3 sep
25

3D Faktorisering

Faktoriser $3x^2 - 12$

(benytter konjugatsetningen)

$= 3(x^2 - 4) = 3(x^2 - 2^2)$

$= 3(x+2)(x-2)$

* $-x^2 + 14x - 49 = -(x^2 \underbrace{-14x}_{2\cdot(-7)} + 49)$

$7^2 = (-7)^2$

$b^2 + 2ab + a^2$

hvor $b = x$ og $a = -7$

(kvadratsetningen)

$= \underline{-(x-7)^2}$

Et uttrykk på formen $(x+d)^2$ kalles et <u>fullstendig</u> kvadrat.

$$(x+d)^2 = x^2 + 2dx + d^2$$

Når er $x^2 + bx + c$ et fullstendig kvadrat?

$b = 2d$ og $c = d^2$

$d = b/2$ $\quad c = (b/2)^2$.

$x^2 + bx + c$ er lik et fullstendig kvadrat $\iff c = (b/2)^2$.

Det er da lik $(x + b/2)^2$

Eks. Er $x^2 + \sqrt{12}\,x + 3$ et fullstendig kvadrat?

Sjekker: $b = \sqrt{12} = \sqrt{4 \cdot 3} = \sqrt{4}\sqrt{3} = 2\sqrt{3}$

$c = 3$

$(b/2) = (2\sqrt{3}/2)^2 = (\sqrt{3})^2 = 3 = c$ ✓

<u>Ja</u>. Det er lik $(x+\sqrt{3})^2$.

* x^2 Er et fullstendig kvadrat ($b=0$, $c=0$ $=(b/2)^2$)

* $2x^2+4x+2$

$\neq 1$ så ikke et fullstendig kvadrat.

$$2(\underbrace{x^2+2x+1}_{\text{fullstendig } \square}) = 2(x+1)^2.$$

* $x^2+4x+16$ nei.

$b=4,\ b/2=2$

$(b/2)^2=4\neq 16$

$$x^2+bx = \overbrace{(x+b/2)^2}^{x^2+bx+(b/2)^2} - \overbrace{\left(\frac{b}{2}\right)^2}^{(b/2)^2}$$

legger til c på begge sider av =-tegnet

$$x^2+bx+c = (x+b/2)^2 - \left(\frac{b}{2}\right)^2 + c$$

$$ax^2+bx+c = a\left(x^2+\frac{b}{a}x+\frac{c}{a}\right)$$

$a\neq 0$

Eks $x^2 + 8x = (x+4)^2 - 4^2$

$$x^2 = x^2$$

$$x^2 + x = (x + 1/2)^2 - (1/2)^2$$

$$x^2 + 3x + 3 = (x + 3/2)^2 - (3/2)^2 + 3$$
$$= (x + 3/2)^2 + \frac{3}{4}.$$

Benytter omskriving med fullstendige $\square$ til å faktorisere

$$x^2 + 6x + 8$$
$$= (x+3)^2 - 3^2 + 8$$
$$= (x+3)^2 - 1^2$$
$$= (\underbrace{x+3}_{b} + \underbrace{1}_{a})(\underbrace{x+3}_{b} - \underbrace{1}_{a}) = \underline{(x+4)(x+2)}$$

$b^2 - a^2 = (b+a)(b-a)$ konjugatsetningen

lar vi $b = x+3$, $a = 1$

tester: $(x+4)(x+2) = x^2 + 2x + 4x + 4 \cdot 2$
$$= x^2 + 6x + 8 \checkmark$$

$$\underbrace{x^2+2x}_{(x+1)^2-1^2}+5 = (x+1)^2+5-1$$

$$= (x+1)^2+4. \quad (\geq 4 \text{ for alle } x)$$

kan ikke faktoriseres over $\mathbb{R}$.

(Anta at den er lik $(x+a)(x+c)$, men dette uttrykket er lik 0 når $x=-a$ og når $x=-c$)

$$x^2+bx+c = (x+b/2)^2 - \left((\tfrac{b}{2})^2-c\right)$$

kan ikkje faktoriseres når $(b/2)^2-c<0$

kan faktoriseres når $(b/2)^2-c \geq 0$.

$$(x+b/2)^2 - \left(\sqrt{(b/2)^2-c}\right)^2$$

$$= \left(x+\tfrac{b}{2}+\sqrt{(b/2)^2-c}\right)\left(x+b/2-\sqrt{(b/2)^2-c}\right)$$

Faktoriser (hvis mulig)

$$\underbrace{x^2+2x}_{(x+1)^2-1^2}-15 = (x+1)^2-16 = (x+1)^2-4^2$$

$$= (x+1+4)(x+1-4)$$

$$= \underline{(x+5)(x-3)}$$

$$x^2+x-1 = \left((x+1/2)^2-(1/2)^2\right)-1$$

$$= (x+1/2)^2-\frac{1}{4}-1 = (x+1/2)^2-\frac{5}{4}$$

$$= (x+1/2)^2-\left(\sqrt{\frac{5}{4}}\right)^2$$

konjugal setningen

$$= \left(x+\frac{1}{2}+\sqrt{\frac{5}{4}}\right)\left(x+\frac{1}{2}-\sqrt{\frac{5}{4}}\right)$$

(benytter at $\sqrt{\frac{5}{4}}=\frac{\sqrt{5}}{\sqrt{4}}=\frac{\sqrt{5}}{2}$)

$$= \underline{\left(x+\frac{1+\sqrt{5}}{2}\right)\left(x+\frac{1-\sqrt{5}}{2}\right)}$$

3E Rasjonale uttrykk

$$\frac{x+3}{2x-1}, \quad \frac{1}{x} \quad x^2+x = \frac{x^2+x}{1}$$

$$\frac{\text{polynom}}{\text{polynom}}$$

utvide: $\frac{1}{x+2} = \frac{1}{x+2} \cdot \frac{x-2}{x-2} = \frac{x-2}{(x+2)(x-2)} = \frac{x-2}{x^2-4}$ for $x \neq 2$ og -2.

utvide med $x-2$.

er lik 1 når $x \neq 2$
ikke definert for $x=2$

forkorte $\frac{x^3}{x^2+3x} = \frac{x \cdot x^2}{x(x+3)} = \frac{x}{x} \cdot \frac{x^2}{x+3} = \frac{x^2}{x+3}$

lik 1 når
$x \neq 0$

sum: $\frac{1}{x+2} + \frac{3}{x-1} = \frac{1}{x+2}\frac{(x-1)}{(x-1)} + \frac{3}{x-1}\frac{(x+2)}{(x+2)}$

$$= \frac{x-1}{(x+2)(x-1)} + \frac{3x+6}{(x-1)(x+2)} = \frac{x+3x-1+6}{(x-1)(x+2)}$$

$$= \underline{\frac{4x+5}{(x-1)(x+2)}}$$

Opg 9 Finn summen $2 + \frac{-4}{x+2} \to 2\,\frac{x+2}{x+2} + \frac{-4}{x+2}$

$$\frac{2(x+2)-4}{x+2} = \frac{2x+4-4}{x+2} = \underline{\frac{2x}{x+2}}$$

Forkort $\frac{x^2+6x+9}{x^2-9}$ ← fullstendig kvadrat $\left(\frac{6}{2}\right)^2=9$ ← benytter konjugatsetningen

$$= \frac{(x+3)^2}{(x+3)(x-3)} = \underbrace{\frac{(x+3)}{(x+3)}}_{1 \text{ for } x \neq -3} \cdot \frac{x+3}{x-3}$$

$$= \underline{\frac{x+3}{x-3}} \quad \text{for } x \neq -3 \text{ (og 3)}.$$

Øving

3.50 b) $\underbrace{a^2 - 12a}_{b=-12} + \underbrace{36}_{(b/2)^2 = (-12/2)^2 = 36}$. er det lik $(a+d)^2$ fullstendig kvadrat

$= (a-6)^2$

3.51 d) $a^2 - 10a + 25$ Fullstendig kvadrat $\left(-\frac{10}{2}\right)^2 = 25$

$\underline{= (a-5)^2}$

b) $n^2 + 49 = n^2 + 7^2$ kan ikke faktoriseres

(a) Hadde det vært $n^2 - 49$ så hadde vi fått $(n-7)(n+7)$

$(n+7)^2 = n^2 + 7^2 + 2 \cdot 7 \cdot n = n^2 + 14n + 49$

3.38 a) $\sqrt{24} - \sqrt{6} = \sqrt{6}$ Vis dette.

$24 = 4 \cdot 6 = 2^2 \cdot 6$

$$\sqrt{24} - \sqrt{6} = \sqrt{2^2 \cdot 6} - \sqrt{6}$$
$$= \sqrt{2^2} \cdot \sqrt{6} - \sqrt{6}$$
$$= 2\sqrt{6} - \sqrt{6} = (2-1)\sqrt{6} = \underline{\sqrt{6}}$$

$\left(\sqrt{a \cdot b} = \sqrt{a}\sqrt{b} \right.$
fordi $(\sqrt{a}\sqrt{b})^2$
$= (\sqrt{a}\sqrt{b}\sqrt{a}\sqrt{b})$
$\left. = (\sqrt{a})^2(\sqrt{b})^2 = a \cdot b \right)$

$(\sqrt{25} - \sqrt{6} = 5 - \sqrt{6} \ldots)$

* $3\sqrt{7} - 2\sqrt{7} - \sqrt{7} = 0$

$\sqrt{3^2 \cdot 7} - \sqrt{2^2 \cdot 7} - \sqrt{7} = 0$

$\sqrt{63} - \sqrt{28} - \sqrt{7} = 0$

"Lager en tilsvarende oppgave" :

3.38 d) variant

Forenkl: $(3-x)^2 + (x+2)\underbrace{(x-3)}_{-(3-x)}$

Faktoriserer først:

$$(3-x)\Big(\underbrace{(3-x) + (-1)(x+2)}_{(3-x-x-2)}\Big)$$

$$= (3-x)(1-2x) = \underline{(x-3)(2x-1)}$$

3.37. b) $5x^3 - x^2 = x^2(5x-1)$

$$= 5x^2\left(x - \tfrac{1}{5}\right)$$

3.30 b) $-9\underbrace{(x+1)(x-1)}_{(x^2-1)} + (3x-1)^2$

$= -9x^2 + 9 + \underbrace{(3x-1)^2}_{\underbrace{(3x)^2 + 2\cdot 3x(-1) + (-1)^2}_{9x^2 - 6x + 1}}$

$= -9x^2 + 9 + 9x^2 - 6x + 1$

$= \underline{-6x + 10} = -2(3x-5)$

$= -6(x - 5/3)$

3.30 a) $4-(5x+2)^2 = 4-((5x)^2 + 2\cdot 5x\cdot 2 + 2^2)$

$= 4 - 25x^2 - 20x - 4 = \underline{-25x^2 - 20x} = \underline{-5x(5x+4)}$

Alternativt: $2^2-(5x+2)^2 = (2+5x+2)(2-(5x+2))$

$= (5x+4)(-5x+\overbrace{2-2}^{0}) = \underline{-5x(5x+4)}$