22.10
25

7A Logaritmer

$1000 = 10^3$

$\sqrt{10} = 10^{1/2}$

$\frac{1}{10} = 10^{-1}$

$10 = 10^1$

$\sqrt[5]{10^3} = (10^3)^{1/5} = 10^{3/5}$

$\frac{1}{100\sqrt{10}} = (10^2 \cdot 10^{1/2})^{-1} = 10^{-5/2}$.

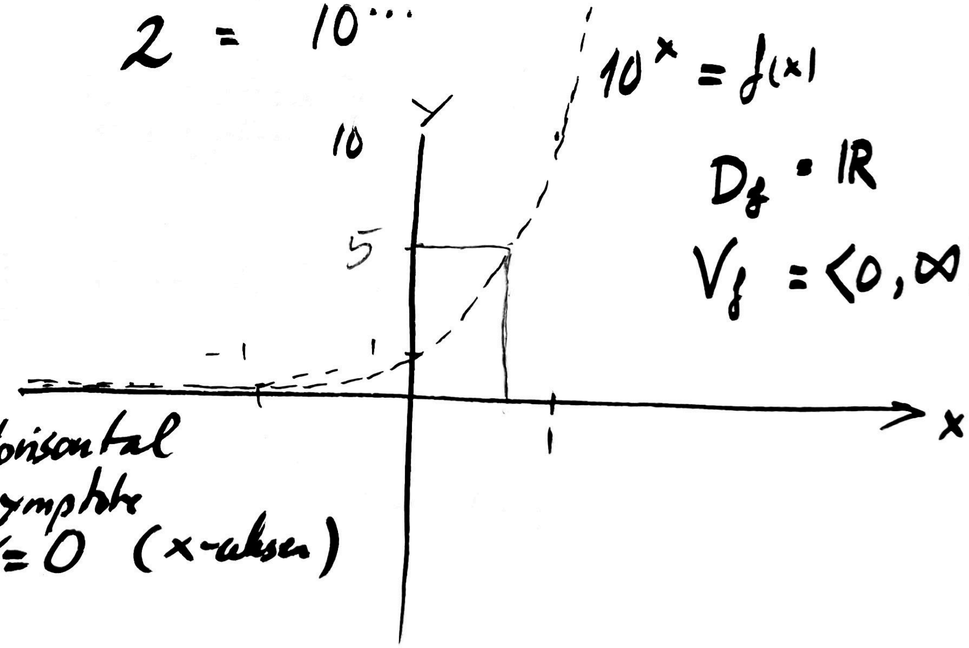
$1\,000\,000 = 10^6$

$1 = 10^0$

$5 = 10^{\cdots}$

$\pi = 10^{\cdots}$

$2 = 10^{\cdots}$



$10^x = f(x)$

$D_f = \mathbb{R}$

$V_f = \langle 0, \infty \rangle$

Horisontal asymptote
$y = 0$ (x-aksen)

(10-er) logaritmen $\log$ (eller $\lg$)

er en funksjon med def. mengde $\langle 0, \infty \rangle$

$10^{\log(y)} = y$ | log er bare definert for positive tall.

$\log(1000) = 3$ $\log(10) = 1$ $\log(1000000) = 6$

$\log(\sqrt{10}) = 1/2$ $\log(\sqrt[5]{10^3}) = 3/5$ $\log(1) = 0$

$\log(\frac{1}{10}) = -1$ $\log\left(\frac{1}{100\sqrt{10}}\right) = -5/2$

$5 = 10^{\log 5}$ $\log 5 \sim 0.69897\ldots$

$5 \sim 10^{0.69897}$

$\pi = 10^{\log \pi}$ $\log \pi \sim 0.49714987\ldots$

$\sim 10^{0.497}$

oppg. $\log(200) \sim 2.301\ldots$

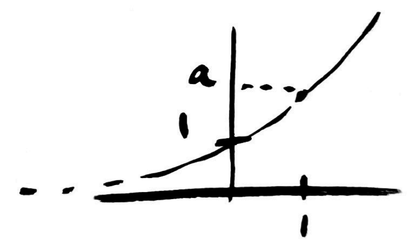
$\log(1/40) \sim -1.602$

$\log(0)$, $\log(-3)$ gir feilmelding.

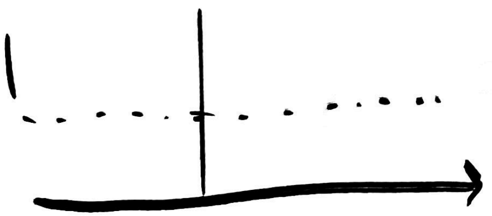
10-er logaritme log kalles Briggsk logaritme

Andre grunntall enn 10

$y = a^x$ $a > 1$



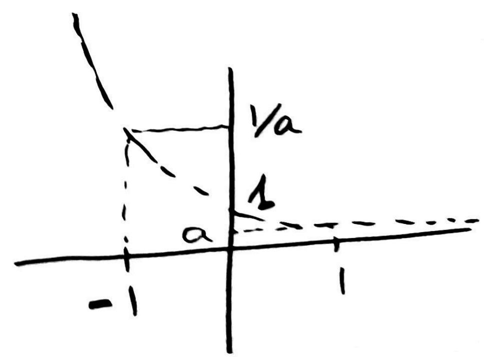
$a = 1$



$0 < a < 1$

$a^{-1} > 1$

$a^x = \left(\frac{1}{a}\right)^{-x}$



a-logaritmen $\log_a$ er definert ved

$$a^{\log_a y} = y \qquad 0 < a \text{ og } a \neq 1$$

2-logaritme.

$$\log_2 8 = \log_2(2^3) = 3$$

$$\log_2(256) = \log_2(2^8) = 8 \qquad \log_2(1) = 0$$

$$\log_2\left(\tfrac{1}{4}\right) = -2.$$

Naturlig logaritme er e-logaritme, hvor $e \approx 2{,}718\ldots$ er Euler tallet.

$\log_e$ skrives ofte som $\ln$ (uttales ℓ-n)

(ofte benyttes log for naturlig logaritme … i programmeringsspråk)

$\ln(1) = 0 \qquad \ln(e) = 1 \qquad \ln(e^3) = 3$

$\ln(10) \sim 2.302585\ldots, \quad e^{2,30} \cong 10$

$\ln(100) = \ln(10 \cdot 10) \sim 2.30 + 2.30 \sim \underline{4.60}$

Geogebra

	$\log_a(x)$	skrives	log(a, x)
Naturlig log	ln	—"—	ln
Briggsk log	log	—"—	log.

Python

må bruke en mattepakke — import numpy as np

	ln(x)	skrives som	np.log(x)
Briggsk		—— " ——	np.log10(x)
	2-log		np.log2(x)

np.e Euler tallet $e^x = \exp(x)$

10-log: $\log 5 + \log 2 = 1$ Hvorfor ?

$$10^{\log 5 + \log 2} = 10^{\log 5} \cdot 10^{\log 2}$$

$$= 5 \cdot 2 = 10 = 10^1$$

Derfor må $\underline{\log 5 + \log 2} = 1$.

$0.69897\ldots + 0.3010299\ldots$

$a > 0$, $\neq 1$

$a^{\log_a y} = y$ $\quad y > 0$

$\log_a(a^x) = x$ $\quad x \in \mathbb{R}$

$\log_a(1) = 0$

$\log_a(a) = 1$

oppg.

$$e^{-\ln 3} = e^{(-1)\cdot(\ln 3)} = (e^{\ln 3})^{-1} = 3^{-1} = \underline{1/3}$$

$$10^{3\cdot\log 5} = (10^{\log 5})^3 = 5^3 = \underbrace{5\cdot 5}_{25}\cdot 5 = \underline{125}$$

$a > 0$, $a \neq 1$

$$a = e^{\ln a}$$

$$\underline{a^x} = (e^{\ln a})^x = \underline{e^{(\ln a)\cdot x}}$$

$$0 < y = a^{\log_a y} = (e^{\ln a})^{\log_a y} = e^{\ln a \cdot \log_a y}$$

$$y = e^{\ln y}$$

Derfor må

$$\ln y = \ln a \cdot \log_a y$$

så $\boxed{\log_a(y) = \frac{\ln(y)}{\ln(a)}}$

Alle logaritmer er på forma $k \cdot \ln(y)$

$$\log_a(x) = k \ln(x)$$

$x = a$
gir
$\log_a(a) = 1 = k \cdot \ln(a)$
så $k = 1/\ln(a)$

$$\operatorname{Log}(x) = \frac{\ln(x)}{\ln(10)} \approx \frac{\ln(x)}{2.30}$$

$$\log_a(x) = \frac{\ln(x)}{\ln(a)}$$

$$\log_{1/a}(x) = \frac{\ln(x)}{\ln(1/a)} = \frac{\ln x}{-\ln(a)}$$

$$\log_{1/a}(x) = -\log_a(x)$$

r kontinuerlig rente
årlig rente $r_{p.a.}$

$$e^r = 1 + r_{p.a}$$
så $r = \ln(1 + r_{p.a})$

$r_{p.a}$		r
$r_{p.a} = 10\% = 0.1$	:	$\ln(1{,}1) \sim 0.0953\ldots \approx 9.53\%$
$r_{p.a} = 100\% = 1$	:	$\ln(1+1) = \ln 2 \sim 0.693 \approx 69.3\%$

7.4 3^x og 0.5^x som e^{kx}

$$\boxed{a = e^{\ln a}}$$

$$a^x = \left(e^{\ln a}\right)^x$$
$$\underline{a^x = e^{(\ln a)\cdot x}}$$

$$5^x = e^{\ln 5 \cdot x}$$

$$3^x = e^{(\ln 3)\cdot x} \sim \underline{e}^{1.0986 \cdot x}$$

$$\left(\tfrac{1}{2}\right)^x = e^{\ln(1/2)\, x} \sim e^{-0.6931\, x}.$$

7.5

$P_0 = 250\,000$ kr

$r_{p.a} = 25\% = 0.25$

a) $P(t) = (1 + r_{p.a})^t \cdot P_0$

b) $P(t) = e^{\ln(1+r_{p.a})\cdot t}\, P_0$

$= e^{r\cdot t} \cdot P_0$

$e^r = 1 + r_{p.a}$

$r = \ln(1 + r_{p.a})$

$r = \ln(1 + r_{p.a})$ kontinuerlig rente.

$= 0.22.31\ldots$

$\sim 22.3\%$

7.7 d) $e^{\ln 10} = 10$

7.8 d) $\ln e^3 - e^{\ln 3}$

$3 - 3 = \underline{\underline{0}}$

(fra definisjon av ln)

$a^{\log_a y} = y$

$\log_a(a^x) = x$

7.12 b) $\log(\ln e) = \log(1) = 0$

c) $e^{\ln 2 + \ln 3} = \underbrace{e^{\ln 2}}_{2} \cdot \underbrace{e^{\ln 3}}_{3} = \underline{6}$

c)* $e^{\ln 4 - \ln 5} = e^{\ln 4} \cdot e^{-\ln 5}$

$= 4 \cdot (e^{\ln 5})^{-1} = 4 \cdot 5^{-1} = \underline{4/5}$

$$\left(\begin{aligned} e^{\ln(s)+\ln(t)} &= s \cdot t \qquad\qquad s, t > 0 \\ e^{\ln(s)-\ln(t)} &= s/t \end{aligned} \right)$$

Oppg. 1) $e^{2\ln 3} = (e^{\ln 3})^2 = (3)^2 = \underline{9}$

2) $(\sqrt{2})^{\log_2 5} = (2^{1/2})^{\log_2 5}$

$= 2^{\frac{1}{2} \cdot \log_2(5)} = (2^{\log_2 5})^{1/2}$

$= (5)^{1/2} = \underline{\sqrt{5}}$

3) $100^{\log 7} = (10^2)^{\log 7} = 10^{2 \cdot \log 7}$

$= (10^{\log 7})^2 = (7)^2 = \underline{49}$

4) $\ln\left(\frac{1}{e^{0.37}}\right) = \ln\left((e^{0.37})^{-1}\right) = \ln(e^{-0.37}) = \underline{-0.37}$