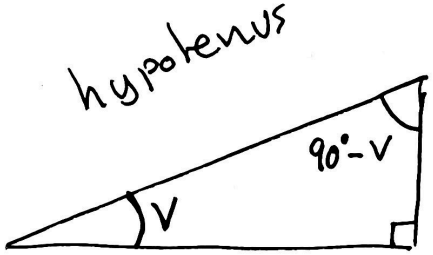
12 januar 2026

10 Trigonometri

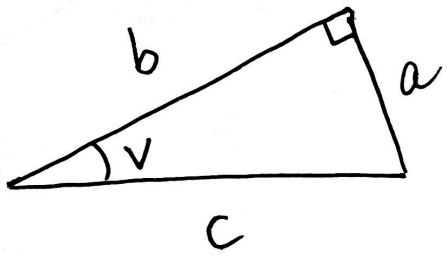
10A Sinus, cosinus og tangens funksjonene



$$\sin(V) = \frac{\text{motstående katet}}{\text{hypotenus}} \quad (= \cos(90°-V))$$

$$\cos(V) = \frac{\text{hosliggende katet}}{\text{hypotenus}} \quad (= \sin(90°-V))$$

$$\tan(V) = \frac{\text{motstående katet}}{\text{hosliggende katet}} = \frac{\sin(V)}{\cos(V)}$$



$$\sin(v) = \frac{a}{c}$$

$$\cos(v) = \frac{b}{c}$$

$$\tan(v) = \frac{a}{b} = \frac{a/c}{b/c} = \frac{\sin(v)}{\cos(v)}.$$

0° ⟶

$\sin(0^\circ) = 0$

$\cos(0^\circ) = 1$

$\tan(0^\circ) = 0$

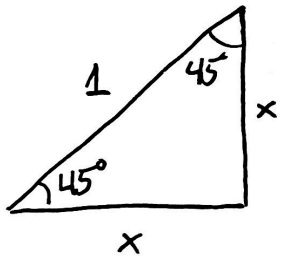
90° |

$\sin(90^\circ) = 1$

$\cos(90^\circ) = 0$

$\tan(90^\circ)$ ikke definert

45°



$\sin(45^\circ) =$

$\cos 45^\circ = \underline{0.7071\ldots}$

$\tan(45^\circ) = 1$

vi finner eksakte verdier:

Pytagoras

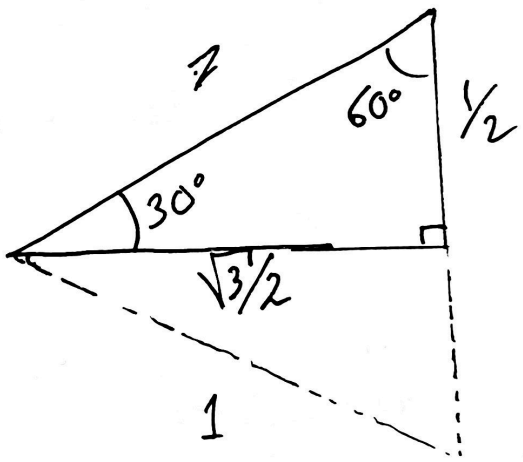
$$x^2 + x^2 = 1^2 = 1$$

$$2x^2 = 1$$

$$x^2 = \frac{1}{2} \qquad (x > 0)$$

$$x = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$\sim 0.707$$



$$\left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 = \frac{1}{4} + \frac{3}{4} = 1$$

Pythagoras

$$\sin(30°) = \cos(60°) = \frac{1}{2} = \underline{0.5}$$

$$\sin(60°) = \cos(30°) = \frac{\sqrt{3}}{2} = \underline{0.866}$$

$$\tan(30°) = 1/\sqrt{3}$$

$$\tan(60°) = \sqrt{3}.$$

$\sin(61°) = 0.8746\ldots$

(forventer en verdi litt større enn 0.866)

$\sin(40°) \approx 0.643$, $\cos(40°) \approx 0.766$

$\sin(30°) = 0.5$

$\sin(45°) = \frac{1}{\sqrt{2}} \sim 0.707$

($45° - 40° = 5°$, $40° - 30° = 10°$) estimat for $\sin(40°)$:

$$\frac{2 \cdot \sin 45° + 1 \cdot \sin 30°}{3}$$

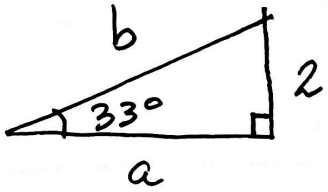
$$= \frac{2/\sqrt{2} + 1/2}{3} = \frac{\sqrt{2} + 1/2}{3} \sim 0.638$$)

Forskjellige vinkelenheter i bruk.

grader	deg	1 omløp	360°
		rett vinkel	90°
radianer	rad	1 omløp	2π
		rett vinkel	$\pi/2$
gradianer (gon)	grad	1 omløp	400
		rett vinkel	100

Pass på hva kalkulatoren er innstilt på (hvilke enheter for vinkler den bruker)

(Test med sin 90 ...)

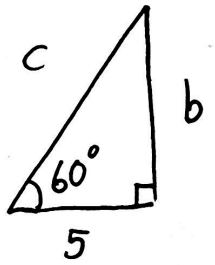


$$\sin(33^\circ) = \frac{2}{b}$$

$$\text{så} \quad b = \frac{2}{\sin(33^\circ)} \approx \underline{3.672}$$

$$\tan(33^\circ) = \frac{2}{a}$$

$$\text{så} \quad a = \frac{2}{\tan(33^\circ)} \sim \underline{3.0797}$$

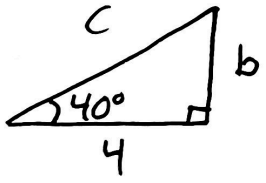


Finn b og c eksakt.

$$\cos 60^\circ = \frac{5}{c} \quad \text{så} \quad c = \frac{5}{\cos(60^\circ)} = \frac{5}{1/2} = \underline{10}.$$

$$\tan(60^\circ) = \frac{b}{5} \quad \text{så} \quad b = 5\underbrace{\tan(60^\circ)}_{\sqrt{3}} = \underline{5\sqrt{3}}$$

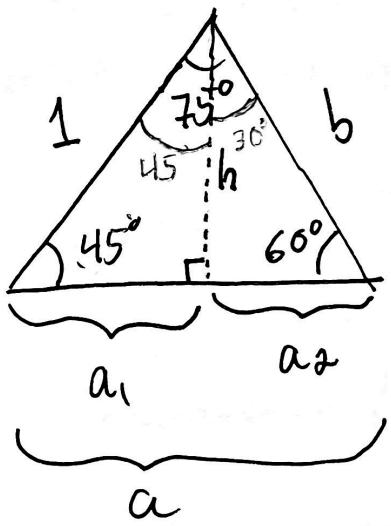
oppg



Finn b og c.

$$\cos(40^\circ) = \frac{4}{c} \quad \text{så} \quad c = \frac{4}{\cos(40^\circ)} \sim \underline{5.2216}$$

$$\tan(40^\circ) = \frac{b}{4} \quad \text{så} \quad b = 4\tan(40) \sim \underline{3.3564}$$



Finn lengdene a og b.

$h = 1 \cdot \sin 45° = \frac{1}{\sqrt{2}}$

$a_1 = 1 \cdot \cos(45°) = \frac{1}{\sqrt{2}}$.

$\sin(60°) = \frac{h}{b}$ så $b = \frac{h}{\sin 60°} = \frac{1/\sqrt{2}}{\sqrt{3}/2} \cdot \frac{2}{2}$

$= \frac{\sqrt{2}}{\sqrt{3}} = \underline{\sqrt{\frac{2}{3}}} \approx 0.8165$

$\cos(60°) = \frac{a_2}{b}$

$a_2 = b \cdot \cos(60°) = \sqrt{\frac{2}{3}} \cdot \frac{1}{2} = \frac{\sqrt{2}}{2} \cdot \frac{1}{\sqrt{3}} = \sqrt{\frac{1}{6}}$

$a = a_1 + a_2 = \underline{\frac{1}{\sqrt{2}} + \sqrt{\frac{1}{6}}} \sim 1.1153$

Inverse trigonometriske funksjoner

$\sin^{-1}(x) = \arcsin(x)$

"arcus"

$\sin(\sin^{-1}(x)) = x$

tilsvarende for $\cos^{-1}$ og $\tan^{-1}$.

$\sin^{-1}(0.3) \sim 17.46^\circ$

$\sin^{-1}(0.5) \sim 30^\circ$

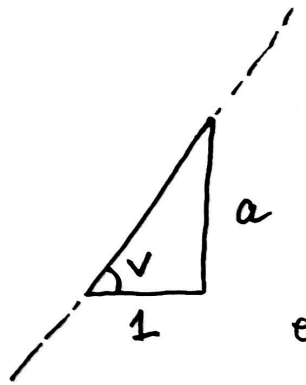
$\sin^{-1}(0.9) \sim 64.16^\circ$

$\sin^{-1}(0.99) \sim 81.89^\circ$

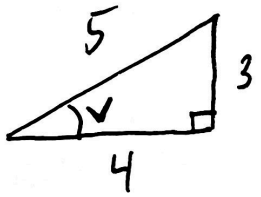
$\tan^{-1}(1) = 45^\circ$

$\tan^{-1}(2) \sim 63.43^\circ$

$\tan^{-1}(10) = 84.29^\circ$



stigningstall a svarer til en stigning med vinkel $v = \tan^{-1}(a)$



$$v = \cos^{-1}\left(\frac{4}{5}\right) \sim 36.869^\circ$$

$$= \sin^{-1}\left(\frac{3}{5}\right) \quad \text{—''—}$$

$$= \tan^{-1}\left(\frac{3}{4}\right) \quad \text{—''—}$$

Integrasjon

$\int 2x\,dx = x^2 + C$ ubestemt integral

$\int f(x)\,dx$ = alle funksjoner som deriverte lik $f(x)$

$= F(x) + C$ hvor $F'(x) = f(x)$

integral-tegn — integranden — differensial — antiderivert til $f(x)$

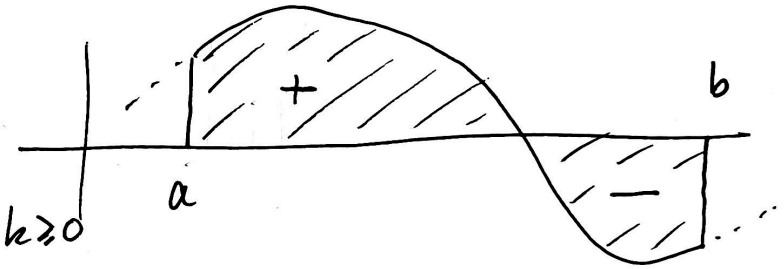
$\int 3x^2 + 4e^x\,dx = x^3 + 4e^x + C$

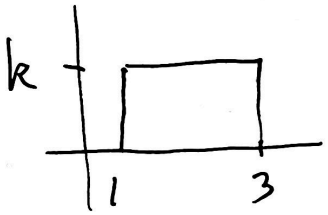
$\int \frac{1}{x}\,dx = \ln|x| + C$

Bestemte integral

$\int_a^b f(x)\,dx$ = arealet med fortegn mellom grafen til $f(x)$ og x-aksen fra $x=a$ til $x=b$

integralet av $f(x)$ fra a til b





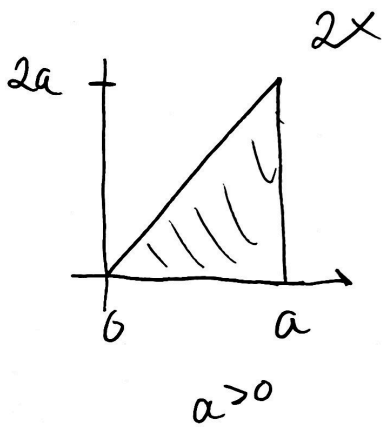
$$\int_1^3 k\,dx = \begin{cases} 2k & k \geq 0 \\ -2k & k < 0 \end{cases}$$

(resultet)

Fundamentalteoremet i kalkulus

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

hvor $F'(x) = f(x)$.



arealet er $\frac{2a \cdot a}{2} = a^2$.

$$\int_0^a 2x\,dx = a^2.$$

$$\int_0^1 x^2\,dx = \frac{x^3}{3}\Big|_0^1 = \frac{1}{3} - \frac{0}{3} = \underline{\underline{\frac{1}{3}}}$$