25 august

25

Fauck

oblig 1 18 sep.

oblig 2 16 okt

oblig 3 18 nov. ?

2B Desimaltall på standardform

$\pm a \cdot 10^n \qquad 1 \leq a < 10$

$1000 = 10^3$ kilo

kilogram $= 10^3$ gram

(kilobyte $2^{10} = 1024$ byte ...)

10^6 mega

10^{-3} milli

10^{-6} mikro

10^{-9} nano ...

$5 \cdot 10^6 \cdot 7 \cdot 10^{-10} = 5 \cdot 7 \cdot 10^{6-10}$

$= \underbrace{35}_{3.5 \cdot 10^1} \cdot 10^{-4}$

$3.5 \cdot 10^1 \cdot 10^{-4} = \underline{3.5 \cdot 10^{-3}}$

10^1, $10^2 = 100$, $10^n = \overbrace{\underbrace{100\ldots0}_{n+1}}^{n}$

$10^{-1} = \frac{1}{10} = 0.1$, $10^{-2} = 0.01$

$10^{-n} = 0.\overbrace{\underbrace{00\ldots01}_{n}}^{n-1}$

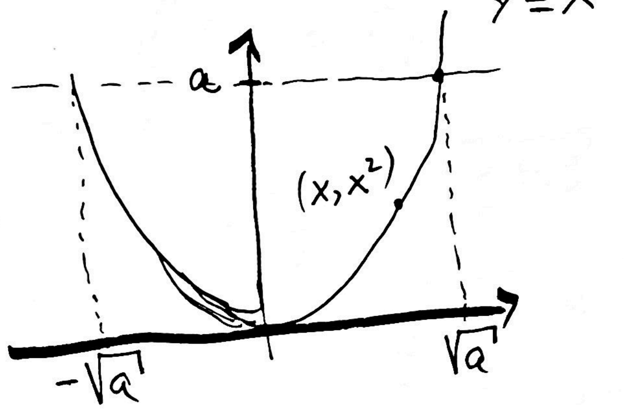
0.0000374

$= \underline{3.74 \cdot 10^{-5}}$

3C Røtter

$\sqrt{a}$ kvadratroten til et ikke-negativt tall a er et ikke-negativt tall $\sqrt{a}$ slik at kvadratet er lik a.

$a \geq 0$ $\sqrt{a} \geq 0$ og $(\sqrt{a})^2 = a$.



$\sqrt{9} = 3$ (men løsningen til $x^2 = 9$ er $x = \sqrt{9} = 3$ og $x = -\sqrt{9} = -3$)

$\sqrt{1} = 1$

$\sqrt{0.04} = 0.2$

$\sqrt{\frac{32}{50}} = \sqrt{\frac{16 \cdot 2}{25 \cdot 2}} = \sqrt{\frac{16}{25}} = \frac{4}{5}$

$\sqrt{2} \approx 1,41\ldots$ irrasjonalt tall.

($\sqrt{(-\frac{2}{10})^2} = \sqrt{(-0.2)^2} = (-0.2)^2 = 0.04$.

løsningen til $x^2 = 0.04$ er $x = \pm 0.2 \ldots$)

$$\sqrt{0.4} = \sqrt{4 \cdot \frac{1}{10}} = \frac{\sqrt{4}}{\sqrt{10}} = \frac{2}{\sqrt{10}}$$ irrasjonalt.

$*$ $$\sqrt{a \cdot b} = \sqrt{a} \cdot \sqrt{b} \qquad a, b \geq 0$$

fordi $\sqrt{a}\sqrt{b} \geq 0$ og $(\sqrt{a}\sqrt{b})^2 = (\sqrt{a})^2 (\sqrt{b})^2 = a \cdot b$

$*$ $$\sqrt{\frac{1}{a}} = \frac{1}{\sqrt{a}} \qquad a > 0$$

$$\sqrt{50} = \sqrt{25 \cdot 2} = \sqrt{25}\sqrt{2} = 5\sqrt{2}$$

$$\sqrt{27} = \sqrt{3 \cdot 3^2} = \sqrt{3} \cdot \sqrt{3^2} = \sqrt{3} \cdot 3 = \underline{3\sqrt{3}}$$

$$\sqrt{\frac{49}{2}} = \frac{\sqrt{49}}{\sqrt{2}} = \frac{7}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{7\sqrt{2}}{2}$$

$$\frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$\sqrt{6}\sqrt{15}\sqrt{10} = \sqrt{6\cdot 15\cdot 10}$$

$$= \sqrt{(2\cdot 3)(3\cdot 5)(2\cdot 5)} = \sqrt{2^2\cdot 3^2\, 5^2} = \sqrt{2^2}\sqrt{3^2}\sqrt{5^2}$$

$$= 2\cdot 3\cdot 5 = \underline{30}$$

oppg. Skriv $\sqrt{20}$ enklere.

$$\sqrt{2^2\cdot 5} = \sqrt{2^2}\sqrt{5} = \underline{2\sqrt{5}}$$

$\sqrt{a+b}$?

$$\sqrt{a+b} \leq \sqrt{a} + \sqrt{b} \qquad a,b>0$$

$$\sqrt{1+1} = \sqrt{2} \sim 1.41 < \sqrt{1}+\sqrt{1} = 2$$

$$\sqrt{a+b} = \sqrt{25} = \underline{5} < \sqrt{a}+\sqrt{b} = \sqrt{9}+\sqrt{16}$$

$$= 3+4 = \underline{7}.$$

$a=9,\ b=16$

$a+b=25$

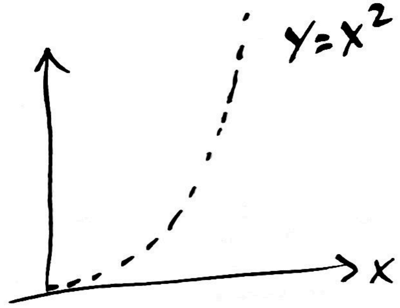
$(\sqrt{a})^2 = a$

$a^2 \geq 0$

$$\sqrt{a^2} = |a|$$

$$\sqrt{2^2} = \sqrt{4} = 2 = |2|$$

$$\sqrt{(-2)^2} = \sqrt{4} = 2 = |-2|$$



$\sqrt{b} > \sqrt{a} \geq 0$ når $b > a \geq 0$

$\sqrt{10} = 3.1622776601\ldots$

$\sqrt{1+x} \underset{x \text{ liten}}{\sim} 1 + \frac{x}{2}$ (en bedre tilnærming er $1 + \frac{x}{2} - \frac{x^2}{8}$)

Resultat:

$$\sqrt{10} = \sqrt{9+1} = \sqrt{9(1+\tfrac{1}{9})} = \sqrt{9}\sqrt{1+\tfrac{1}{9}} \sim 3\left(1 + \tfrac{1}{2} \cdot \tfrac{1}{9}\right) = 3 + \tfrac{1}{2}\tfrac{3}{9} = 3 + \tfrac{1}{6} = 3.1666\ldots$$

$\sqrt{103}$ $\qquad 10^2 = 100 \qquad 11^2 = 121$

$10 < \sqrt{103} < 11$

$$\sqrt{103} = \sqrt{100\left(1+\frac{3}{100}\right)} \sim \sqrt{100}\left(\sqrt{1+\frac{3}{100}}\right) = 10\left(1+\frac{1}{2}\cdot\frac{3}{100}\right)$$

$$\sim 10\left(1+\frac{3}{200}\right) \approx 10 + \frac{3\cdot 5}{20\cdot 5} = 10 + \frac{15}{100}$$

$$\sim \underline{10.15}$$

$\sqrt{103} = 10.148891\ldots$

$49 = 7^2$ og $64 = 8^2$

Øvg. Estimer $\sqrt{50}$.

forventer et tall litt større enn 7.

$$\sqrt{50} = \sqrt{49+1} = \sqrt{49\left(1+\frac{1}{49}\right)} = \sqrt{49}\sqrt{1+\frac{1}{49}}$$

$$= 7\sqrt{1+\frac{1}{49}} \sim 7\left(1+\frac{1}{2\cdot 49}\right) = 7 + \frac{7}{2\cdot 7^2} = \underline{7+\frac{1}{14}}$$

$$\left(7 + \frac{7}{7\cdot 7\cdot 2} \qquad 7 + \frac{7}{7}\cdot\frac{1}{14} = 7 + \frac{1}{14}\right)$$

$\sqrt{50} = 7.0710678\ldots$

$7 + \frac{1}{14} = 7.0714285\ldots$ $\quad$ ganske bra tilnærming.

$$\sqrt{4 \cdot 9}$$
$$= \sqrt{4} \; \sqrt{9}$$
$$= 2 \cdot 3 = 6$$

$$\sqrt{4} \cdot 9$$
$$= 2 \cdot 9$$
$$= 18$$

$\sqrt{\quad 4 \cdot}\, 9$? uklart

$\sqrt{}(4 \cdot 9)$ tar roten av alt i parentesen.

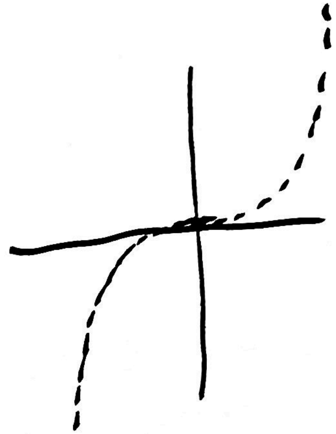
$\sqrt{}$ rotsymbolet

n-te røtter

$\sqrt[n]{a}$ n-te roten til a.

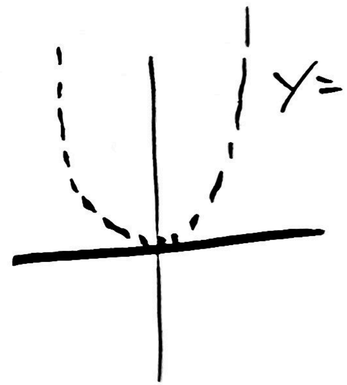
$$\left(\sqrt[n]{a}\right)^n = a$$

$n \in \mathbb{N}$



n odd: alle a er tillatt

$\sqrt[n]{a}$ er den entydige løsningen $x^n = a$.



n jevn (partall)

$\sqrt[n]{a}$ er definert for $a \geq 0$

$\sqrt[n]{a} \geq 0$ og $\left(\sqrt[n]{a}\right)^n = a$

(den ikke-negative løsningen til $x^n = a$)

$\sqrt[2]{a}$ skrives bare som $\sqrt{a}$

$\left(\sqrt[1]{a} = a\right)$

$\sqrt[3]{-8} = -2$ $\left(\underbrace{(-2)\cdot(-2)}_{4} \cdot (-2) = -8 \right)$.

$\sqrt[6]{64} = \sqrt[6]{2^6} = 2.$ $\left(\underbrace{(-2)(-2)}_{4} \cdot \underbrace{(-2)(-2)}_{4} = 16 \right)$

$\sqrt[4]{-16}$ ikke definert

$(-2)^4 = 16$

$2^4 = 16$

$\sqrt[4]{16} = 2$

$\sqrt[3]{125} = 5$

$\sqrt[5]{-100\,000} = -10$

Øving

Oppg. 2.46 c) Vis at $\sqrt[6]{a^7} = a\cdot\sqrt[6]{a}$.

Kun definert for $a \geq 0$ (siden $a^7 < 0$ hvis $a < 0$)

$$\sqrt[6]{a^7} = \sqrt[6]{a^6\cdot a} = \sqrt[6]{a^6}\,\sqrt[6]{a}$$

$$= |a|\sqrt[6]{a} = a\cdot\sqrt[6]{a} \quad \text{siden } a \geq 0$$

Generelt $\sqrt[n]{a^n} = a$ $\quad$ n odde

$\sqrt[n]{a^n} = |a|$ $\quad$ n jevn

$\sqrt[6]{a^8}$ definert for <u>alle</u> a.

$$= \sqrt[6]{a^6\cdot a^2} = \sqrt[6]{a^6}\,\sqrt[6]{a^2}$$

$$= |a|\sqrt[6]{a^2}.$$

Er $\sqrt[6]{a^2} = \sqrt[3]{a}$?

Begge er Definert for alle a.

$\left(\sqrt[6]{a^2}\right)^6 = a^2$ og $\left(\sqrt[3]{a}\right)^6 = \left(\left(\sqrt[3]{a}\right)^3\right)^2$

$= (a)^2 = a^2$.

$$\sqrt[6]{a^2} = \begin{cases} \sqrt[3]{a} & a \geq 0 \\ -\sqrt[3]{a} & a < 0 \end{cases}$$

Like kun for $a \geq 0$
motsatt fortegn for $a < 0$.

2.47c $\sqrt[4]{\dfrac{16y^4}{10000}} = \dfrac{\sqrt[4]{16} \cdot \sqrt[4]{y^4}}{\sqrt[4]{10000}} = \dfrac{\sqrt[4]{2^4}\ \sqrt[4]{y^4}}{\sqrt[4]{10^4}}$

Definert for alle y.

$= \dfrac{2 \cdot \sqrt[4]{y^4}}{10} = \dfrac{1}{5}\sqrt[4]{y^4} = \dfrac{1}{5}|y|$

d) variant $\sqrt[3]{4} \cdot \sqrt[3]{16} = \sqrt[3]{4 \cdot 16}$ $\left(= \sqrt[3]{64} =\right)$

$= \sqrt[3]{4 \cdot 4 \cdot 4} = \underline{4}$

Skriv så enkelt som mulig.

2.54 a) $5\sqrt{2} - \sqrt{50}$

$= 5\sqrt{2} - \sqrt{25 \cdot 2} = 5\sqrt{2} - \underbrace{\sqrt{25}}_{5}\sqrt{2}$

$= 5\sqrt{2} - 5\sqrt{2}$

$= \underline{0}$

Annet eksempel:

$\underline{5\sqrt{2} + \sqrt{6}} = 5\sqrt{2} + \sqrt{2 \cdot 3}$

$= 5\sqrt{2} + \sqrt{2} \cdot \sqrt{3}$

$= \underline{\sqrt{2}(5 + \sqrt{3})}$ Hva er "enklest"?

Variasjon av 2.35

*a) -0.00378

$= -\underline{3.78 \cdot 10^{-3}}$

2.36

$$\frac{5.0 \cdot 10^{7}}{3.5 \cdot 10^{3} \cdot 2.0 \cdot 10^{-5}}$$

$$= \frac{5.0}{3.5 \cdot 2.0} \cdot \frac{10^{7}}{10^{3} \cdot 10^{-5}}$$

$$= \underbrace{\frac{5.0}{7.0}}_{= 0.714\ldots} \cdot \underbrace{10^{7} \cdot 10^{-3} \underbrace{(10^{-5})^{-1}}_{10^{5}}}_{10^{7-3+5}}$$

$$= 0.714 \cdot 10^{9}$$

$$= 7.14 \cdot 10^{-1} \cdot 10^{9}$$

$$= \underline{\underline{7.1 \cdot 10^{8}}}$$