26.02
25

12G Trevektorprodukt.

2×2-determinant

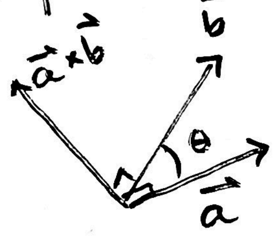
$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$

$$\left| \begin{bmatrix} a \\ c \end{bmatrix}, \begin{bmatrix} b \\ d \end{bmatrix} \right| = \begin{vmatrix} [a, b] \\ [c, d] \end{vmatrix}$$

søylevektorer radvektorer

$$\begin{vmatrix} \sqrt{2} & \sqrt{3} \\ \sqrt{3} & \sqrt{2} \end{vmatrix} = \sqrt{2}^2 - \sqrt{3}^2 = 2 - 3 = \underline{\underline{-1}}$$

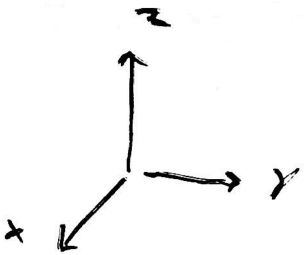
Kryss-produkt



$\vec{a} \times \vec{b}$ vinkelrett på $\vec{a}$ og $\vec{b}$.

$\vec{a}, \vec{b}, \vec{a} \times \vec{b}$ høyrehåndsystem

$|\vec{a} \times \vec{b}| = |\vec{a}||\vec{b}| \sin \theta$



$\vec{e}_1 \times \vec{e}_2 = \vec{e}_3$

$\vec{e}_1 \times \vec{e}_3 = -\vec{e}_2$

$\vec{e}_2 \times \vec{e}_3 = \vec{e}_1$

Kryssproduktet er lineært i begge vektorene.

$\vec{a} \times \vec{b} = - \vec{b} \times \vec{a}$ antisymmetrisk.

$(\vec{a} \times \vec{b}) \times \vec{c} \neq \vec{a} \times (\vec{b} \times \vec{c})$ ikke assosiativt.

eksempel:

$\vec{e}_1 \times (\vec{e}_1 \times \vec{e}_2) = \vec{e}_1 \times \vec{e}_3 = -\vec{e}_2$

$(\vec{e}_1 \times \vec{e}_1) \times \vec{e}_2 = \vec{0} \times \vec{e}_2 = \vec{0}$

Koordinatform

$$[x_1\ y_1\ z_1] \times [x_2\ y_2\ z_2] = \begin{array}{c} \begin{array}{ccc} + & - & + \end{array} \\ \begin{vmatrix} \vec{e}_1 & \vec{e}_2 & \vec{e}_3 \\ x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \end{vmatrix} \end{array}$$

$$= \begin{vmatrix} y_1 & z_1 \\ y_2 & z_2 \end{vmatrix} \vec{e}_1 - \begin{vmatrix} x_1 & z_1 \\ x_2 & z_2 \end{vmatrix} \vec{e}_2 + \begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \end{vmatrix} \vec{e}_3$$

Finn alle vektorer som er vinkelrette til

$$[1, 0, 2] \text{ og } [7, -14, 21] = 7[1, -2, 3]$$

En normal vektor er

$$[1, 0, 2] \times [1, -2, 3] = \begin{vmatrix} \vec{e_1} & \vec{e_2} & \vec{e_3} \\ 1 & 0 & 2 \\ 1 & -2 & 3 \end{vmatrix}$$

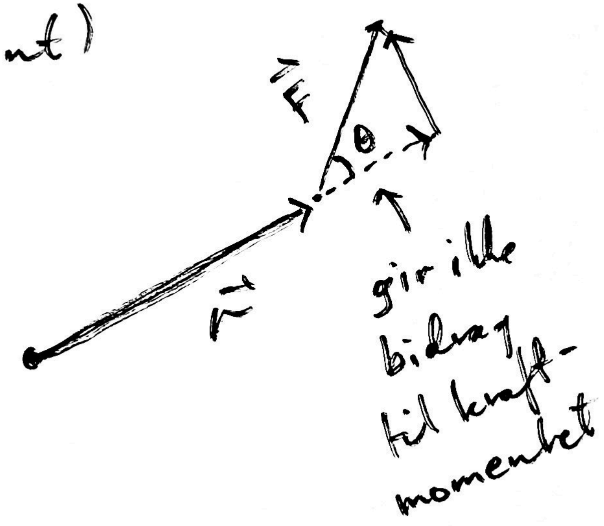
$$= \left[\begin{vmatrix} 0 & 2 \\ -2 & 3 \end{vmatrix}, - \begin{vmatrix} 1 & 2 \\ 1 & 3 \end{vmatrix}, \begin{vmatrix} 1 & 0 \\ 1 & -2 \end{vmatrix} \right]$$

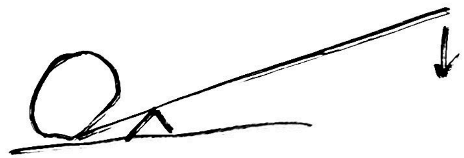
$$= \underline{[4, -1, -2]}$$

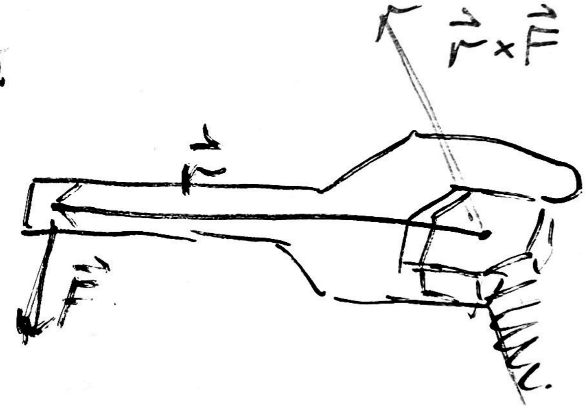
Alle normalvektorene er $k[4, -1, -2]$, $k \in \mathbb{R}$

Kraftmoment
(Dreiemoment)

$\vec{r} \times \vec{F}$







impuls (eng. momentum)

$\vec{p} = m\vec{v}$ masse · hastighet.

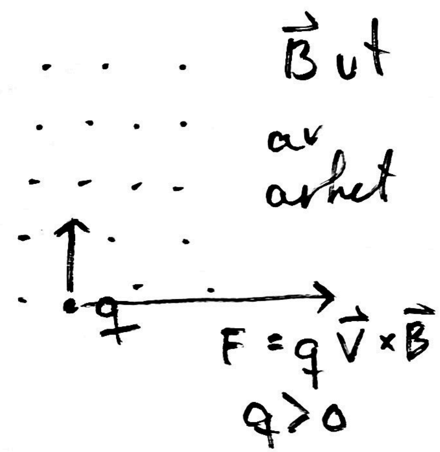
impulsmoment $\vec{r} \times \vec{p}$

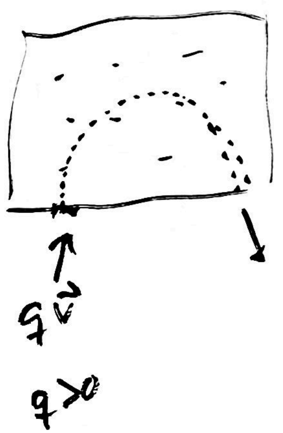
Kraft på en ladd partikkel som beveger seg i et magnetfelt

$$\vec{F} = q\,\vec{v} \times \vec{B}$$ ← magnetfelt.

q: elektrisk ladning

$\vec{v}$: hastighet



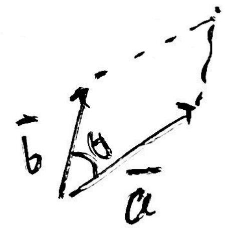


$$\vec{v} = \frac{d\vec{s}}{dt}$$

$$\Delta\vec{s} = \vec{v}\,\Delta t$$

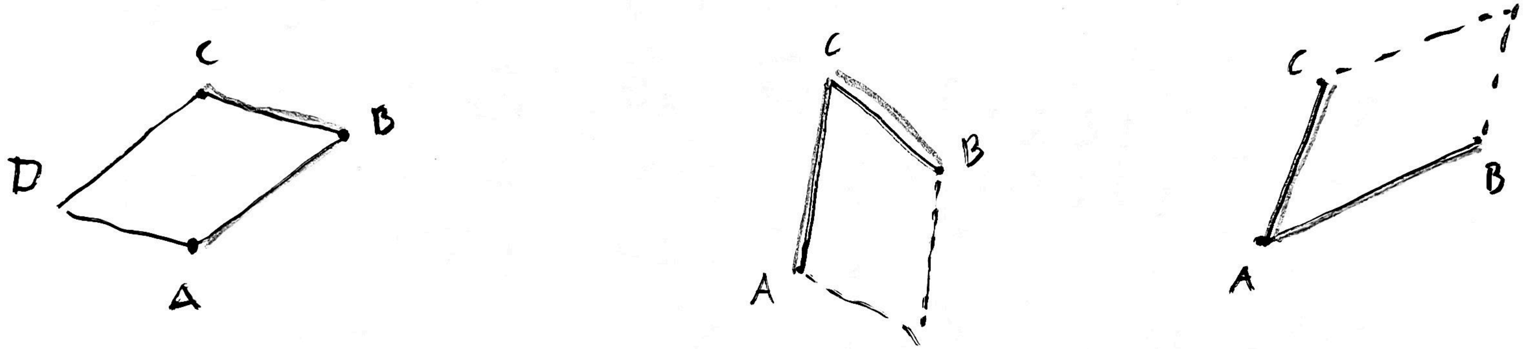
$$\Delta\vec{s} \cdot \vec{F} = \vec{0}$$

Det utføres ikkje noe arbeid.



$$|\vec{a}\times\vec{b}| = |\vec{a}|\,|\vec{b}|\sin\theta$$

arealet til parallellogrammet utspent av $\vec{a}$ og $\vec{b}$



Tre parallellogram som har A, B og C som hjørner.

Arealene er de samme for alle 3, (2 · areal til ΔABC)!

$\vec{AB}$ og $\vec{AC}$ eller $\vec{CB}$ og $\vec{CA}$.

Arealet $A = \text{abs}\begin{vmatrix} \vec{e}_1 & \vec{e}_2 & \vec{e}_3 \\ & \vec{OA}-\vec{OB} & \\ & \vec{OC}-\vec{OB} & \end{vmatrix}$ eller tilsvarende med

Oppg Finn arealet til parallellogrammet som har $A(0,1,0)$ $B(2,3,3)$ $C(-1,-1,2)$ som hjørner.

$$\vec{AB} = \vec{OB} - \vec{OA} = [2,3,3] - [0,1,0] = [2,2,3]$$

$$\vec{AC} = \vec{OC} - \vec{OA} = [-1,-1,2] - [0,1,0] = [-1,-2,2].$$

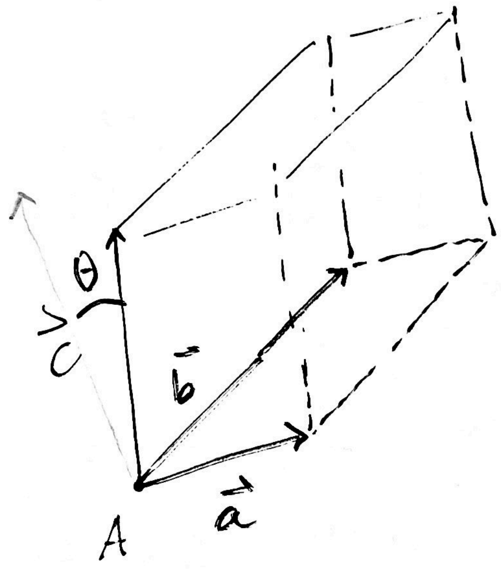
$$\vec{AB} \times \vec{AC} = \begin{vmatrix} \vec{e}_1 & \vec{e}_2 & \vec{e}_3 \\ 2 & 2 & 3 \\ -1 & -2 & 2 \end{vmatrix} = \left[\begin{vmatrix} 2 & 3 \\ -2 & 2 \end{vmatrix}, -\begin{vmatrix} 2 & 3 \\ -1 & 2 \end{vmatrix}, \begin{vmatrix} 2 & 2 \\ -1 & -2 \end{vmatrix} \right]$$

$$= \left[4-(-6), -\left(4-(-3)\right), -4-(-2) \right]$$

$$= [10, -7, -2].$$

Arealet er $|\vec{AB} \times \vec{AC}| = \sqrt{10^2 + (-7)^2 + 2^2} = \underline{\sqrt{153}}$

parallellepiped



P er i parallellepipedet

$$\vec{AP} = s_1\vec{a} + s_2\vec{b} + s_3\vec{c}$$

$$0 \le s_1, s_2, s_3 \le 1$$

Volum = Areal grunnflate · høyden

$$V = |(\vec{a}\times\vec{b})\cdot\vec{c}| = \underbrace{|\vec{a}\times\vec{b}|}_{\text{areal grunnflate.}} \cdot \underbrace{|\vec{c}||\cos\Theta|}_{\text{høyden}}$$

$$= \text{abs}\begin{vmatrix}\vec{c}\\ \vec{a}\\ \vec{b}\end{vmatrix} = \text{abs}\begin{vmatrix}\vec{a}\\ \vec{b}\\ \vec{c}\end{vmatrix}$$

(Trippelproduktet) Trevektor-produktet av $\vec{a}, \vec{b}$ og $\vec{c}$ er $\begin{vmatrix}\vec{a}\\ \vec{b}\\ \vec{c}\end{vmatrix}$

Et parallellepipeder er utspent av

$\vec{a} = [-1, -2, -4]$ $\quad$ $\vec{b} = [5, 10, -5]$

$\vec{c} = [3, -9, 12]$

Finn trippelproduktet av $\vec{a}, \vec{b}$ og $\vec{c}$ og finn volumet

$$\left|\begin{matrix} \vec{a} \\ \vec{b} \\ \vec{c} \end{matrix}\right| = \left|\begin{matrix} -1 & -2 & -4 \\ 5 & 10 & -5 \\ 3 & -9 & 12 \end{matrix}\right| = (-1)5\cdot 3 \left|\begin{matrix} 1 & 2 & 4 \\ 1 & 2 & -1 \\ 1 & -3 & 4 \end{matrix}\right|$$

trekker første rad fra rad 2 og 3

$$= -15 \left|\begin{matrix} 1 & 2 & 4 \\ 0 & 0 & -5 \\ 0 & -5 & 0 \end{matrix}\right|$$

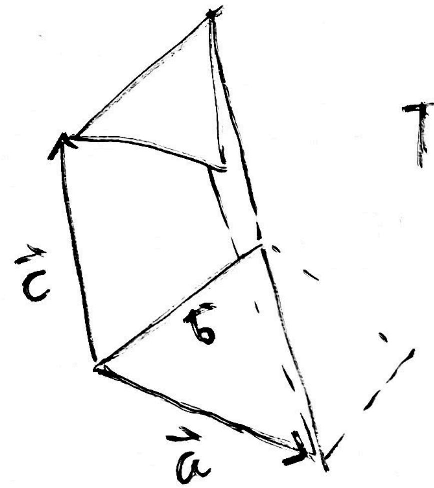
$$= -15\cdot 25 \left|\begin{matrix} 1 & 2 & 4 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{matrix}\right| = -15\cdot 25\,(-1) \overbrace{\left|\begin{matrix} 0 & 0 & 1 \\ 1 & 2 & 4 \\ 0 & 1 & 0 \end{matrix}\right|}^{1}$$

byttet rad 1 og 2.

$$= 15\cdot 25$$

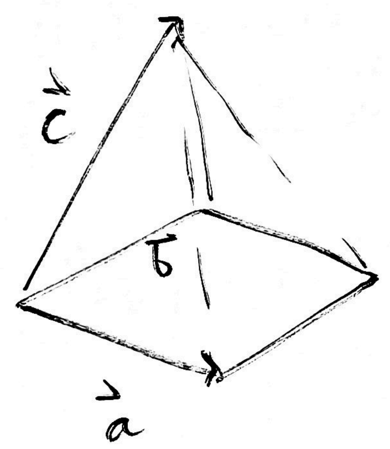
$$= 250 + 125 = \underline{375}$$

Volumet er $|375| = \underline{\underline{375}}$



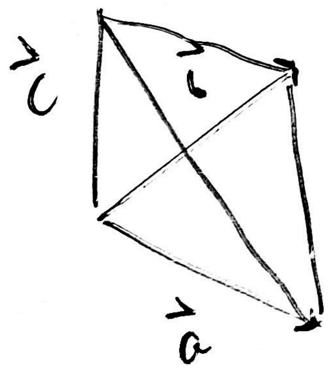
Trekant prisme

$$V = \frac{1}{2}\left|(\vec{a}\times\vec{b})\cdot\vec{c}\right|$$



Firkant pyramide

$$V = \frac{1}{3}\left|(\vec{a}\times\vec{b})\cdot\vec{c}\right|$$



Tetraeder

Volum $V = \frac{1}{6}\left|(\vec{a}\times\vec{b})\cdot\vec{c}\right|$.

ØVING

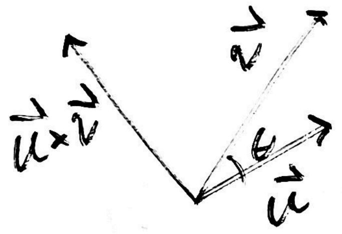
$\vec{u} \cdot \vec{v}$ en skalar $= |\vec{u}||\vec{v}|\cos\theta$

$[x_1, y_1] \cdot [x_2, y_2] = x_1 x_2 + y_1 \cdot y_2$

$\vec{e}_1 \cdot \vec{e}_1 = 1$

$\vec{e}_1 \cdot \vec{e}_2 = 0$

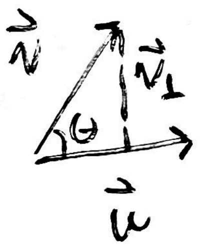
$\vec{u} \times \vec{v}$ vektor



$\vec{u}, \vec{v}$ og $\vec{u} \times \vec{v}$ høyrehandsystem

$\vec{u}$ og $\vec{v}$ $\perp$ $\vec{u} \times \vec{v}$.

$|\vec{u} \times \vec{v}| = |\vec{u}||\vec{v}_\perp| = |\vec{u}||\vec{v}|\sin\theta$.



12.99 a) $\vec{u}\cdot\vec{v} - \underbrace{\vec{v}\cdot\vec{u}}_{\vec{u}\cdot\vec{v}} = 0$

b) $\vec{u}\times\vec{v} - \underbrace{\vec{v}\times\vec{u}}_{-\vec{u}\times\vec{v}} = \vec{u}\times\vec{v} - \left(-(\vec{u}\times\vec{v})\right) = \vec{u}\times\vec{v} + \vec{u}\times\vec{v}$

$= 2\vec{u}\times\vec{v}$

c) $3\vec{u}\times(2\vec{v} - 4\vec{u}) = (3\vec{u})\times(2\vec{v}) + \underbrace{3\vec{u}\times(-4\vec{u})}_{\vec{0}} = \underline{6\vec{u}\times\vec{v}}$

parallelle

d) $(\vec{u}+\vec{v})\times(\vec{u}+\vec{v}) = \vec{0}$

like

$(\vec{u}+\vec{v})\times(\vec{u}-\vec{v}) = \underbrace{\vec{u}\times\vec{u}}_{\vec{0}} \underbrace{-\vec{v}\times\vec{v}}_{\vec{0}} - \vec{u}\times\vec{v} + \underbrace{\vec{v}\times\vec{u}}_{-\vec{u}\times\vec{v}} = \underline{-2\vec{u}\times\vec{v}}$

e) $\vec{u},\vec{v}$ er parallelle (så $\Theta = \begin{matrix} 0° \text{ samm.ret.} \\ 180° \text{ motsatt ret.} \end{matrix}$)

$\vec{u}\times\vec{v} = \vec{0}$