24 sep
25

4H Ulikheter

Eksempler

$$\left.\begin{array}{l} 2x - 3 > 4 \\ x^2 > 2x \end{array}\right\}$$ polynom ulikheter

$\frac{x+1}{x-1} > 2$ rasjonal ulikhet

$x < 3x+1$ og $3x+1 \leq 5x+2$

kombineres til $x < 3x+1 \leq 5x+2$
dobbel ulikhet

$a < b$ a er mindre enn b $\iff b > a$ b er større enn a.

$a \leq b$ a er mindre enn eller lik b $\iff b \geq a$ b er større enn eller lik a.

$\left(\begin{array}{l} a \leqq b \\ a \leqq b \end{array}\right)$

$$a > b \iff a + c > b + c$$

$$\iff a - b > 0 \iff 0 > b - a$$

$c > 0$

$$a > b \iff ac > bc$$

$$a - b > 0 \qquad \underbrace{ac - bc}_{(a-b)\cdot c} > 0$$

$c < 0$

$$a > b \iff ac < bc$$

ulikheten snues.

$$a - b > 0 \iff \underbrace{(a-b)c}_{ac - bc} < 0$$

$3 > 2$ ganger med 4 : $12 > 8$

-4 : $-12 < -8$

snur fortegnet.

$a > b$ ganger med -1 $\quad -a < -b$

$a - b > 0 \qquad -b > -a$

Lineære ulikheter $\qquad 2x + 1 > 4$

Løsningen til ulikheten er mengden av alle x som gjør ulikhet (påstanden) sann.

$2x + 1 > 4 \iff 2x > 4 - 1 = 3$ $\quad$ gange med $\frac{1}{2}$ på begge sider av ulikheten (deler med 2..)

$\iff x > \frac{3}{2}$

Løsningene er alle x s.a. $x > 3/2$

Alternativt: Løsningsmengden er $\underline{\langle 3/2, \infty \rangle}$

$$-3x+4 \leq -8+x$$

$$-3x-x \leq -8-4$$

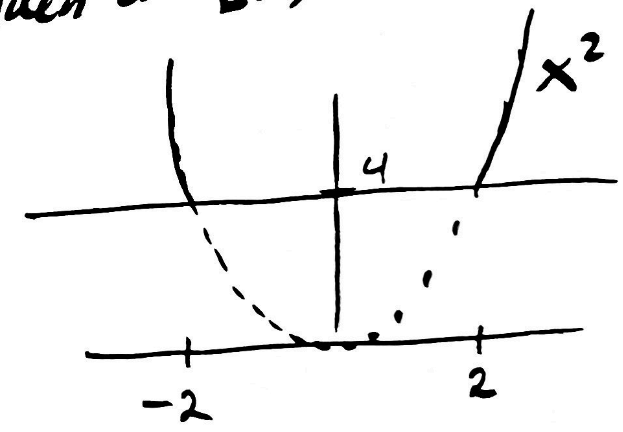
$$-4x \leq -12$$

gange med $\frac{-1}{4}$ på begge sider av ulikhetstegnet.

$$x \geq \frac{-12}{-4} = 3$$

$$\underline{x \geq 3}$$

alternativt Løsningsmengden er $[3, \infty\rangle$



Polynomulikhet:

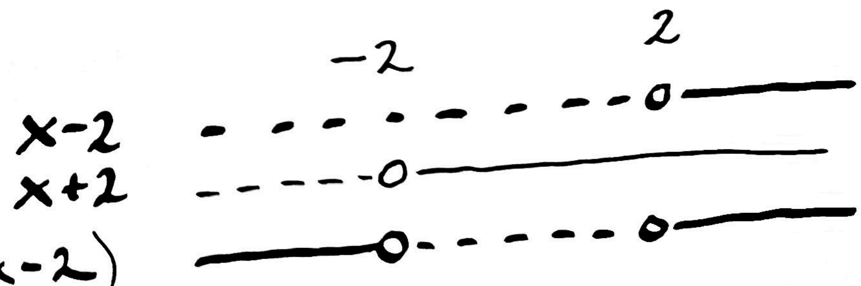
$$x^2 - \underbrace{4}_{2^2} > 0$$

$$x^2 > 4$$

faktoriserer

$$(x+2)(x-2) > 0$$

Fortegnsskjema

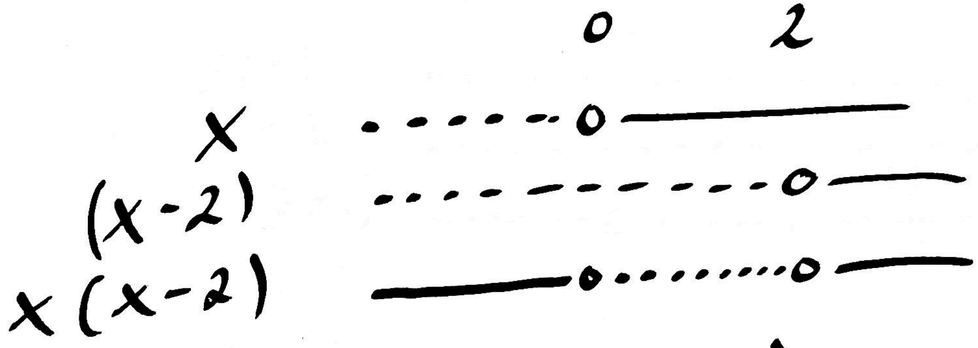


Løsningsmengden er $\langle -\infty, -2\rangle \cup \langle 2, \infty\rangle$

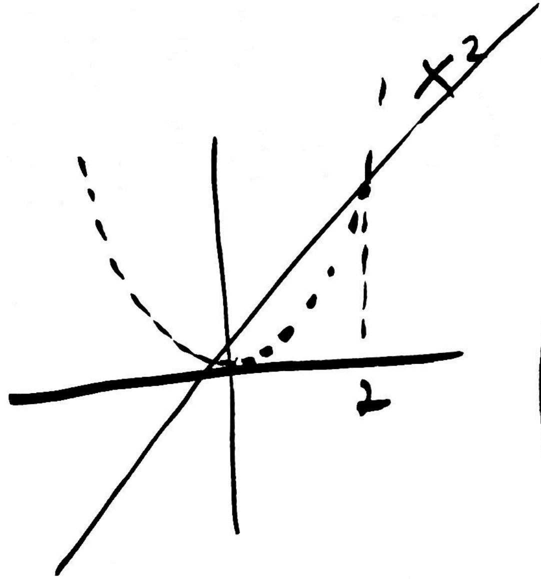
Oppg. Løs ulikheten $x^2 > 2x$

$$x^2 - 2x > 0$$

$$x(x-2) > 0$$



Løsningsmengden er $\langle -\infty, 0\rangle \cup \langle 2, \infty\rangle$



$x^2 > 2x$ deler med x

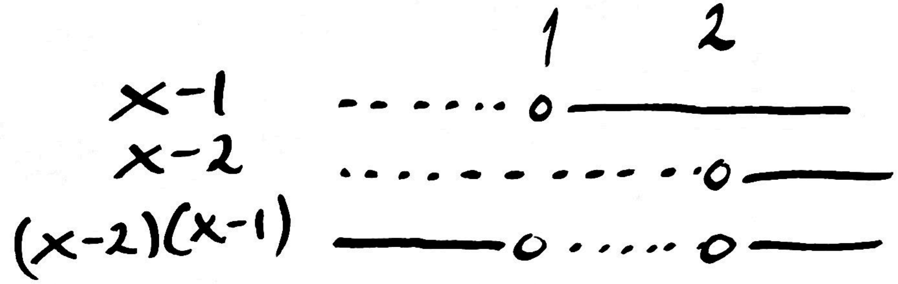
når $x>0$ $\quad x > 2$ $\quad \langle 2, \infty\rangle$

når $x<0$ $\quad x < 2$ $\quad \langle -\infty, 0\rangle$

(når $x=0$: $0>0$ galt)

* $x^2 + x + 2 > 4x \iff x^2 - 3x + 2 > 0$

$(x-2)(x-1) > 0$

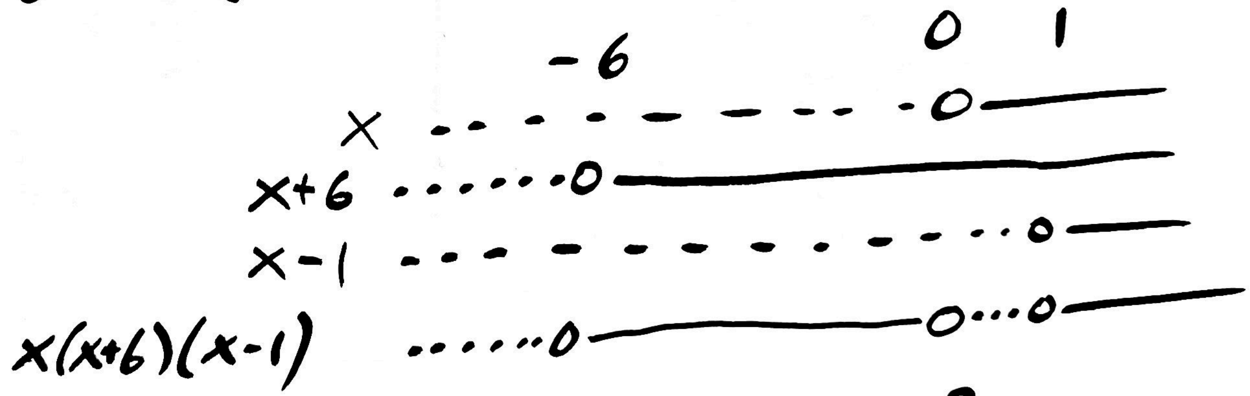


Løsningsmengden er

$\langle -\infty, 1 \rangle \cup \langle 2, \infty \rangle$

* $x^3 + 5x^2 - 6x \leq 0 \iff x(x+6)(x-1) \leq 0$

$x(x^2 + 5x - 6) \leq 0$



Løsningsmengden er $\langle -\infty, -6] \cup [0, 1]$

Doble ulikheter

$$2x - 1 < 4x \leq 10 - x$$

betyr $\quad 2x - 1 < 4x \quad$ og $\quad 4x \leq 10 - x$

felles løsning til de to ulikhetene.

$2x - 1 < 4x$

$2x - 4x < 1$

$-2x < 1$ deler med $-2 (< 0)$

$x > \frac{1}{-2} = \frac{-1}{2}.$

snur ulikheten

$\underline{\langle \frac{-1}{2}, \infty \rangle}$

$4x \leq 10 - x$

$4x + x \leq 10$

$5x \leq 10$ deler med 5

$x \leq \frac{10}{5} = 2$

$x \leq 2$

$\underline{\langle -\infty, 2]}$

Løsningene til den doble ulikheten er $\langle \frac{-1}{2}, \infty \rangle \cap \langle -\infty, 2] = \underline{\langle \frac{-1}{2}, 2]}$

Rasjonale ulikheter

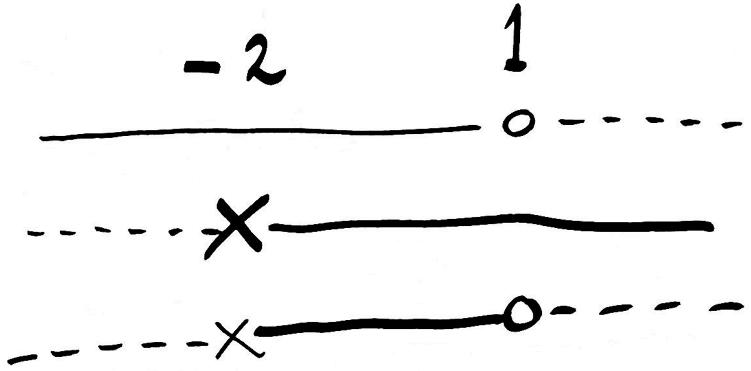
$$\frac{1-x}{2+x} \geq 0$$

$\begin{pmatrix} 2+x=0 \\ x=-2 \end{pmatrix}$

$1-x$

$1/(2+x)$

$\frac{1-x}{2+x}$



Løsningene $\underline{\langle -2, 1]}$

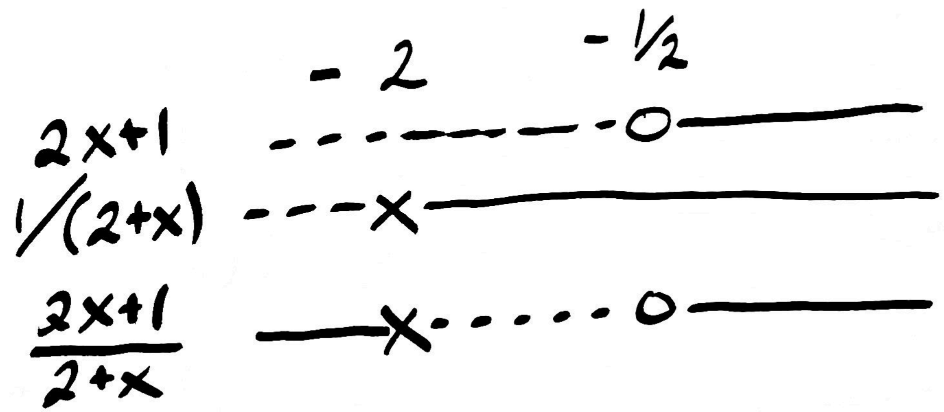
* $\frac{1-x}{2+x} > 1$ $\quad x \neq -2$, $\left(\begin{array}{ll} \text{gange med } 2+x & \begin{array}{l} 2+x>0 : 1-x > 2+x \\ 2+x<0 : 1-x < 2+x \\ \qquad\qquad \text{snur ulikheten} \end{array} \end{array} \right)$

$$\frac{1-x}{2+x} - 1 > 0 \iff \frac{1-x}{2+x} - \frac{2+x}{2+x} > 0$$

$$\frac{1-x-(2+x)}{2+x} > 0 \iff \frac{-1-2x}{2+x} > 0$$

ganger med -1 $\quad \frac{2x+1}{2+x} < 0$

snur ulikheten



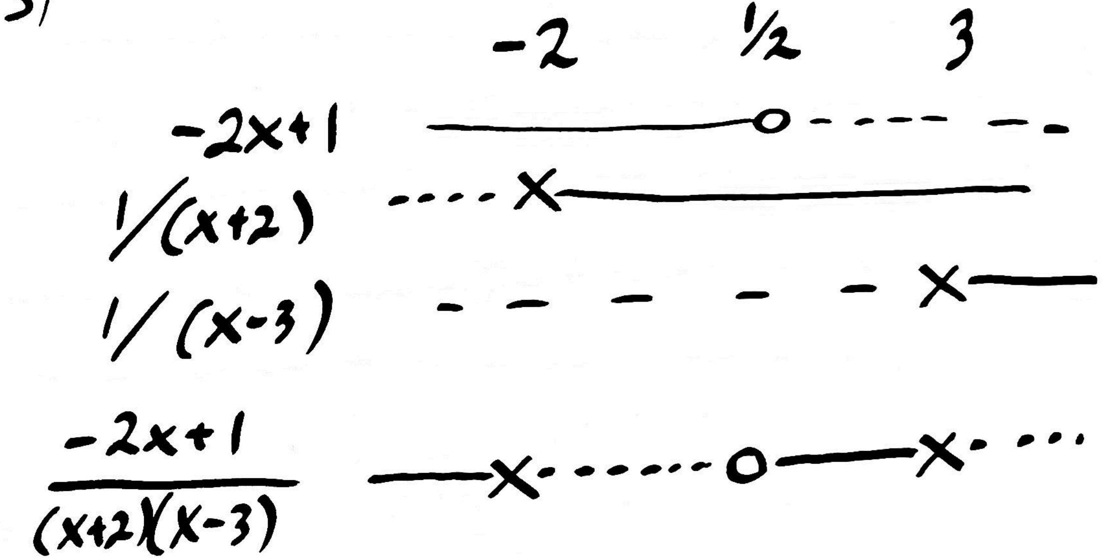
Løsningsmengden er $\langle -2, -\frac{1}{2}]$

$$\frac{x+1}{x+2} \geq \frac{x-2}{x-3} \iff \frac{x+1}{x+2} - \frac{x-2}{x-3} \geq 0$$

$$\frac{(x+1)(x-3)-(x-2)(x+2)}{(x+2)(x-3)} \geq 0$$

$$\frac{x^2+x-3x-3-(x^2-4)}{(x+2)(x-3)} \geq 0$$

$$\frac{-2x+1}{(x+2)(x-3)} \geq 0$$

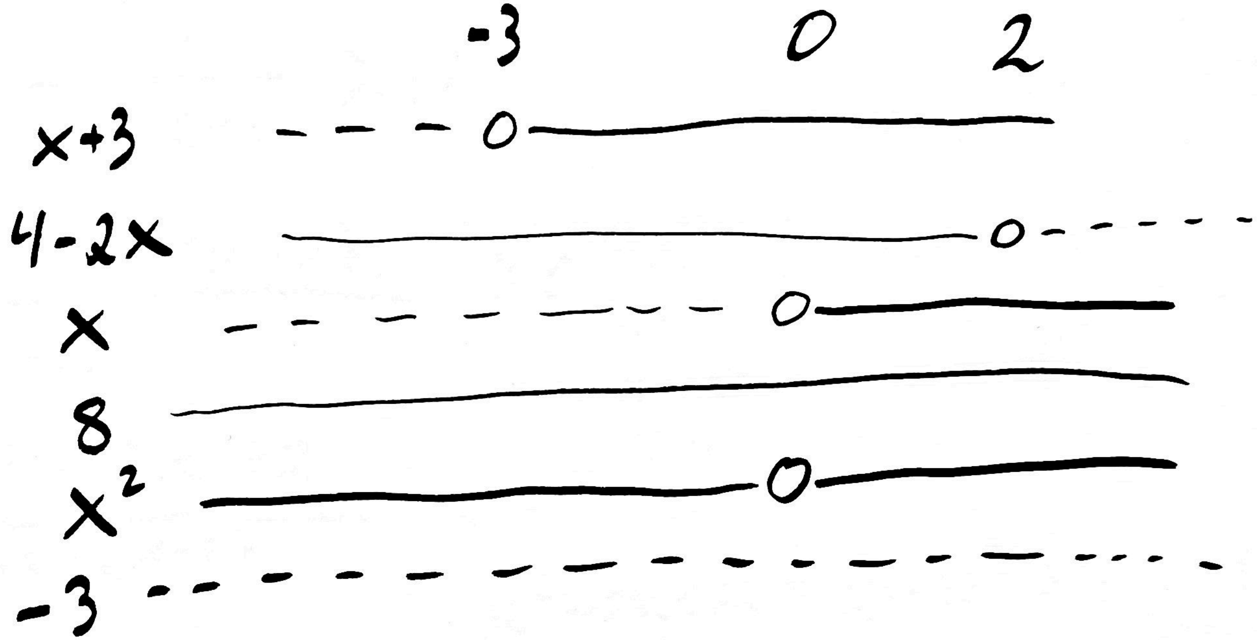


Løsningsmengden er $\underline{\langle -\infty, -2\rangle \cup [\tfrac{1}{2}, 3\rangle}$

4.111

Fortegnslinjer

Øving



$x^2 \geq 0$ har den løsning alle x

$x^2 > 0$ løsningen $\langle -\infty, 0\rangle \cup \langle 0, \infty\rangle$
$= \mathbb{R} \setminus \{0\}$

$x^2 \leq 0$ har løsning $\underline{\{0\}}$

$x^2 < 0$ —— $\emptyset$

* Løs $\frac{x}{x-2} \geq \frac{1}{x-1}$

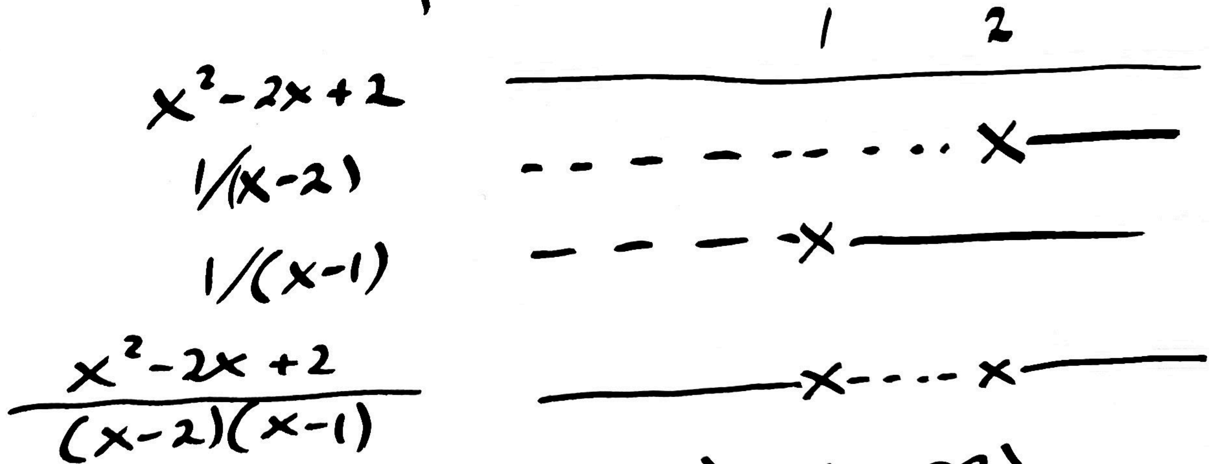
$$\frac{x}{x-2} - \frac{1}{x-1} \geq 0$$

$$\frac{x(x-1) - (x-2)}{(x-2)(x-1)} \geq 0$$

$$\frac{x^2 - 2x + 2}{(x-2)(x-1)} \geq 0$$

$x^2 - 2x + 2 = (x-1)^2 + 1 \geq 1$ for alle x.

positiv for alle x.



Løsningene er $\underline{\langle -\infty, 1\rangle \cup \langle 2, \infty\rangle}$

eksamensoppg.
26 mai 25

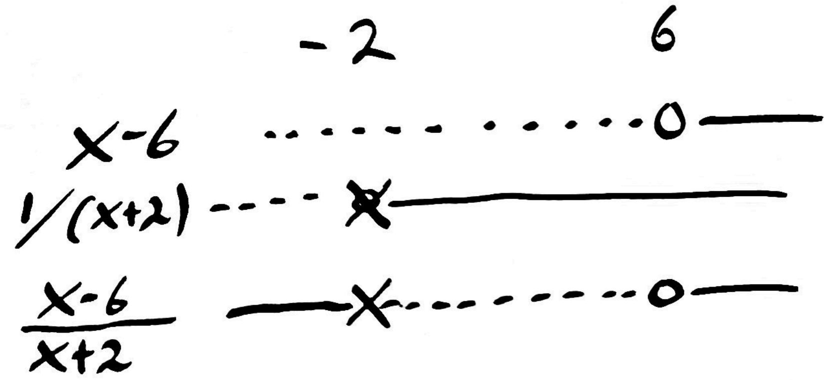
$$\frac{2x-4}{x+2} \geq 1$$

$$\frac{2x-4}{x+2} - 1 \geq 0$$

$$\frac{2x-4}{x+2} - \frac{x+2}{x+2} \geq 0$$

$$\frac{2x-4-(x+2)}{x+2} \geq 0$$

$$\frac{x-6}{x+2} \geq 0$$



Løsningene er $\langle \leftarrow, -2\rangle \cup [6, \rightarrow\rangle$

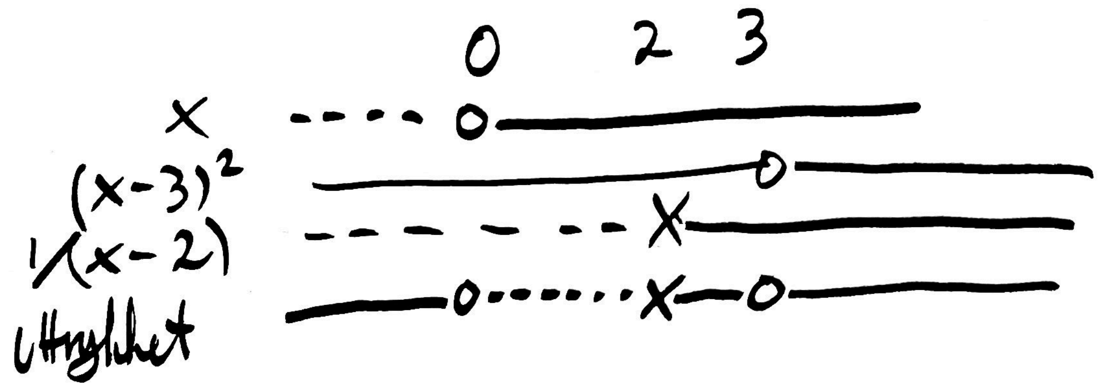
$$\frac{x^3-3x^2+3x}{x-2} \le 3x$$

$$\frac{x^3-3x^2+3x}{x-2} - 3x \le 0$$

$$\frac{x^3-3x^2+3x \;-3x(x-2)}{x-2} \le 0$$

$$\frac{x(x^2-3x+3 \;-3x \;-3(-2))}{x-2} \le 0$$

$$\frac{x(x^2-6x+9)}{x-2} \le 0 \iff \frac{x(x-3)^2}{x-2} \le 0$$



Løsningene er

$[0, 2\rangle \cup \{3\}$