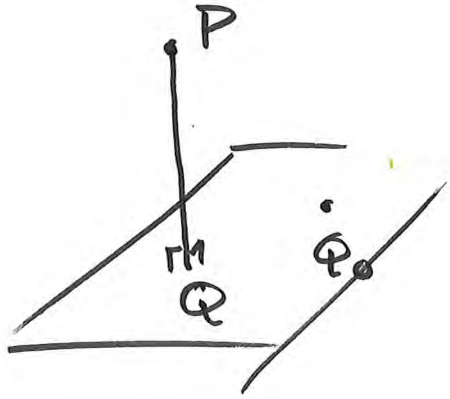
11 mars
25

13P



$$3x - y + 3z = 7$$

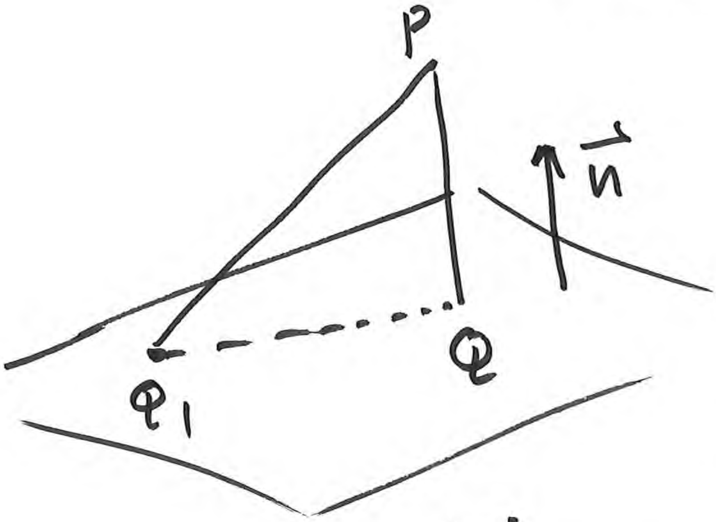
$P(3, 4, 2)$

En normalvektor til planet er

$$\vec{n} = [3, -1, 3]$$

Et punkt i planet er $Q_0(0, -7, 0)$

et annet $Q_1(3, 2, 0)$



$\overrightarrow{QP}$ er komponenten $\overrightarrow{Q_1P_{\parallel}}$ til $\overrightarrow{Q_1P}$ langs $\vec{n}$

$$\overrightarrow{Q_1P_{\parallel}} = \frac{\overrightarrow{Q_1P} \cdot \vec{n}}{|\vec{n}|^2}\vec{n}$$

$$\overrightarrow{Q_1P} - \overrightarrow{Q_1P_{\parallel}} = \overrightarrow{Q_1P_{\perp}}$$

står vinkelrett på $\vec{n}$ siden $\overrightarrow{Q_1P_{\perp}} \cdot \vec{n} = 0$

Avstanden mellom P og planet er lik $|\overrightarrow{Q_1P_{\parallel}}|$.

$$\overrightarrow{Q_1P} = \overrightarrow{OP} - \overrightarrow{OQ_1} = [3,4,2] - [3,2,0] = [0,2,2] = \underline{2[0,1,1]}$$

$$\vec{n} = [3,-1,3]$$

$$\overrightarrow{QP} = \frac{\overrightarrow{Q_1P} \cdot \vec{n}}{|\vec{n}|^2}\vec{n}$$

$$= \frac{2[0,1,1] \cdot [3,-1,3]}{3^2+(-1)^2+3^2}[3,-1,3]$$

$$= \frac{2(0-1+3)}{19}[3,-1,3]$$

$$\overrightarrow{QP} = \underline{\frac{4}{19}[3,-1,3]}$$

Avstanden er $\left|\frac{4}{19}[3,-1,3]\right|$

$$= \frac{4}{19}\sqrt{19} = \underline{\frac{4}{\sqrt{19}}}$$

$$\overrightarrow{OP} - \overrightarrow{OQ} = \frac{4}{19}[3,-1,3]$$

$$\overrightarrow{OQ} = \overrightarrow{OP} - \frac{4}{19}[3,-1,3]$$

$$[3,4,2] - \frac{4}{19}[3,-1,3]$$

$$= \frac{1}{19}[57-12, 76+4, 38-12]$$

$$= \frac{1}{19}[45, 80, 26]$$

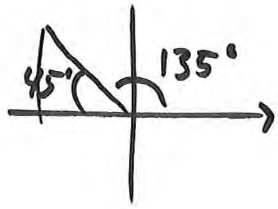
Punktet i planet nærmest P er $\underline{Q\left(\frac{45}{19}, \frac{80}{19}, \frac{26}{19}\right)}$

1 $\quad \vec{a} = [-1, 2, 2] \qquad \vec{b} = [1, 0, -1]$

$$\vec{a} \cdot \vec{b} = (-1)\cdot 1 + 2\cdot 0 + 2\cdot(-1)$$
$$= \underline{-3}$$

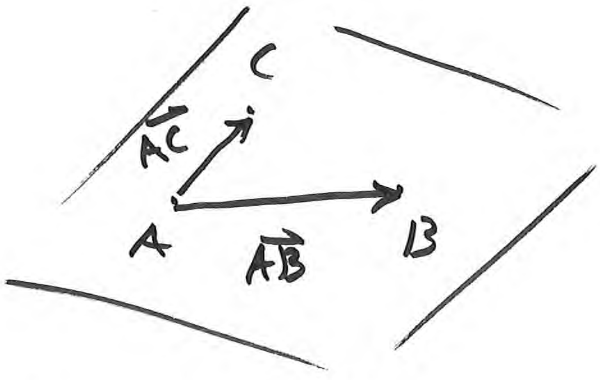
$$\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}| \cos(v)$$
$$|\vec{a}| = \sqrt{(-1)^2 + 2^2 + 2^2} = \sqrt{9} = 3$$
$$|\vec{b}| = \sqrt{1^2 + 0^2 + (-1)^2} = \sqrt{2}$$

$$\cos v = \frac{-3}{3 \cdot \sqrt{2}} = \frac{-1}{\sqrt{2}}$$



$$\underline{v = 135°}$$

2. $\quad A(0,0,2)$
$\quad B(1,-2,3)$
$\quad C(4,1,1)$



$$\vec{AB} = \vec{OB} - \vec{OA} = [1,-2,3] - [0,0,2]$$
$$= [1,-2,1]$$
$$\vec{AC} = \underline{\overline{[4,1,-1]}}$$

$$\vec{AB} \times \vec{AC} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & -2 & 1 \\ 4 & 1 & -1 \end{vmatrix}$$

normal til planet

$$= [1, -(-5), 9] = \underline{[1, 5, 9]}$$

planet er på formen

$$1 \cdot x + 5 \cdot y + 9 \cdot z = d$$

setter inn koordinatene

til $A(0,0,2)$ for å bestemme d.

$$0 + 0 + 9 \cdot 2 = d \quad \text{så} \quad d = 18$$

$$x + 5y + 9z = 18$$

Arealet til ΔABC

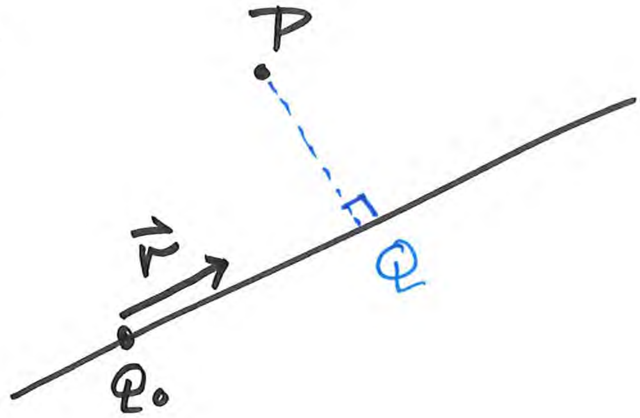
er lik $\quad \frac{1}{2} \left| \vec{AB} \times \vec{AC} \right|$

$$= \frac{1}{2} \left| [1, 5, 9] \right|$$

$$= \frac{1}{2} \sqrt{1^2 + 5^2 + 9^2}$$

$$= \frac{\sqrt{107}}{2}$$

Avstand mellom punkt og linje



$\vec{QP}$ er kortest når $\vec{QP} \perp \vec{r}$.

$\vec{QP} = \vec{OP} - \vec{OQ}$

$$\Updownarrow$$

$$\vec{QP} \cdot \vec{r} = 0$$

$$\Leftrightarrow \vec{OP} \cdot \vec{r} = \vec{OQ} \cdot \vec{r}$$

eksempel:

linje $[x, y, z] = [-1, 2, 2]\,t + [4, 3, 2]$

parametrisering

$P(5, 3, 1)$.

$$\vec{OP} = [5, 3, 1]$$
$$\vec{OQ} = [4-t,\ 3-2t,\ 2-2t]$$
$$\vec{r} = [-1, 2, 2]$$

$$\vec{OP} \cdot \vec{r} = [5, 3, 1] \cdot [-1, 2, 2] = -5 + 6 + 2 = 3$$

$$\vec{OQ} \cdot \vec{r} = -1(4-t) + 2(3-2t) + 2(2-2t)$$
$$= -4 + 6 + 4 + t - 4t - 4t$$
$$= 6 + -7t$$

$$\vec{OP} \cdot \vec{r} = \vec{OQ} \cdot \vec{r}$$
$$6 + (-7)t = 3$$
$$-7t = 3 - 6 = -3$$
$$\underline{t = 3/7}$$

Punktet på linjen nærmest P er

$$\vec{OQ} = [-1, 2, 2]\frac{3}{7} + [4, 3, 2]$$

$$= \left[4 - \frac{3}{7},\ 3 + \frac{6}{7},\ 2 + \frac{6}{7}\right]$$

$$= \frac{1}{7}[28 - 3,\ 21 + 6,\ 14 + 6]$$

$$= \frac{1}{7}[25, 27, 20]$$

$$\underline{Q\left(\frac{25}{7}, \frac{27}{7}, \frac{20}{7}\right)}$$

Korteste avstand er lengden til vektor

$$\vec{QP} = \vec{OP} - \vec{OQ}$$

$$= [5, 3, 1] - \left([4, 3, 2] + \frac{3}{7}[-1, 2, 2]\right)$$

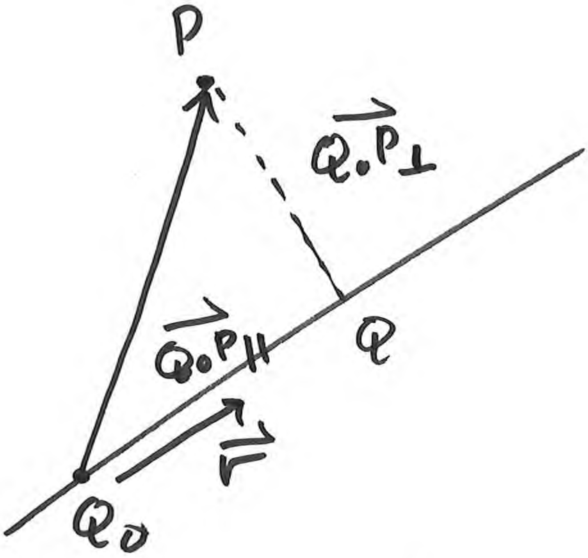
$$= [1, 0, -1] - \left[-\frac{3}{7}, \frac{6}{7}, \frac{6}{7}\right]$$

$$= \left[\frac{10}{7}, \frac{-6}{7}, -\frac{13}{7}\right]$$

$$|\vec{QP}| = \frac{1}{7}\sqrt{10^2 + 6^2 + 13^2}$$

$$= \frac{1}{7}\sqrt{100 + 36 + 169}$$

$$= \underline{\underline{\frac{1}{7}\sqrt{305}}}$$

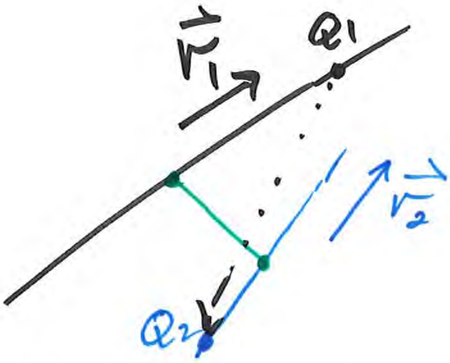


$$\overrightarrow{Q_0P_\perp} = \overrightarrow{Q_0P} - \frac{\overrightarrow{Q_0P} \cdot \vec{r}}{|\vec{r}|^2}\vec{r}.$$

Hvordan finne korteste avstand mellom to linjer.

parallelle

mange par som realiserer korteste avstand.



Korteste linjestykke må være ortogonalt til både $\vec{r}_1$ og $\vec{r}_2$ og må derfor være parallell til $\underline{\vec{r}_1 \times \vec{r}_2} = \vec{n}$

anta $\vec{r}_1$ og $\vec{r}_2$ ikke er parallelle

$$\left| \frac{\overrightarrow{Q_1Q_2} \cdot \vec{n}}{|\vec{n}|} \right|$$ er korteste avstanden mellom linjene.