1.sep 25

3B Kvadrat- og konjugatsetningene

$$(a+b)(a+b) = (a+b)^2$$

$$a(a+b) + b(a+b) = a\cdot a + a\cdot b + \underbrace{b\cdot a}_{ab} + b\cdot b$$

$$(a+b)^2 = a^2 + 2ab + b^2$$ kvadratsetningen

Kryssleddet

(1. kvadratsetning)

$$(a-b)^2 = (a+(-b))^2 = a^2 + 2a(-b) + (-b)^2$$
$$= a^2 - 2ab + b^2$$

(2. kvadratsetning)

eks

$$(13)^2 = (10+3)^2 = 10^2 + 2\cdot 10\cdot 3 + 3^2$$
$$= 100 + 60 + 9$$
$$= \underline{169}$$

* $(19)^2 = (20-1)^2 = (20+(-1))^2$

$= (20)^2 + 2\cdot 20(-1) + (-1)^2$

$= 400 - 40 + 1 = \underline{361}$

* $(\sqrt{2}+5)^2 = (\sqrt{2})^2 + 2\cdot\sqrt{2}\cdot 5 + 5^2$

$= 2 + 10\sqrt{2} + 25$

$= \underline{27 + 10\sqrt{2}}$

* $(2x - y^2)^2 = (2x)^2 + 2\cdot 2x(-y^2) + (-y^2)^2$

$= 4x^2 - 4xy^2 + y^4$

* $x^2 + 6x + 9 = x^2 + 2\cdot 3\cdot x + 3^2$

$= (x^2+3)^2$

$a = x$
$b = 3$

$$(a+b)(a-b) = a\cdot a + \underbrace{a(-b) + b\cdot a}_{-a\cdot b + a\cdot b} + b(-b)$$

(a-b konjugert til a+b)

Konjugatsetningen $(a+b)(a-b) = a^2 - b^2$

3. kvadratsetning.

* $13\cdot 7 = (10+3)(10-3)$
$= 10^2 - 3^2 = 100 - 9 = \underline{91}$

* $23\cdot 17 = (20+3)(20-3)$
$= (20)^2 - 3^2 = 400 - 9$
$= \underline{391}$

* $(\sqrt{13} + \sqrt{11})(\sqrt{13} - \sqrt{11})$
$= (\sqrt{13})^2 - (\sqrt{11})^2$
$= 13 - 11 = \underline{2}$

Så

$$\frac{1}{\sqrt{13}+\sqrt{11}} = \frac{\sqrt{13}-\sqrt{11}}{2}$$

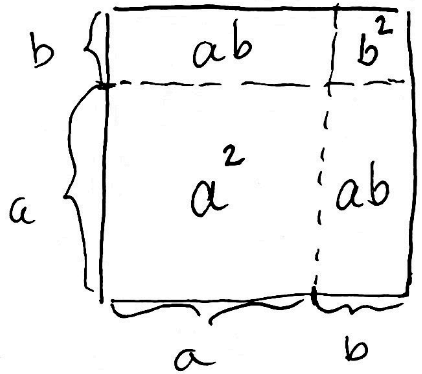
$$9x^2 - 4z^2$$
$$= (3x)^2 - (2z)^2$$
$$= (3x + 2z)(3x - 2z)$$

$$9919 = 10\,000 - 81$$
$$= 100^2 - 9^2$$
$$= (100 + 9)(100 - 9)$$
$$= \underline{109 \cdot 91}$$

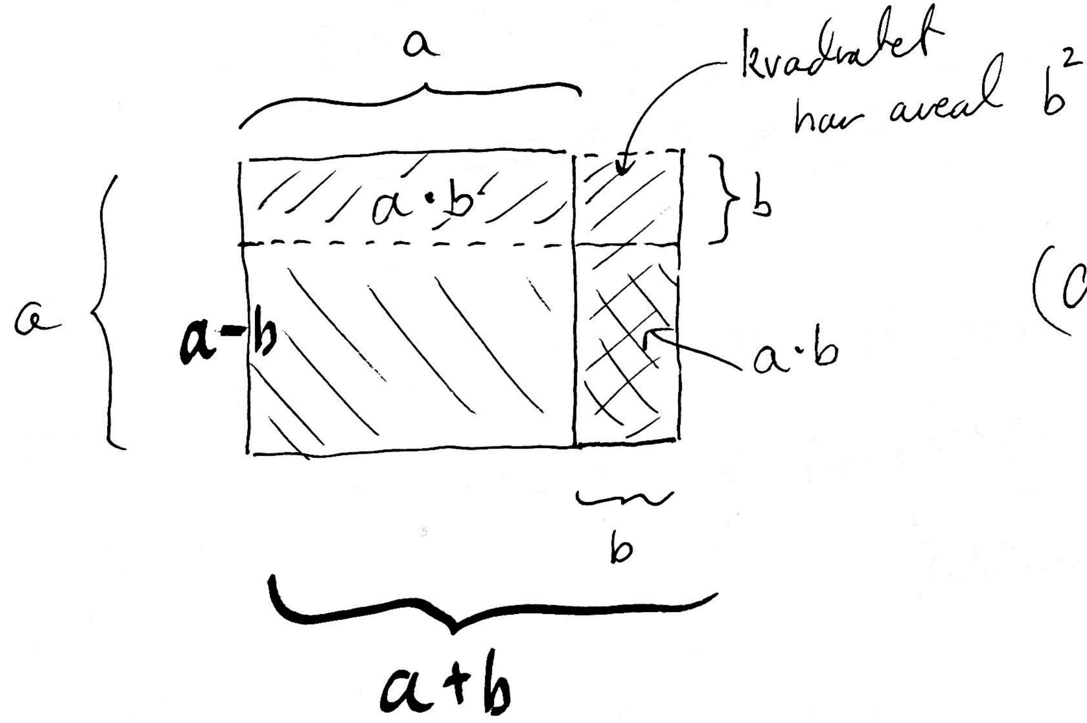
$$153 = 169 - 16 = 13^2 - 4^2$$
$$= (13 + 4)(13 - 4) = 17 \cdot 9 = \underline{17 \cdot 3^2}$$

$$319 = 400 - 81 = 20^2 - 9^2$$
$$= (20 + 9)(20 - 9) = \underline{11 \cdot 29}$$

oppg. Faktoriser



Hele kvadratet
har areal
$(a+b)^2 = a^2 + b^2 + ab + ab$



$$(a-b)(a+b) = a^2 - b^2$$

* $(3x+2y-5)(3x-2y-5)$

$= ((3x-5)+2y)((3x-5)-2y)$ konjugat

$= (3x-5)^2 - (2y)^2$

$= (3x)^2 + 2\cdot 3x(-5) + (-5)^2 - 4y^2$

$= 9x^2 - 30x + 25 - 4y^2$

$= \underline{9x^2 - 4y^2 - 30x + 25}$

Når er n^2-4 et primtall for $n \geq 3$?

*

$n^2-4 = n^2 - 2^2 = (n+2)(n-2)$

faktorene ≥ 2 for $n \geq 4$

n^2-4 for $n=3$ er $9-4=5$ ✓

$n=4$ partall

$n=5$ $5^2-4 = 21 = 3\cdot 7$

$n=7$ $49-4 = 45 = 5\cdot 9$

Konklusjo.

n^2-4 $n \geq 3$

er et primtall

kun når $\underline{n=3}$.

★ n^2+1 er et partall når n er et oddetall.

$n=1$: $1^2+1=2$

$n=3$: $3^2+1 = 10 = 2\cdot 5$

$n=5$: $5^2+1 = 26 = 2\cdot 13$

$n=7$: $7^2+1 = 50 = 2\cdot 25$

Det ser ut til at n^2+1 kun har én faktor 2.

Vi forsøker å vise dette:

oddetallen ≥ 0: $n = 2m+1$ $\quad m \geq 0$

$$(2m+1)^2+1 \underset{\text{kvadratsetningen}}{=} (2m)^2 + 2\cdot 2m \cdot 1 + 1^2 + 1$$

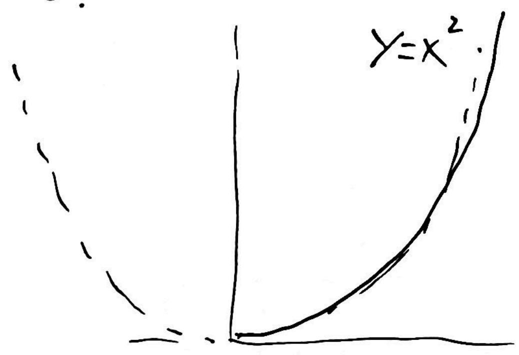
$$= 4m^2+4m+2$$

$$= 2\underbrace{(2m^2+2m+1)}_{\text{et oddetall for alle } m\geq 0.} = 2(2m(m+1)+1)$$

så n^2+1 er delelig med 2, men ikke 4 for oddetall $n \geq 0$.

$$\frac{n^2+1}{2} = \underbrace{2m(m+1)}_{\text{partall}} + 1 \quad \text{er på formen } 4k+1 \quad k \in \mathbb{N}$$

Hvis $a, b \geq 0$, da er $a^2 \leq b^2 \iff a \leq b$.



$$b^2 \geq a^2 \iff b^2 - a^2 \geq 0$$

$$\iff \underbrace{(b+a)}_{\text{antagelse}}(b-a) \geq 0 \iff b-a \geq 0$$

(Hvis a, b ikke nødvendigvis er ikke-negative, er dette galt: $-3 < -2$ men $(-3)^2 > (-2)^2$)

$$\sqrt{1+1} = 1{,}41\ldots < \sqrt{1} + \sqrt{1} = 2$$

$$0 \leq \sqrt{x+y} \leq \sqrt{x} + \sqrt{y}$$

$$\Updownarrow$$

$$x+y = \left(\sqrt{x+y}\right)^2 \underset{\text{Sant}}{\leq} \left(\sqrt{x} + \sqrt{y}\right)^2 = x + y + \overbrace{2\sqrt{x}\sqrt{y}}^{\geq 0} \qquad x, y \geq 0$$

ekte ulikheter når $x, y > 0$.

$8^2 = 64$

$8^2 = (9-1)^2 = 9^2 - 2\cdot 9\cdot 1 + (-1)^2$ tungvint

$= 81 - 18 + 1 = 64$

Digresjon (spørsmål)

Benytter $\sqrt{1+x} \sim 1 + \frac{x}{2}$ x liten.

$\sqrt{8} = \sqrt{9-1} = \sqrt{9(1-\frac{1}{9})}$

$= \sqrt{9}\sqrt{1-\frac{1}{9}} \approx 3(1 + \frac{1}{2}(-\frac{1}{9}))$

$\sqrt{8} \sim 3 - \frac{3}{2\cdot 9} = 3 - \frac{1}{6}$

$\left(\sqrt{11} = \sqrt{9+2} \ \ldots\right)$

$\sqrt{15} = \sqrt{16-1} = \sqrt{16(1-\frac{1}{16})} \sim 4\sqrt{1-\frac{1}{16}} \ldots$

$$(\underbrace{3x^2}_{a} + \underbrace{y+2}_{b})(\underbrace{3x^2}_{a} \underbrace{-y-2}_{-b})$$ konjugatsetning

$$(3x^2)^2 - (y+2)^2$$
$$= 9x^4 - (y^2 + \underbrace{2\cdot y\cdot 2}_{4y} + 2^2)$$

$$= 9x^4 - y^2 - 4y - 4$$

$$(a+b)^3 = (a+b)(a+b)^2$$
$$= (a+b)(a^2 + 2ab + b^2)$$
$$= a(a^2+2ab+b^2) + b(a^2+2ab+b^2)$$
$$= a^3 + 2a^2b + ab^2 + ba^2 + 2ab^2 + b^3$$

→

$$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$

$$(a+b)^n = a^n + na^{n-1}b + \frac{n(n-1)}{2}a^{n-2}b^2 + \dots + b^n$$

Det finnes en formel for $(a+b)^n$ som kalles binomial formelen

Øving

3.15 d) $(5-x)^2 = ((5)+(-x))^2 = 5^2 + (-x)^2 + 2\cdot 5\cdot(-x)$

$= \underline{x^2 - 10x + 25}$

c) $(x+3)(x-3)$

(konjugatsetningen)

$= x^2 - 3^2$

$= \underline{x^2 - 9}$

Variant

$(x+3)\underbrace{(3-x)}_{-(x-3)} = 3^2 - x^2 = 9 - x^2$

$= -(x+3)(x-3) = -(x^2-9) = 9 - x^2.$

3.16 d) $(3x-5)^2 = ((3x)+(-5))^2$

$= (3x)^2 + 2\cdot(3x)(-5) + (-5)^2$

$= \underline{9x^2 - 30x + 25}$

3.17 a) $\left(4x-\frac{1}{2}\right)\left(4x+\frac{1}{2}\right)$

$= (4x)^2 - \left(\frac{1}{2}\right)^2 = \underline{16x^2 - \frac{1}{4}}$

b) $(x^4+3y)(3y-x^4)$

$= (3y+x^4)(3y-x^4)$

$= (3y)^2 - (x^4)^2 = \underline{9y^2 - x^8}$

3.18 a) $(x+2)^2 - (x-2)^2$ (kvadratsetningen ...)

alternativt: $\left(\underbrace{(x+2)+(x-2)}_{2x}\right)\left(\underbrace{(x+2)-(x-2)}_{4}\right)$

kjenne på uttrykket som $a^2-b^2=(a+b)(a-b)$

$= 2x \cdot 4 = \underline{8x}$

b) $(2x-3)^2 - \underbrace{4(x+2)^2}_{(2\cdot(x+2))^2} = (2x-3)^2 - 2^2(x+2)^2$

$(2x-3)^2 - (2\cdot(x+2))^2$ konjugatsetning.

$\big((2x-3) + (2x+4)\big)\big((2x-3) - (2x+4)\big)$

$(4x+1) \cdot (-7)$

$= \underline{-7(4x+1)}$

3.30 b) $-9\underbrace{(x+1)(x-1)}_{} + (3x-1)^2$

$-9(x^2-1)$

$= -9x^2 + 9 + (3x-1)^2$

$= (3x-1)^2 - (3x)^2 + 9$

$= (3x-1+3x)(3x-1-3x) + 9$

$= (6x-1)(-1) + 9 = -6x + 1 + 9 = \underline{10 - 6x}$