Prøveeksamen i forkurs Matematikk, OsloMet
30. april 2025

**Oppgave 1.** Løs ulikheten

$$\frac{2x}{1-x} \geq x$$

**Oppgave 2.** Vi får oppgitt at en vinkel $v$ ligger i andre kvadrant og at $\sin(v) = 1/4$. Finn eksakte verdier for $\cos(v), \cos(2v)$ og $\tan(v)$.

**Oppgave 3.** Løs likningen

$$2\ln(x+2) - \ln(x^2) = 1$$

**Oppgave 4.** Finn de ubestemte integralene

a) $\displaystyle\int x\sin(\pi x)\,dx$

b) $\displaystyle\int \frac{3}{\sqrt{e^x}} + \frac{4x}{3x^2+1}\,dx$

**Oppgave 5.** a) Finn en parametrisering av planet gitt ved likningen

$$2x + 3y - 5z = 7$$

b) Finn ut hvor linjen som går gjennom punktene $A(1, 2, -1)$ og $B(2, 1, -1)$ treffer planet i del a).

**Oppgave 6.** Bestem vinklene og lengdene på sidene i alle trekantene $ABC$ slik at lengden til $AC$ er lik 10 og lengden på side $BC$ er lik $5\sqrt{2}$ samt at vinkel $A$ er lik 30 grader.

**Oppgave 7.** Finn summen av alle positive heltall som er mindre enn eller lik 1000 og som er delelige med 7.

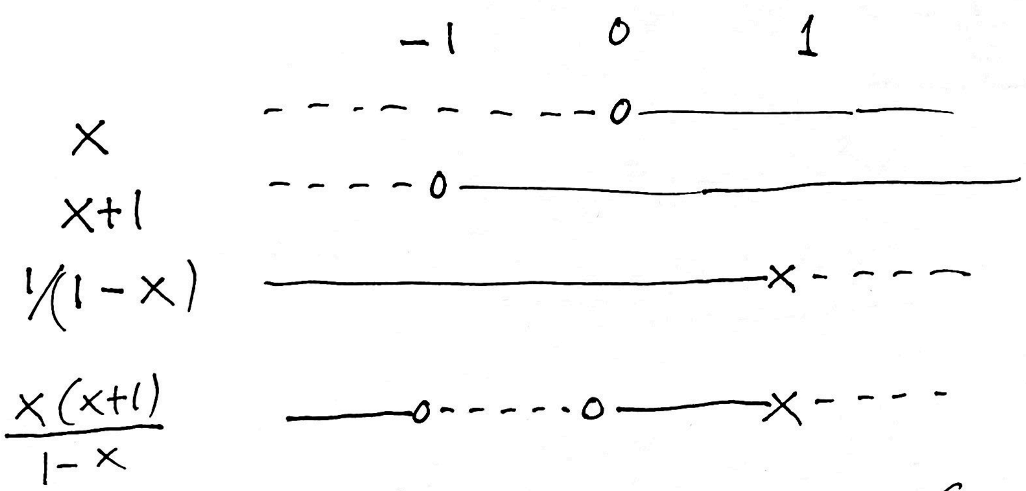
**Oppgave 8.** En stålstreng med lengde 1 meter, deles opp i to deler. Av den ene delen lages et kvadrat og av den andre delen så lages en trekant som er både rettvinklet og likebeinet. Hvordan må strengen deles opp for at summen av arealene deres skal bli minst mulig.

Prøveeksamen 30 april 2025.

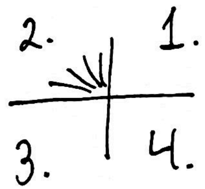
1 $\dfrac{2x}{1-x} \geq x$

$\Leftrightarrow \dfrac{2x}{1-x} - x \geq 0 \Leftrightarrow \dfrac{2x - x(1-x)}{1-x} \geq 0$

$\Leftrightarrow \dfrac{x^2+x}{1-x} \geq 0 \Leftrightarrow \dfrac{x(x+1)}{1-x} \geq 0$




Løsningene er $x \in \underline{\langle -\infty, -1] \cup [0, 1\rangle}$

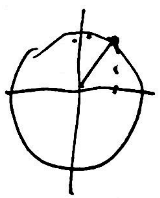



kvadrant

$\sin V = \frac{1}{4}$

$90° \leq V \leq 180°$

så $\cos V \leq 0$



Pytagores

$\cos^2 V + \sin^2 V = 1$

$\cos^2 V = 1 - \sin^2 V = 1 - \left(\frac{1}{4}\right)^2$

$= \frac{16}{16} - \frac{1}{16} = \frac{15}{16}$

$\cos V = -\sqrt{\frac{15}{16}}$

* $\underline{\cos V = -\frac{\sqrt{15}}{4}}$

$\cos(2V) = \cos^2 V - \sin^2 V = \frac{15}{16} - \frac{1}{16} = \frac{14}{16}$

* $\underline{\cos(2V) = \frac{7}{8}}$

* $\tan V = \frac{\sin V}{\cos V} = \frac{1/4}{-\sqrt{15}/4} = \underline{\frac{-1}{\sqrt{15}}}$

3

$$2\ln(x+2) - \ln(x^2) = 1$$

$$x \neq 0$$
$$x > -2.$$

$$\ln((x+2)^2) - \ln x^2 = 1$$

$$\ln(a \cdot b) = \ln a + \ln b$$
$$\ln a^r = r \ln a$$

$$\ln \frac{(x+2)^2}{x^2} = 1$$

$$e$$

$$e$$

$$\left(\frac{x+2}{x}\right)^2 = e$$

$$\frac{x+2}{x} = \pm\sqrt{e}$$

$$x+2 = \pm\sqrt{e}\,x$$

$$2 = (\pm\sqrt{e} - 1)x$$

$$x = \frac{2}{\pm\sqrt{e} - 1}$$

$$x_1 = \frac{2}{\sqrt{e} - 1} > 0$$

$$x_2 = \frac{2}{-\sqrt{e} - 1} = -\frac{2}{1+\sqrt{e}} > -1$$

$$0 > x_2 > -1$$

To løsninger

$$x_1 = \frac{2}{\sqrt{e} - 1}$$

$$x_2 = -\frac{2}{\sqrt{e}+1}$$

4 a) $\int \underbrace{x}_{u} \underbrace{\sin \pi x}_{v'} \, dx$

$u = x$ $\quad$ $v' = \sin \pi x$

$u' = 1$ $\quad$ $v = \dfrac{-\cos(\pi x)}{\pi}$

$$= u \cdot v - \int u' v \, dx$$

$$= x \cdot \frac{-\cos(\pi x)}{\pi} - \int 1 \cdot \frac{-\cos(\pi x)}{\pi} \, dx$$

$$= x \; \frac{-1}{\pi} \cos(\pi x) + \frac{1}{\pi} \int \cos(\pi x) \, dx$$

$$= \underline{-\frac{x}{\pi} \cos(\pi x) + \frac{1}{\pi^2} \sin(\pi x) + C}$$

b) $\int 3 \frac{1}{\sqrt{e^x}} dx + \int \frac{4x}{3x^2+1} dx = 3 \int e^{-x/2} dx + 4 \int \frac{x}{3x^2+1} dx$

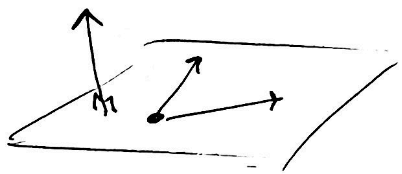
$u = 3x^2+1$

$u' = 6x$

$$= 3 \; \frac{e^{-x/2}}{-1/2} + 4 \int \frac{(1/6) u'}{u} dx$$

$$= -6 e^{-x/2} + \frac{4}{6} \int \frac{du}{u} = \underline{-6 e^{-x/2} + \frac{2}{3} \ln|3x^2+1| + C}$$

5 a) $2x + 3y - 5z = 7$




Parametrisering

Normalvektor : $\vec{n} = [2, 3, -5]$.

$\vec{V_1} = [3, -2, 0]$

$\vec{V_2} = [5, 0, 2]$

ikke parallelle

$\vec{V_1}, \vec{V_2} \perp \vec{n}$

så $\vec{V_1}$ og $V_2$ ligger i planet.

Punkt i planet $P(1, 0, -1)$

$[x, y, z]$ ligger i planet $\Leftrightarrow$

$[x, y, z] = \vec{OP} + s\vec{V_1} + t\vec{V_2}$

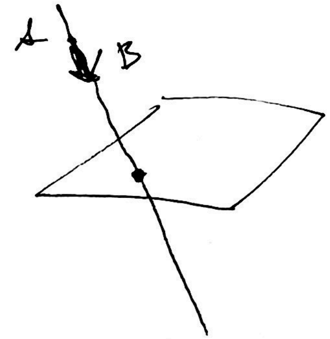
$[x, y, z] = [1, 0, -1] + s[3, -2, 0] + t[5, 0, 2]$

Parametrisering.

$$\begin{aligned} x &= 1 + 3s + 5t \\ y &= -2s \\ z &= -1 + 2t \end{aligned}$$

$s, t \in \mathbb{R}$

5 b) $A(1,2,-1)$ og $B(2,1,-1)$




$$\vec{AB} = \vec{OB} - \vec{OA}$$
$$= [2,1,-1] - [1,2,-1]$$
$$= \cancel{[1,-1,-2]} = \underline{[1,-1,0]}$$

Parametrisering av linjen gjennom A og B

$$[x,y,z] = [1,2,-1] + s[1,-1,0].$$
$$= [1+s,\ 2-s,\ -1]$$

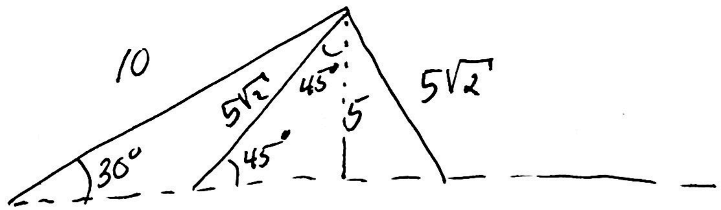
setter vi inn i likningen $2x+3y-5z=7$ for planet.

$$2(1+s) + 3(2-s) - 5(-1) = 7$$
$$2+2s + 6-3s + 5 = 7$$
$$-s + 2+6+5-7 = 0$$
$$-s + 6 = 0$$
$$\underline{s = 6}$$

snittpunktet $[x,y,z] = [1,2,-1] + 6[1,-1,0] = [7,-4,-1]$

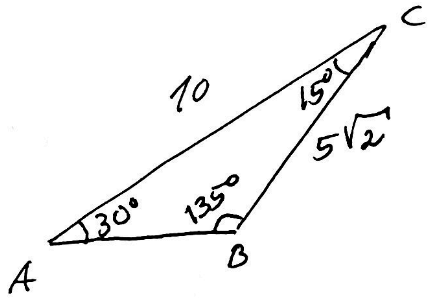
$$(x,y,z) = (\cancel{7,4}, \quad \underline{(7,-4,-1)}$$

6




To løsninger

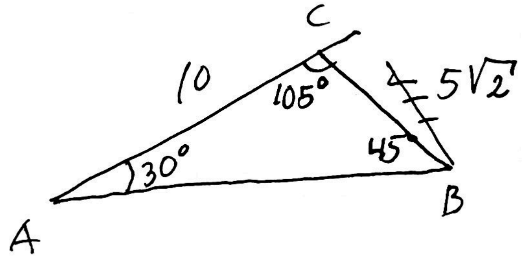
I




$$\frac{AB}{\sin 15^\circ} = \frac{AC}{\sin B} = \frac{10}{1/\sqrt{2}} = 10\cdot\sqrt{2}$$

$$AB = 10\sqrt{2}\cdot\sin 15^\circ \sim \underline{3.66}$$

II




$$\frac{AB}{\sin 105} = \frac{AC}{\sin B} = 10\cdot\sqrt{2}$$

$$AB = 10\sqrt{2}\sin(105) = \underline{13.66}$$

# 7

$7 + 14 + 21 + \cdots \qquad 980 + 987 + 994$

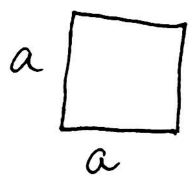
$7 \cdot 140 \qquad \underline{7 \cdot 142}$

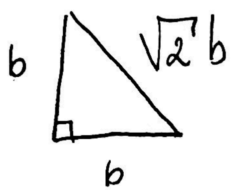
$2 \times 7 \cdot 70 = 490 \times 2 = 980$

$7(1 + 2 + 3 + \cdots + 142) = 7\,\dfrac{142 \cdot 143}{2}$

$= \underline{7 \cdot 71 \cdot 143} = \underline{71071}$

# 8




$4a + (2+\sqrt{2})b = 1\,(m)$

Areal $\quad a^2 + \dfrac{b^2}{2}$. minst mulig.

$a = \frac{1}{4}(1 - (2+\sqrt{2})b)$

$A(b) = \frac{1}{16}(1 - (2+\sqrt{2})b)^2 + \dfrac{b^2}{2}$.

$A'(b) = \frac{1}{16}\,2(1 - (2+\sqrt{2})b)(-(2+\sqrt{2})) + b = 0 \qquad 1 \cdot 8$

$-(2+\sqrt{2})(1 - (2+\sqrt{2})b) + 8b = 0$

$-(2+\sqrt{2}) + (2+\sqrt{2})^2 b + 8b = 0$

$((2+\sqrt{2})^2 + 8)b = 2 + \sqrt{2}$.

$\left( \begin{array}{l} (2+\sqrt{2})^2 \\ 4 + 4\sqrt{2} + 2 \\ = 6 + 4\sqrt{2} \end{array} \right)$

$\underline{B = \dfrac{2+\sqrt{2}}{(2+\sqrt{2})^2 + 8}}$

strengbiten til $\Delta$ må ha lengde

$(2+\sqrt{2})B = \underline{(2+\sqrt{2})^2 / ((2+\sqrt{2})^2 + 8)}$.