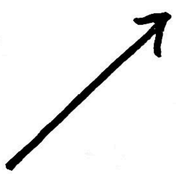
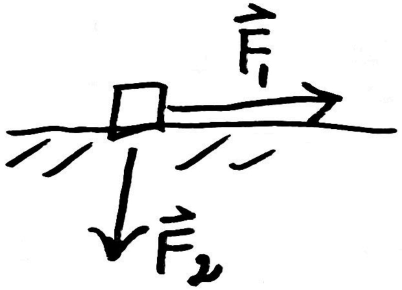
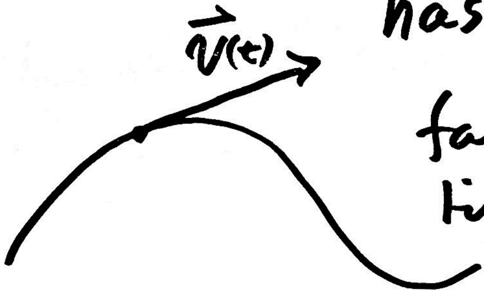
10 feb
25

12 Vektorer

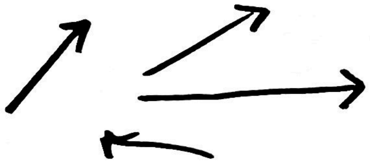


En vektor
har en størrelse (lengden til pilen)
en retning

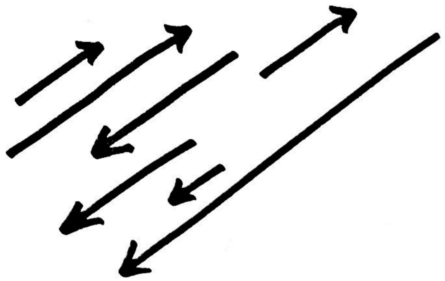




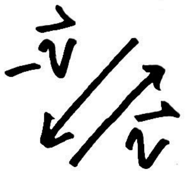
hastighet (hastighsvektor)
fart er størrelsen
til hastighetsvektor



vektorer



parallelle
vektorer



$-\vec{v}$ motsattvektor til $\vec{v}$
samme størrelse
motsatt retning.

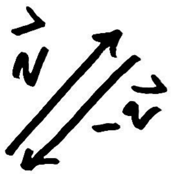
Addere vektorer

$\vec{0}$ nullvektoren . lengde lik 0

"har alle retninger"

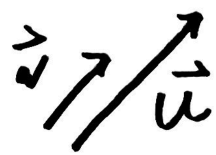
$$\vec{0}+\vec{a} = \vec{a} = \vec{0}+\vec{a}$$

Alle vektorer $\vec{v}$ er parallelle til $\vec{0}$.



$$\vec{v}+(-\vec{v}) = \vec{0}$$

Lengden til $\vec{v}$ skriver vi som $|\vec{v}|$



$$|\vec{u}| = 2\cdot|\vec{v}|$$

$$2\cdot\vec{v} = \vec{v}+\vec{v} = \vec{u}$$

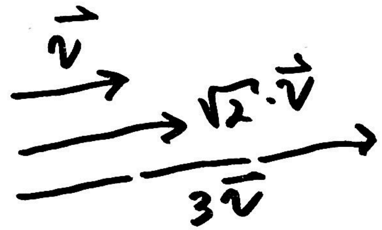
Skalering $\quad k \in \mathbb{R}$

$k\vec{v}$ skalerer $\vec{v}$ med k.

$k = 0 \quad k\vec{v} = 0\vec{v} = \vec{0}$

$k > 0 \quad k\vec{v}$ ha samme retning som $\vec{v}$

størelsen til $k\vec{v}$ er k ganget med størelsen til $\vec{v}$

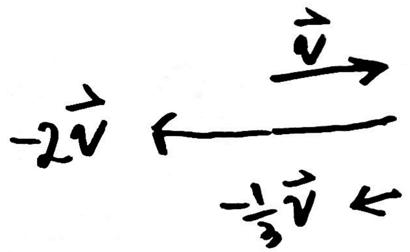


$\frac{1}{2}\vec{v}$

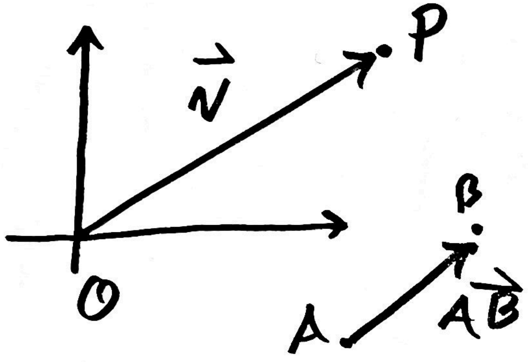
$-1 \cdot \vec{v} = -\vec{v}$ motsattvektoren

$k < 0 \quad k\cdot\vec{v}$ størelse $|k|$ ganget med størelse til $\vec{v}$

motsatt retning til $\vec{v}$



Punkt – vektorer



$\vec{OP}$ vektoren som starter i origo O og ender opp i P.

punkt $\longleftrightarrow$ vektorer

$P \longleftrightarrow \vec{OP}$

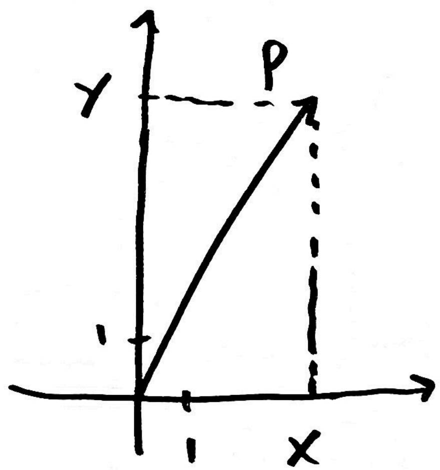
Vektorkoordinater

$P = (a, b)$

$\vec{OP} = [a, b]$ vektorkoordinatene til vektor $\vec{OP}$

(firkantparentesen)

$Q = (1, -2)$ så er $\vec{v} = \vec{OQ} = [1, -2]$.



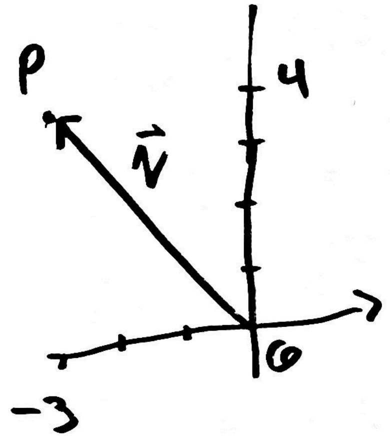
$P(x,y)$

$\vec{OP} = [x, y]$

$|\vec{OP}|$ lengden til linjestykke i xy-planet

Ved Pytagoras $|\vec{OP}|^2 = |x|^2 + |y|^2$

$|\vec{OP}| = \sqrt{x^2 + y^2}$

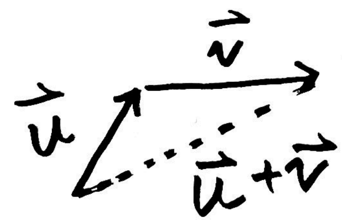


$\vec{v} = \vec{OP} = [-3, 4]$

$|\vec{v}| = \sqrt{(-3)^2 + 4^2} = \sqrt{9+16} = \underline{5}$

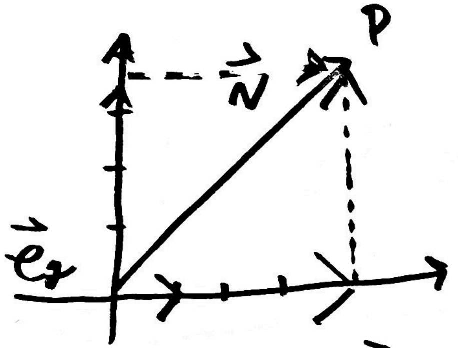
lengden (normen) til $\vec{v}$.

$|\vec{0}| = |\vec{OO}| = |[0,0]| = 0$



$|\vec{u}+\vec{v}| \leq |\vec{u}| + |\vec{v}|$

trekantulikheten.



$$\vec{e}_1 = \overrightarrow{O(1,0)} = [1,0]$$

$$\vec{e}_2 = \overrightarrow{O(0,1)} = [0,1]$$

$$\vec{v} = \overrightarrow{OP} = \overrightarrow{O(x,y)} = [x,y]$$

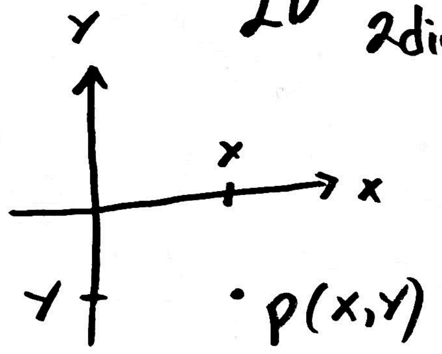
$$\vec{v} = \overrightarrow{O(x,0)} + \overrightarrow{O(0,y)}$$

$$\vec{v} = x \cdot \vec{e}_1 + y \cdot \vec{e}_2 = [x,y]$$

Alle vektorer i planet er kombinasjoner av skaleringer av $\vec{e}_1$ og $\vec{e}_2$ på en entydig måte.

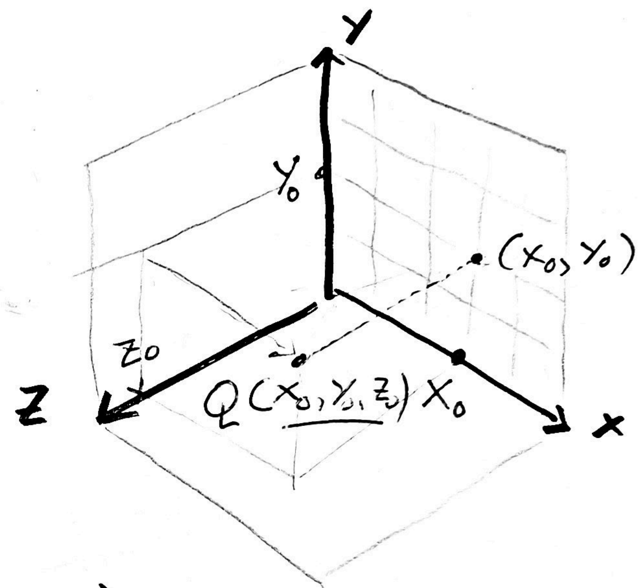
12A Koordinatsystemer.

2D 2dimensjoner



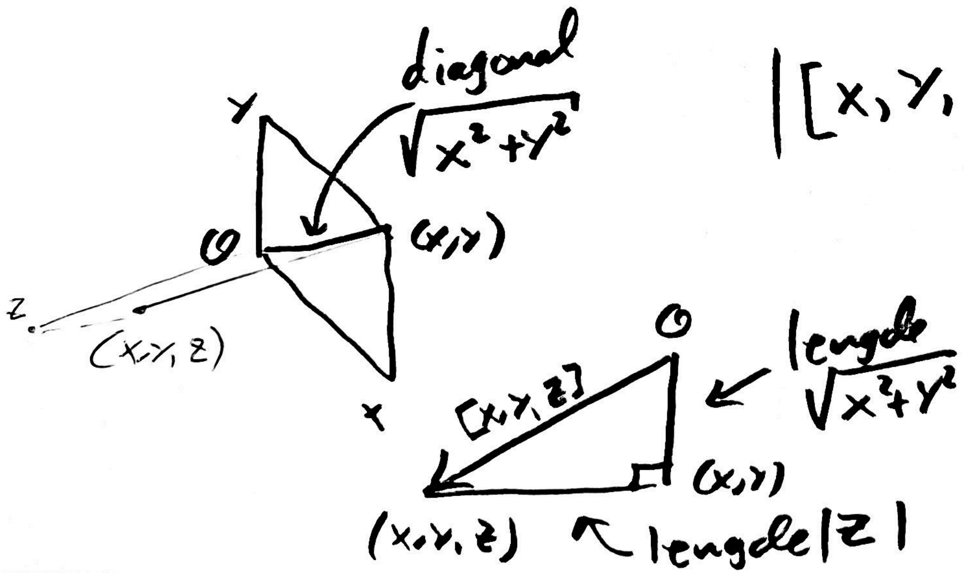
$$|[x,y]| = \sqrt{x^2+y^2}$$

3D 3dimensjoner



$$\vec{OQ} = [x_0, y_0, z_0].$$

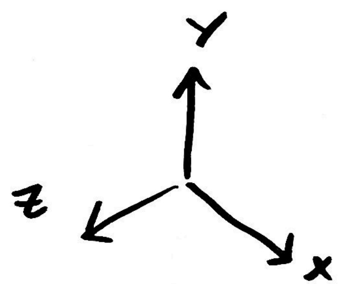
$$|[x, y, z]| = \sqrt{x^2+y^2+z^2}$$



Pyt. $|[x,y,z]|$

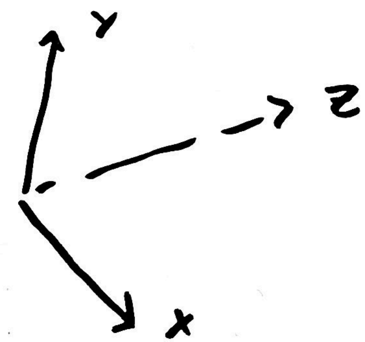
$$= \sqrt{(\sqrt{x^2+y^2})^2 + |z|^2}$$

$$= \sqrt{(x^2+y^2) + z^2}$$



x, y, z koordinatene
er et høyrehåndssystem

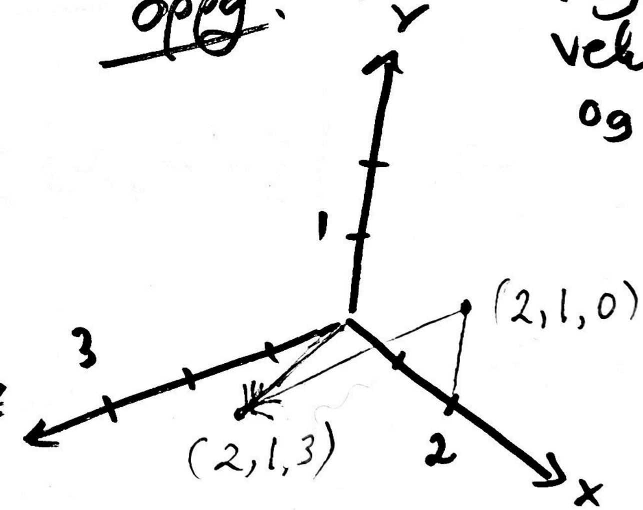
Det er og et
z, x, y høyrehåndssystem
y, z, x ———



x, y, z koordinatene
er et venstrehåndssystem.

~~oppg~~.

Tegn inn
vektoren [2,1,3]
og finn lengden



$$|[2,1,3]| = \sqrt{2^2+1^2+3^2} = \underline{\sqrt{14}}$$

Øving

$[t, 3]$ Hva må t være hvis lengden skal være lik 5.

$$|[t,3]| = \sqrt{t^2+3^2} = 5 \iff t^2+3^2 = 5^2$$

$$t^2 = 5^2 - 3^2 = 25 - 9 = 16$$

$$\underline{t = \pm 4}$$

Oppg Finn t slik at $|[2t, -t]| = a$ når

$a = 0$ $t = 0$

$a = 7$

$$\sqrt{(2t)^2 + t^2} = 7 \iff 5t^2 = 7^2 = 49$$

$$= \sqrt{(2t)^2 + (-t)^2}$$

$$t^2 = 49/5 = 9{,}8$$

$$t = \pm\sqrt{9.8} \sim \pm 3.13049\ldots$$

Bestem t slik at $|[5,-t,6]| = a$

1) $a = 10$

2) $a = 7$

$$|[5,-t,6]| = \sqrt{5^2+(-t)^2+6^2} = \sqrt{25+36+t^2} = \sqrt{61+t^2}$$

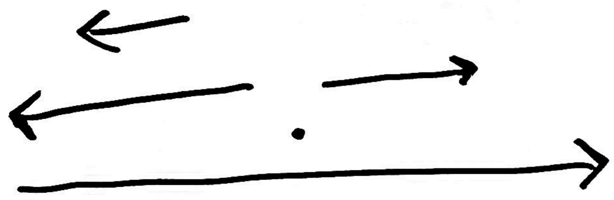
1) $a=10$:

$$61+t^2 = 10^2 = 100$$
$$t^2 = 100-61 = 39$$
$$t = \underline{\sqrt{39}} \text{ og } \underline{-\sqrt{39}}$$

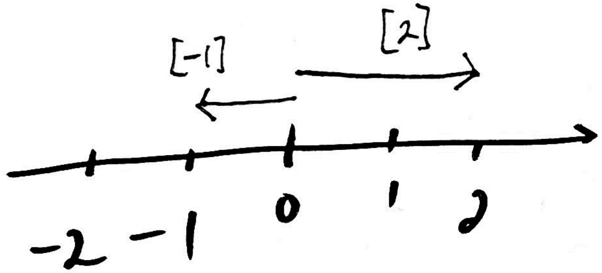
2) $a=7$

$$\sqrt{61+t^2} = 7$$
$$61+t^2 = 7^2 = 49$$
$$t^2 = 49-61 = -12.$$

<u>ingen løsning</u>



1D **én-dimensjonale vektorer.**

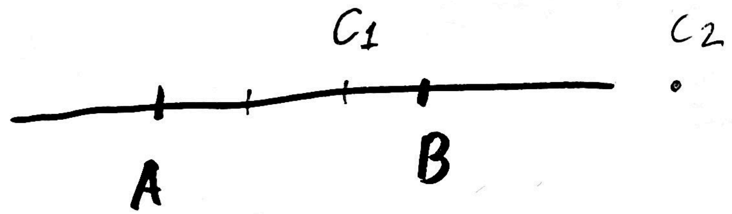


$|[a]| = |a|$ tallstørrelse

fortegnet er retningen

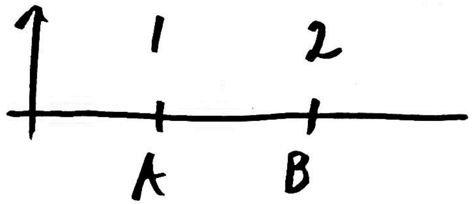
pos : til høyre

neg : til venstre.



Bestem C på linjen gjennom A og B slik at $|\vec{AC}| = 2|\vec{BC}|$.

To løsninger.



$C = [c]$, $|\vec{AC}| = |c-1| = 2|\vec{BC}|$

$= 2|2-c|$

$$|c-1| = 2|c-2|$$

$c \leq 1$ $\quad 1-c = 2(2-c) = 4-2c$

$c = 4-1 = 3$ ikke <1.

$1 \leq c \leq 2$ $\quad c-1 = 2(2-c) = 4-2c$

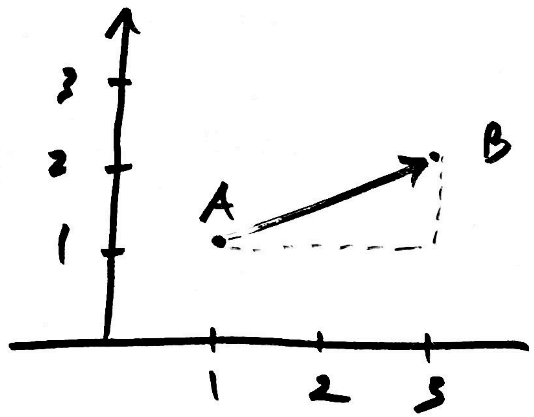
$3c = 1+4 = 5$

$c = 5/3 = \underline{1 + 2/3}$

$c \geq 2$ $\quad c-1 = 2(c-2) = 2c-4$

$\underline{c = 3}$

Løsningene er $\underline{[5/3] \text{ og } [3]}$.



$$\overrightarrow{AB} = [3-1,\ 2-1]$$
$$= [2, 1]$$

$$\overrightarrow{OA} + \overrightarrow{AB} = \overrightarrow{OB}.$$

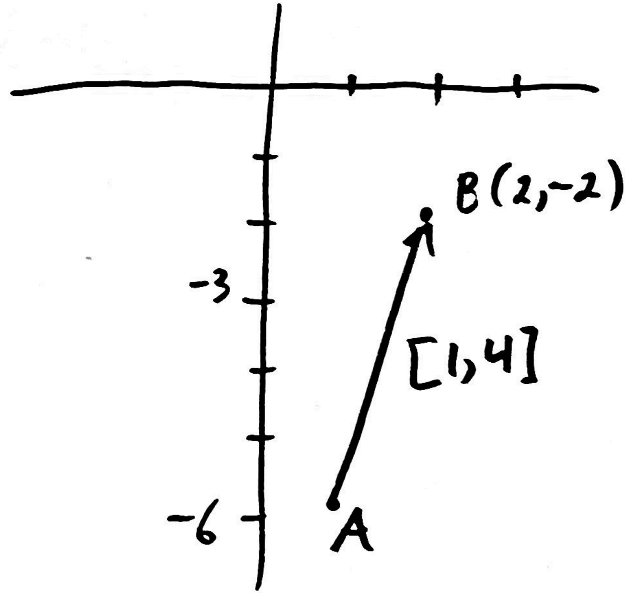
$A(1,1)$ $B(3,2)$

Oppg

$\overrightarrow{AB} = [1,4]$ og $B(2,-2)$

Hva er A?

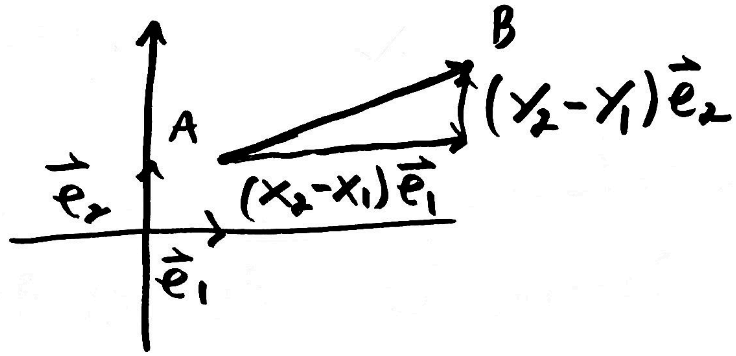
<u>$A(1,-6)$</u>



$A(x_1, y_1)$

$B(x_2, y_2)$

$\vec{AB} = [x_2 - x_1, y_2 - y_1]$



$A(1, -2, -4)$ og $B(0, 5, -7)$

$\vec{AB} = [0-1, 5-(-2), -7-(-4)]$

$= \underline{[-1, 7, -3]}$

$\vec{v}(t) = [t,\ 1-t,\ 2+t]$

For hvilke t er lengden $\vec{v}(t)$ minst mulig?

$|\vec{v}| = \sqrt{t^2+(1-t)^2+(2+t)^2}$

≥ 0

er minst mulig når

$|v|^2 = t^2+(1-t)^2+(2+t)^2$ er minst mulig.

$$\frac{d}{dt}|v(t)|^2 = 2t + 2(1-t)\underbrace{(1-t)'}_{-1} + 2(2+t)\cdot\underbrace{(2+t)'}_{1}$$

$$= 2t - 2(1-t) + 2(2+t) = 0$$

$$2t - 2 + 2t + 4 + 2t = 0$$

$$3\cdot 2\cdot t = -4-(-2) = -2$$

$$\underline{t = -1/3}.$$

Lengden er da:

$$\left|\left[\frac{1}{3}, \frac{4}{3}, \frac{5}{3}\right]\right| = \frac{1}{3}\sqrt{1^2+4^2+5^2} = \frac{1}{3}\sqrt{1+16+25} = \frac{\sqrt{42}}{3} \sim \underline{2.1602..}$$

$\vec{v}(t) = [t+1, \ t^2]$

Når er $|\vec{v}|$ minst mulig?

$|\vec{v}(t)| = \sqrt{(t+1)^2 + (t^2)^2}$ minst når $|\vec{v}(t)|^2 = (t+1)^2 + t^4$ er minst.

$$\frac{d}{dt}|\vec{v}(t)|^2 = 2(t+1)\underbrace{(t+1)'}_{1} + 4t^3 = 2(t+1) + 4t^3$$

$$= \underline{2(2t^3 + t + 1)} = 0$$

$2t^3 + t + 1 = 0.$

Vi finner løsningen til å være

$t = \underline{-0.58975\ldots}$