfrac{\sin C}{4.3}$

$\sin C_1 = \dfrac{4.3}{3.9} \sin 51^\circ = 0.85685\ldots$

$C_1 = \arcsin(0.85685\ldots) \approx 58^\circ$

$C_2 = 180^\circ - C_1 = 122^\circ$