11.02
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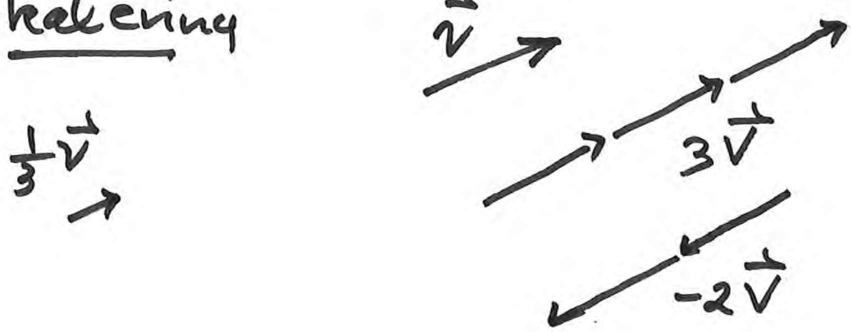
12C og D Addisjon, skalering
Parallellitet

(onsdag 12E skalarproduktet)

parallellforskyvd

like vektorer : samme retning
— størrelse.

## Skalering




$(-1)\vec{v} = -\vec{v}$
motsattvektoren

$1 \cdot \vec{v} = \vec{v}$

$0 \cdot \vec{v} = \vec{0}$

$$(a+b)\vec{v} = a\vec{v} + b\vec{v}$$
$$a(b\vec{v}) = (a \cdot b)\vec{v}$$

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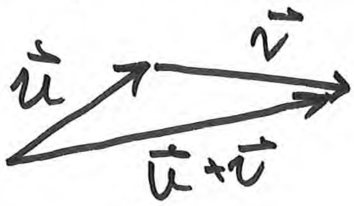
Norm $\quad |\vec{v}| \geq 0 \quad$ lengden til vektoren.

bare lik 0 for $\vec{v} = \vec{0}$.

$$|k\vec{v}| = |k| \cdot |\vec{v}| \qquad k \in \mathbb{R}$$

Trekantulikheten $\quad |\vec{u}+\vec{v}| \leq |\vec{u}| + |\vec{v}|$



En enhetsvektor har norm 1.

De svarer til retninger.

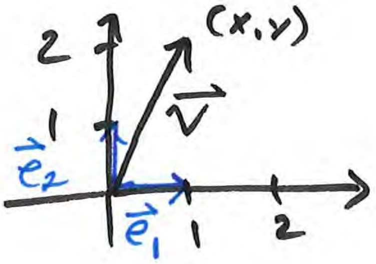
$\vec{v} \neq \vec{0}$ $\qquad \vec{v} = |\vec{v}| \cdot \dfrac{\vec{v}}{|\vec{v}|}$

$\left( |\vec{v}| \cdot \dfrac{1}{|\vec{v}|} \cdot \vec{v} \right)$

størrelse · retning.

$\dfrac{\vec{v}}{|\vec{v}|}$ enhetsvektoren tilordnet $\vec{v}$

samme retning som $\vec{v}$, lengde 1.




Euklidsk norm $|[x,y]| = \sqrt{x^2+y^2}$

$|\vec{e}_1| = 1$

$|\vec{e}_2| = 1$

$\vec{e}_1$ enhetsvektor langs 1. aksen

$\vec{e}_2$ ——— 2. aksen

$\vec{v} = [x, y] = \overrightarrow{O\,(x,y)}$

$= x \cdot \vec{e}_1 + y \vec{e}_2$

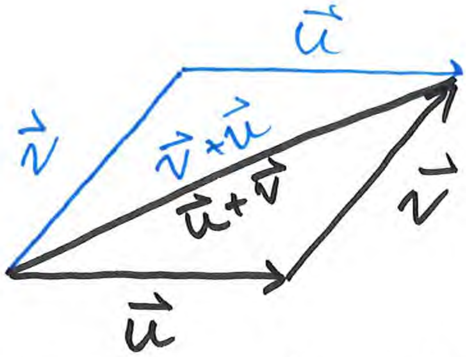
Skalering på koordinatform

$k[x,y] = [kx, ky]$

$3[1, 0, -5] = [3, 0, -15]$

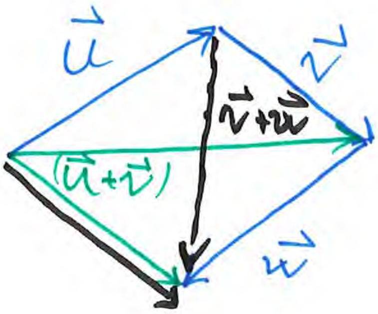
$-2[3, -4] = [-6, 8]$.

## Addisjon



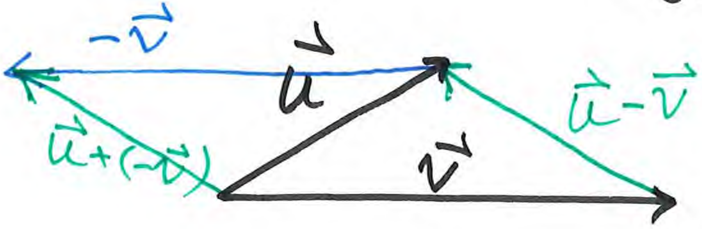

$$\vec{u}+\vec{v} = \vec{v}+\vec{u} \quad \text{(kommutativ)}$$

$$(\vec{u}+\vec{v}) + \vec{w} = \vec{u} + (\vec{v}+\vec{w})$$
(assosiativ)




## subtraksjon

$$\vec{u} - \vec{v}$$
$$= \vec{u} + (-1)\vec{v}$$




## Addisjon på koordinatform

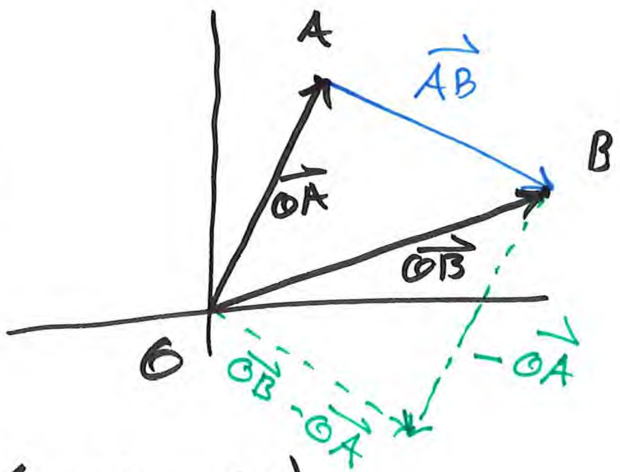
$$[x_1, y_1] + [x_2, y_2] = [x_1+x_2, y_1+y_2]$$

$$\left( x_1\vec{e}_1 + y_1\vec{e}_2 + x_2\vec{e}_1 + y_2\vec{e}_2 = (x_1+x_2)\vec{e}_1 + (y_1+y_2)\vec{e}_2 \right)$$

* $[2, 3] + [-5, 0] = [2+(-5),\ 3+0]$
$= [-3, 3]$.

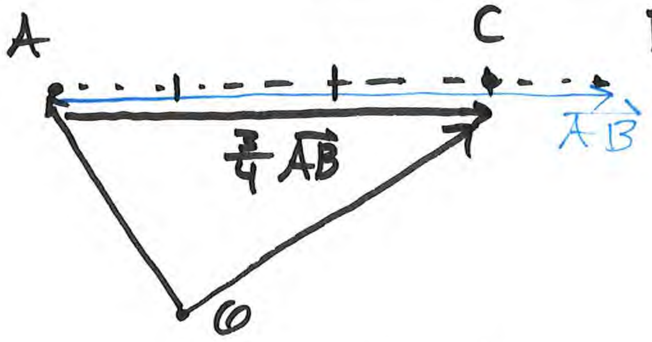
* $2[3, 5, 1] - [7, 8, 3]$
$= [6, 10, 2] + [-7, -8, -3]$
$= [6-7,\ 10-8,\ 2-3] = \underline{[-1, 2, -1]}$

$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$
$(\Leftrightarrow \overrightarrow{OA} + \overrightarrow{AB} = \overrightarrow{OB})$




$A(1, -3)$ og $B(-4, -5)$
$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$
$= [-4, -5] - [1, -3]$
$= [-4-1,\ -5+3] = \underline{[-5, -2]}$

eks. Finn punktet $C$ på linjestykke mellom $A(1,3,5)$ og $B(7,5,1)$ slik at $\underline{|AC| = 3|BC|}$




$$\vec{OC} = \vec{OA} + \frac{3}{4}\vec{AB}$$

$$\vec{AB} = \vec{OB} - \vec{OA} = [7,5,1] - [1,3,5]$$

$$= [7-1,\ 5-3,\ 1-5] = \underline{[6,\ 2,-4]}$$

$$\frac{3}{4}\vec{AB} = \frac{3}{4}[6,\ 2,\ -4] = \left[\frac{3\cdot 6}{4},\ \frac{3\cdot 2}{4},\ \frac{-4\cdot 3}{4}\right]$$

$$= \left[\frac{9}{2},\ \frac{3}{2},\ -3\right].$$

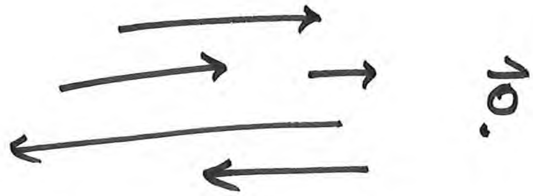
$$\vec{OC} = \vec{OA} + \frac{3}{4}\vec{AB}$$

$$= [1,3,5] + \left[\frac{9}{2},\frac{3}{2},-3\right]$$

$$= \underline{\left[\frac{11}{2},\ \frac{9}{2},\ 2\right]}$$

så $\underline{C\left(\frac{11}{2},\frac{9}{2},2\right)}$

# Parallellitet




$\vec{u}$ og $\vec{v}$ er parallelle

hvis $\vec{u} = k\vec{v}$ $\quad k, l \in \mathbb{R}$.

eller $\vec{v} = l\vec{u}$

$\vec{u}$ og $\vec{v}$ er ikke parallelle hvis

$$a\vec{u} + b\vec{v} = \vec{0}$$

bare har løsningen $a = b = 0$.

eks Bestem $t$ slik at

$[t, 3]$ og $[4, 5]$

blir parallelle.

Parallelle $\Leftrightarrow$ finnes en skalar $s$

$$[t, 3] = s[4, 5] = [4s, 5s]$$

$\Leftrightarrow$ $t = 4s$ $\quad$ (samme x og y koordinater)

$3 = 5s$

så $s = \frac{3}{5}$ og $t = 4s = 4 \cdot \frac{3}{5}$

$$t = \frac{12}{5} = \underline{2.4}$$

$\vec{u} = 2\vec{a} + 3\vec{b}$ $\qquad$ $\vec{v} = -\vec{a} + \vec{b}$

Beskriv $3\vec{u} - 4\vec{v}$ som en lineær kombinasjon av $\vec{a}$ og $\vec{b}$

Anta $\vec{a}$ og $\vec{b}$ ikke er parallelle.

$$3(2\vec{a} + 3\vec{b}) - 4(-\vec{a} + \vec{b})$$
$$= 6\vec{a} + 9\vec{b} + 4\vec{a} - 4\vec{b}$$
$$= (6+4)\vec{a} + (9-4)\vec{b}$$
$$= \underline{10\vec{a} + 5\vec{b}} = 5(2\vec{a} + \vec{b})$$

Bestem s og t slik at

$$s\vec{u} + t\vec{v} = \vec{a} + \vec{b}$$

$$s(2\vec{a} + 3\vec{b}) + t(-\vec{a} + \vec{b}) = \vec{a} + \vec{b}$$
$$(2s - t)\vec{a} + (3s + t)\vec{b} = \vec{a} + \vec{b}.$$
$$(2s - t - 1)\vec{a} + (3s + t - 1)\vec{b} = \vec{0}$$

$\vec{a}, \vec{b}$ ikke parallelle $\Rightarrow$

$$2s - t = 1$$
$$3s + t = 1$$

L1 + L2 (likning 1 etc)

$5s = 2$ så $s = \frac{2}{5}$

$t = 1 - 3s = 1 - \frac{6}{5} = \frac{-1}{5}$

$$\underline{s = \frac{2}{5} \text{ og } t = \frac{-1}{5}}$$