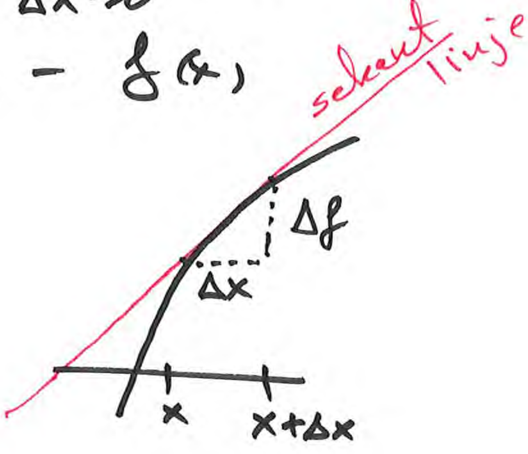
4. feb
25

Derivasjon

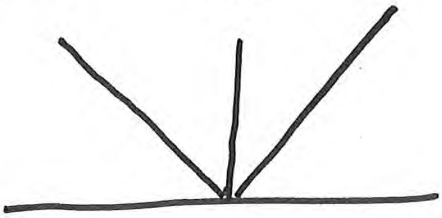
Den deriverte $f'(x) = \lim_{\Delta x \to 0} \frac{\Delta f}{\Delta x}$

hvor $\Delta f = f(x+\Delta x) - f(x)$



$f'(x)$ er stigningstallet til tangentlinjen til f i $(x, f(x))$.

Leibniz notasjonen $f'(x) = \frac{df}{dx}(x)$



* $|x| = \begin{cases} x & x \geq 0 \\ -x & x < 0 \end{cases}$

$(|x|)' = \begin{cases} 1 & x > 0 \\ -1 & x < 0 \end{cases}$

ikke deriverbar i $x = 0$.

$(|x|)' = \frac{|x|}{x}$

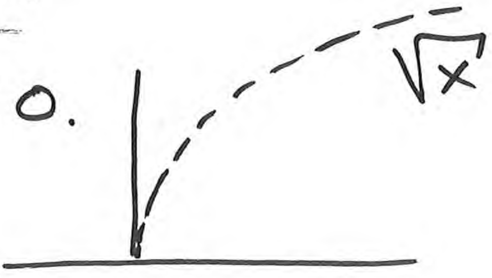
* $\sqrt{x}$

$(\sqrt{x})' = (x^{1/2})' = \frac{1}{2} x^{\frac{1}{2}-1}$

$= \frac{1}{2} x^{-1/2} = \frac{1}{2x^{1/2}}$

$(\sqrt{x})' = \frac{1}{2\sqrt{x}}$

ikke deriverbar i 0.



I går :

$$(\sin x)' = \cos x$$

$$(\cos x)' = -\sin x$$

$$(\tan x)' = \frac{1}{\cos^2 x}$$

$$= \frac{\cos^2 x + \sin^2 x}{\cos^2 x}$$

$$= 1 + \tan^2 x$$

Vinkel
med enhet
radianer

Deriver $\sin x + \cos(4)$

$$(\sin x + \cos(4))' = (\sin x)' + (\underbrace{\cos(4)}_{\text{konstant}})'$$

$$= \cos(x) + 0 = \underline{\cos(x)}$$

<u>Produktregelen</u>

$$(f \cdot g)(x) = f(x)\, g(x)$$

$$(f \cdot g)' = f' \cdot g + f \cdot g'$$

Eks : $(x \cdot x)' = (x)' \cdot x + x \cdot (x)'$

$$1 \cdot x + x \cdot 1 = 2x$$

$$\left(\text{ikke } (x)' \cdot (x)' = 1 \cdot 1 = 1 \;!! \right)$$

oppg. $(3x^2 \cos(x))' = 3(x^2 \cos x)'$

$$3((x^2)' \cos x + x^2 \cdot (\cos x)')$$

$$= 3(2x \cos x + x^2(-\sin x))$$

$$= \underline{3x(2\cos x - x\sin x)}$$

* $(4\sin x)' = 4(\sin x)' = 4\cos x$

(Unødvendig å benytte produktregelen

$\underbrace{(4)'}_{0}\sin x + 4\cdot(\sin x)' = 4\cos x$)

* Deriver $\dfrac{\tan x}{\sqrt{x}}$

$$\left(\frac{\tan x}{x^{1/2}}\right)' = \left(x^{-1/2}\cdot\tan x\right)'$$

$$= (x^{-1/2})'\tan x + x^{-1/2}(\tan x)'$$

$$= -\frac{1}{2}x^{-3/2}\tan x + x^{-1/2}(1+\tan^2 x)$$

$$= \underline{\frac{-\tan x + 2x(1+\tan^2 x)}{2x\sqrt{x}}}$$

Kjerneregelen

$f \circ u$ - først u og så f

$$(f\circ u)(x) = \underset{\substack{\uparrow \\ \text{ytre funksjon}}}{f}(\underset{\substack{\uparrow \\ \text{kjerne}}}{u(x)})$$

$\sin^3 x = (\sin x)^3$

kjerne $\sin x$

ytre funksjon er $f(u) = u^3$

$$\left(f(u(x))\right)' = f'(u(x)) \cdot u'(x)$$

$$\frac{d\, f \circ u}{dx} = \frac{df}{du} \cdot \frac{du}{dx}$$

$$\left(\frac{\Delta f}{\Delta x} = \frac{\Delta f}{\Delta u} \cdot \frac{\Delta u}{\Delta x} \right.$$

Problematisk hvis $\Delta u = 0$)

Eks $$(\sin^3 x)' = \frac{d\, u^3}{du} \cdot \frac{d \sin x}{dx}$$

$$= 3u^2 \cdot \cos x$$

$$= \underline{3\sin^2 x \cdot \cos x}$$

oppg $$(\sin(-4x+7))'$$

$$= \cos(-4x+7) \cdot (-4x+7)'$$

$$= \underline{-4\cos(-4x+7)}$$

* $$\frac{1}{\cos^4(x^2-x)}$$

$$= \left(\cos(x^2-x)\right)^{-4}$$

$$= h(g(f(x)))$$

$f(x) = x^2 - x$

$g(x) = \cos x$

$h(x) = x^{-4}$

Den deriverte :

$$\left(h(g(f(x)))\right)' = h'(g\circ f(x))\cdot(g\circ f(x))'$$

$$= h'(g\circ f(x))\cdot g'(f(x))\cdot f'(x).$$

$$\left((\cos(x^2-x))^{-4}\right)'$$

$$= -4\left(\cos(x^2-x)\right)^{-5}\left(\cos(x^2-x)\right)'$$

$$= \;-\,''\;\text{———}\;\left(-\sin(x^2-x)\cdot\underbrace{(x^2-x)'}_{(2x-1)}\right)$$

$$= \underline{4(2x-1)\sin(x^2-x)\left(\cos(x^2-x)\right)^{-5}}$$

* Deriver $x(2-5x)^7$

Benytter produkt - og kjerneregelene

$$\left(x(2-5x)^7\right)' = (x)'(2-5x)^7 + x\left((2-5x)^7\right)'$$

$$= (2-5x)^7 + x\left(7(2-5x)^6\underbrace{(2-5x)'}_{-5}\right)$$

$$= (2-5x)^7 + x(-35)(2-5x)^6$$

$$= (2-5x)^6\left(\underbrace{(2-5x)-35x}_{2-40x}\right)$$

$$= \underline{2(1-20x)(2-5x)^6}$$

Funksjonsdrøfting.

$$f(x) = 2\sin x - \sqrt{3}\,x$$

Finn ekstremalpunktene.

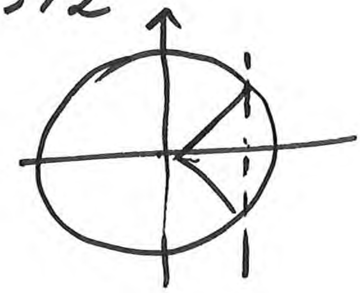
$$f'(x) = 0$$

$$f'(x) = 2(\sin x)' - \sqrt{3}(x)'$$

$$= 2\cos x - \sqrt{3}.$$

$f'(x) = 0 \iff 2\cos x - \sqrt{3} = 0$

$\cos x = \sqrt{3}/2$



$$x = \pm \pi/6 + 2\pi \cdot n$$

$$f''(x) = \left(2\cos x - \sqrt{3}\right)'$$

$$= -2\sin x$$

$f''(x)$ er negativ : $\frac{\pi}{6} + 2\pi n$

positiv : $-\frac{\pi}{6} + 2\pi n$

Maksimumspunkt $x = \frac{\pi}{6} + 2\pi \cdot n$

Minimumspunkt $x = \frac{-\pi}{6} + 2\pi \cdot n$

$$f(x) = \begin{cases} 0 & x \leq 0 \\ x^2 \sin\left(\frac{1}{x}\right) & x > 0 \end{cases}$$

Eksempel.

Påstand : $f(x)$ er deriverbar i alle punkt, men $f'(x)$ er ikkje kontinuerlig i $x=0$.

$$f'(x) = \left(x^2 \sin\left(\tfrac{1}{x}\right)\right)' \qquad x>0$$

$$= (x^2)' \sin\left(\tfrac{1}{x}\right) + x^2 \left(\sin\left(\tfrac{1}{x}\right)\right)'$$

$$= 2x \sin\left(\tfrac{1}{x}\right) + x^2 \left(\cos\left(\tfrac{1}{x}\right) \cdot \underbrace{\left(\tfrac{1}{x}\right)'}_{\frac{-1}{x^2}} \right)$$

$$f'(x) = \underline{2x \sin\left(\tfrac{1}{x}\right) - \cos\left(\tfrac{1}{x}\right)}$$

$$f'(0) = \lim_{h \to 0^+} \frac{h^2 \sin\left(\frac{1}{h}\right) - 0}{h} = 0$$

$$\lim_{h \to 0^-} \frac{0 - 0}{h} = 0$$

$$f'(x) = 0 \qquad x < 0$$

$$f'(x) = \begin{cases} 0 & x \leq 0 \\ 2x \sin\left(\frac{1}{x}\right) - \cos\left(\frac{1}{x}\right) & x > 0 \end{cases}$$

$\lim_{x \to 0^+} f'(x)$ eksisterer ikkje. ($\cos\left(\frac{1}{x}\right)$ svinger mellom -1 og 1 vilkårlig nært 0)