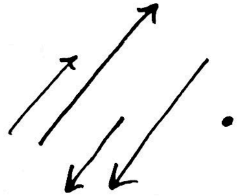
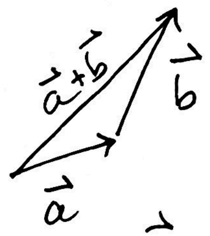
12 mars
25

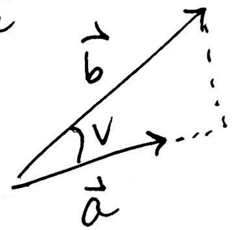
$\vec{a}$  $\cdot\vec{0}$

skalere  $\cdot$

parallelle vektorer

addere





Skalarprodukt
prikk-produkt

$\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}|\cos(v)$

$[x_1, y_1] \cdot [x_2, y_2] = x_1 x_2 + y_1 y_2$

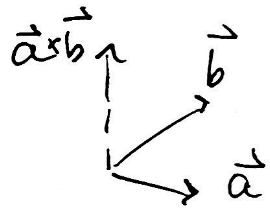
Kryssprodukt
vektorprodukt

$|\vec{a} \times \vec{b}| = |\vec{a}||\vec{b}| \sin v$

$\vec{a}, \vec{b}, \vec{a} \times \vec{b}$ høyrehåndsystem

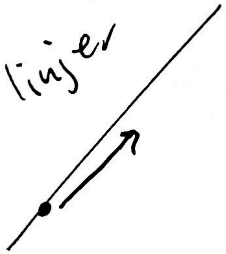
$\vec{a} \times \vec{b}$ ortogonal $\vec{a}$ og $\vec{b}$

$\vec{a} \times \vec{b} = \vec{0} \Leftrightarrow \vec{a}, \vec{b}$ parallelle



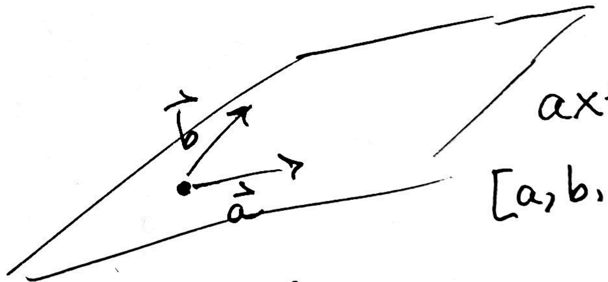
i $\mathbb{R}^3$

linjer



2 likninger

parametriseri-
1 parameter



1 likning

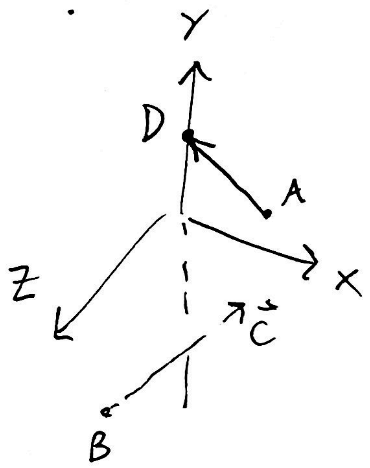
$ax + by + cz = d$

$[a, b, c]$ normalvektor

parametrisere
2 parametre

V2024

d) Finn D på y-aksen slik at $\vec{AD} \perp \vec{BC}$



$A(1, 2, -1)$ $\qquad$ $D(0, d, 0)$

$\vec{BC} = [2, 3, 4]$

$\vec{AD} = \vec{OD} - \vec{OA}$ er ortogonal til $\vec{BC}$

når $\vec{AD} \cdot \vec{BC} = 0 \iff \vec{OD} \cdot \vec{BC} = \vec{OA} \cdot \vec{BC}$

$\left((\vec{OD} - \vec{OA}) \cdot \vec{BC} = \vec{0} \right.$

$\left. \vec{OD} \cdot \vec{BC} - \vec{OA} \cdot \vec{BC} = \vec{0} \right)$

$3d = 1 \cdot 2 + 2 \cdot 3 + (-1)4$

$3d = 4$

$d = 4/3$

Koordinaten til D er $(0, \frac{4}{3}, 0)$

e) Finn E slik at $\vec{AE} = 3\vec{AB} - 2\vec{BC}$ $\qquad$ $\left(\vec{AB} + \vec{BC} = \vec{AC} \text{ etc.} \right)$

$(\vec{OE} - \vec{OA}) = 3(\vec{OB} - \vec{OA}) - 2(\vec{OC} - \vec{OB})$

$\vec{OE} = \vec{OA} + 3\vec{OB} - 3\vec{OA} - 2\vec{OC} + 2\vec{OB}$

$= -2\vec{OA} + 5\vec{OB} - 2\vec{OC}$ etc.

V2023

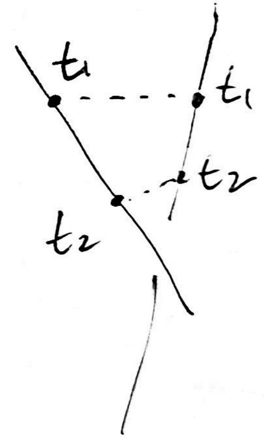
a) $\vec{s}_1(t) = \overbrace{[20, 10, 5]}^{\vec{v}(t)}_{\text{km/min}} t + [4, 7, 0]$ km

Finn farten i km/time. $|\vec{v}(t)| = |5[4,2,1]|$

$= 5\sqrt{4^2+2^2+1^2}$ km/min $= 5\sqrt{21}$ km/min $\cdot \frac{60 \text{ min}}{1 \text{ time}}$

$= \underline{300\sqrt{21} \text{ km/time}} \sim 1374$ km/t

c) $\vec{s}_2(t) = [30, -20, 10]t + [-1, -3, 0]$

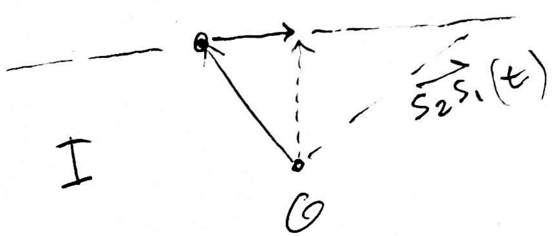


$\overrightarrow{S_2S_1} = \overrightarrow{OS_1} - \overrightarrow{OS_2} = [-10, 30, -5]t + [5, 10, 0]$

når er $\overrightarrow{S_2S_1}$ minst når

$\overrightarrow{S_2S_1}(t) \perp 5[-2, 6, -1]$

I



Alternativ

II $\frac{d}{dt} |\overrightarrow{S_2S_1}(t)| = 0 \ldots$

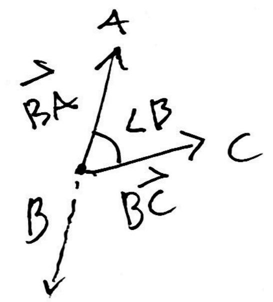
$$\overrightarrow{S_2S_1} \cdot [-2,6,-1] = 0$$

$$5[-2,6,-1] \cdot [-2,6,-1]\, t + 5[1,2,0] \cdot [-2,6,-1] = 0$$

$$5\left(\left((-2)^2 + 6^2 + (-1)^2\right) t + (-2+12)\right) = 0$$

$$41t + 10 = 0$$

$$t = \frac{-10}{41}$$



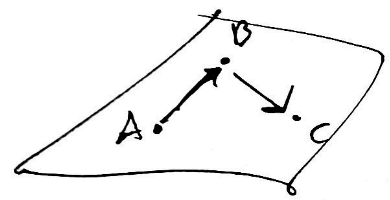
8

a)

lengden AB er $|\overrightarrow{AB}|$

BC $|\overrightarrow{BC}|$

$$\cos(\angle B) = \frac{\overrightarrow{BA} \cdot \overrightarrow{BC}}{|\overrightarrow{BA}|\,|\overrightarrow{BC}|} = -\frac{\overrightarrow{AB} \cdot \overrightarrow{BC}}{\sim}$$



b) Plan gjennom A, B og C

Normalvektor

$[a, b, c] = \overrightarrow{BC} \times \overrightarrow{AB}$

$$ax + by + cz = d$$

sett inn en av koordinatene til A, B eller C for å bestemme d.

$\vec{a} = [-2, 2, 3]$ $\vec{b} = [6, 3, -3] = 3[2, 1, -1]$

a) Finn en likning for planet gjennom $P(3, 7, 11)$ som inneholder $\vec{a}$ og $\vec{b}$. ($\vec{a}, \vec{b}$ er parallelle til planet)

$$\vec{a} \times \vec{b} = 3\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -2 & 2 & 3 \\ 2 & 1 & -1 \end{vmatrix} = 3\left[\begin{vmatrix} 2 & 3 \\ 1 & -1 \end{vmatrix}, -\begin{vmatrix} -2 & 3 \\ 2 & -1 \end{vmatrix}, \begin{vmatrix} -2 & 2 \\ 2 & 1 \end{vmatrix}\right] = 3[-5, -(-4), -6]$$

$$= 3\underbrace{[-5, 4, -6]}_{\vec{n} \text{ normalvektor til planet}}$$

$$\underline{-5x + 4y - 6z} = \underbrace{-5(3) + 4 \cdot 7}_{-1+14} \underbrace{- 6 \cdot 11}_{-66} = \underline{-53}$$

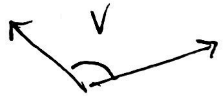
b) Finn en parametrisering for planet i a)

$$\vec{OP} + s\vec{a} + t\vec{b}/3$$

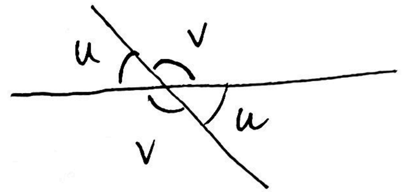
$$\begin{cases} x = 3 - 2s + 2t \\ y = 7 + 2s + t \\ z = 11 + 3s - t \end{cases}$$

Hint til oblig 6

1.



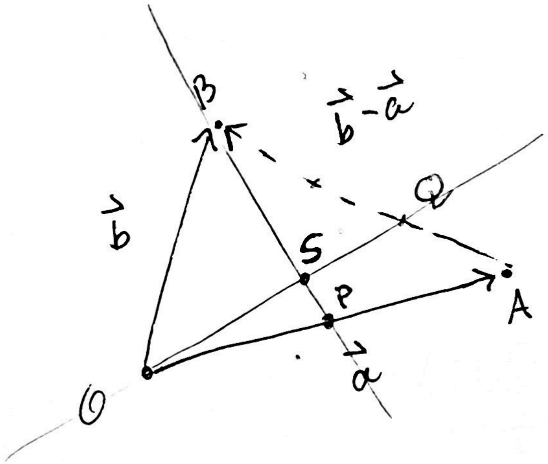
$v \in [0^\circ, 180^\circ]$



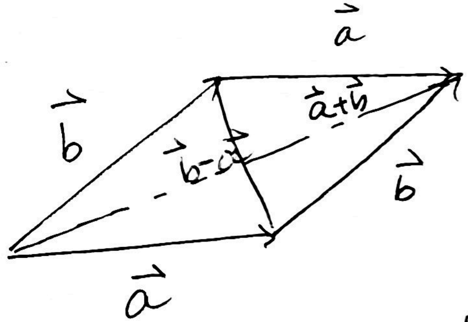
$u + v = 180^\circ$

$u \in [0, 90^\circ]$

3.

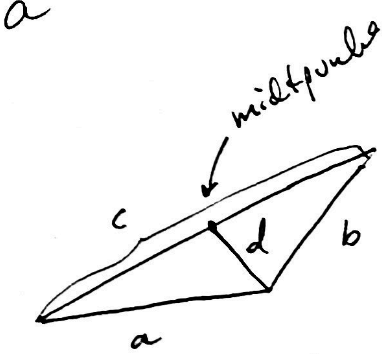


4



$\vec{d_1} = \vec{b} - \vec{a}$

$\vec{d_2} = \vec{b} + \vec{a}$



$2d^2 + \frac{c^2}{2} = a^2 + b^2$

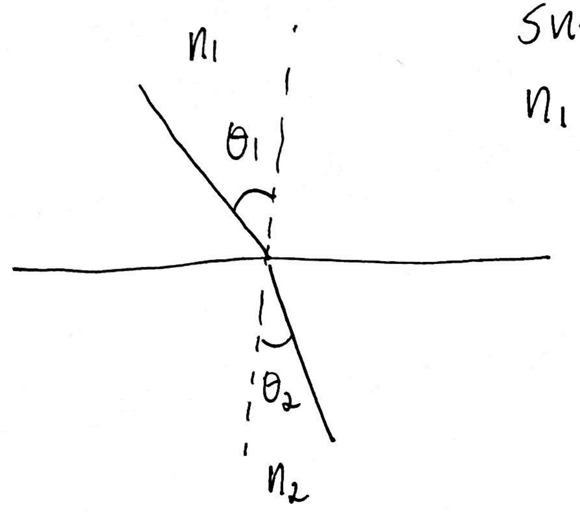
+1

11 Benytt parametrisering...

12

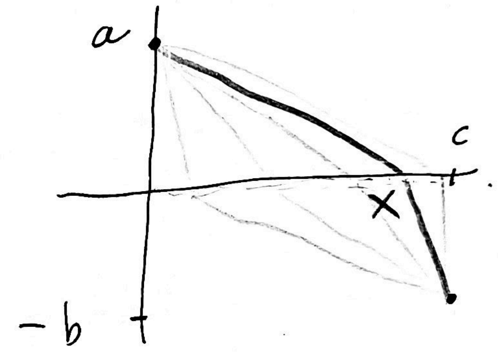
$\frac{c}{n_1}$

$\frac{c}{n_2}$



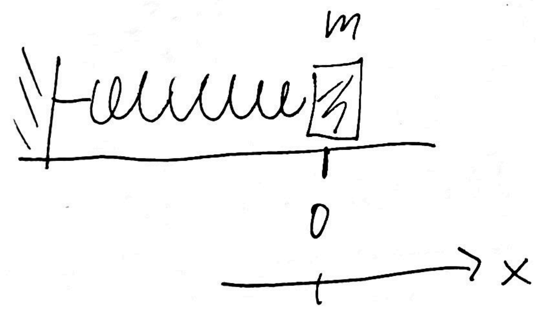
Snell

$n_1 \sin\theta_1 = n_2 \sin\theta_2$



$a, b, c > 0$

13.



$F = -kx$

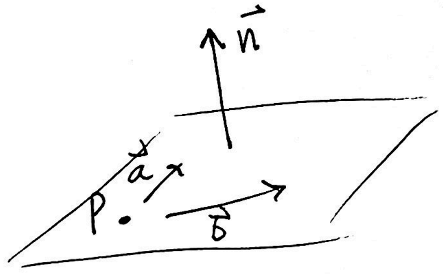
$= m x''$

$k > 0$

$m\, x''(t) = -k \cdot x$

$\underline{x''(t) + \frac{k}{m} x(t) = 0}$

Plan $\quad 2x - 5y + 3z = 11$



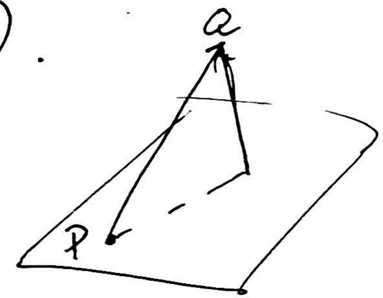
a) Finn en parametrisering av planet.

$\vec{n} = [2, -5, 3]$ $\quad$ $P(4,0,1)$ ligger i planet.

Behøver $\vec{a} \nparallel \vec{b}$ som er ortogonale til $\vec{n}$: $\quad \vec{a} = [5,2,0]$, $\quad \vec{b} = [3,0,-2]$

Parametrisering : $[x,y,z] = \underbrace{[4,0,1]}_{\vec{OP}} + s\underbrace{[5,2,0]}_{\vec{a}} + t\underbrace{[3,0,-2]}_{\vec{b}}$.

b) Finn avstanden fra planet til punktet $Q(3,0,5)$.



$\vec{PQ} = [3,0,5] - [4,0,1] = [-1,0,4]$

$$\frac{|\vec{PQ} \cdot \vec{n}|}{|\vec{n}|} = \left| \frac{[-1,0,4] \cdot [2,-5,3]}{\sqrt{2^2 + (-5)^2 + 3^2}} \right| = \frac{|-2+12|}{\sqrt{38}} = \underline{\frac{10}{\sqrt{38}}}$$

+1