29.01
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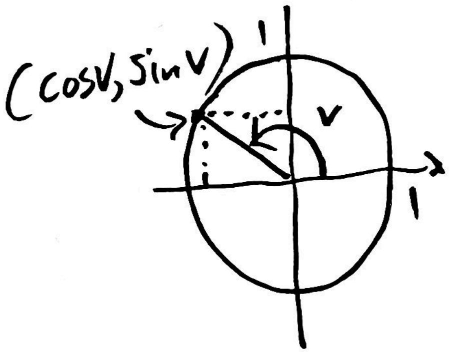
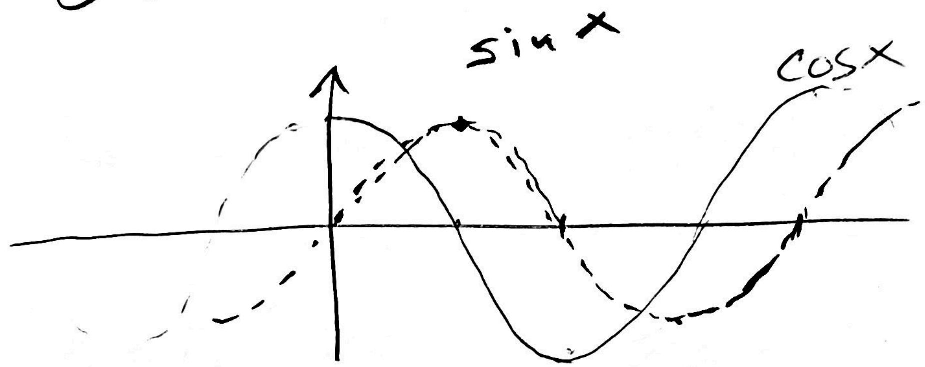
11D Trigonometriske funksjoner

Skal se på grafene til $\sin x$, $\cos x$ og $\tan x$





$$\sin x = \cos\left(\frac{\pi}{2} - x\right)$$

$$\cos x = \sin\left(\frac{\pi}{2} - x\right)$$

$$\cos x = \cos(-x) = \sin\left(\frac{\pi}{2} - (-x)\right)$$

$$\underline{\cos(x) = \sin\left(x + \frac{\pi}{2}\right)}$$

Grafen til $\cos(x)$ er lik grafen til $\sin x$ forskyvd med $-\frac{\pi}{2}$ mot venstre.

Grafen til $f(x+d)$ er lik grafen til
$f(x)$ forskyvd med d til venstre

La $f(x)$ ha en symetrisk definisjonsmengde
(ser lik ut på begge sider av 0)

$x \in D_f \iff -x \in D_f$.

$[-2, 2]$, $\mathbb{R}$ $\{-3, -1, 1, 3\}$ symetrisk

$[-2, 3\rangle$ ikke symmetrisk.

$f(x)$ er en <u>jevn</u> funksjon hvis $f(-x) = f(x)$ $x \in D_f$
Grafen til f reflekteres om y-aksen

$f(x)$ er en <u>odde</u> funksjon hvis $f(-x) = -f(x)$ $x \in D_f$.
Grafen til f er "symmetrisk om origo"

$f(x) = x^n$ ⟨ jevn funksjon $\iff$ n jevn tall (partall)
odde — $\iff$ n oddetall.

$\sin(-x) = -\sin(x)$ odde funksjon

$\cos(x) = \cos(-x)$ jevn funksjon.

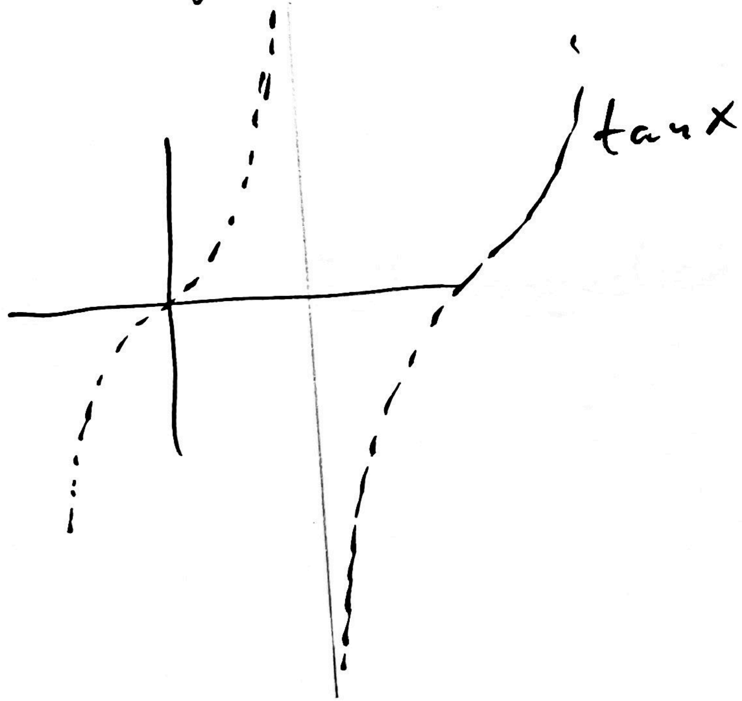
$\tan(x) = \dfrac{\sin x}{\cos x}$

definert når $\cos x \neq 0$:

alle x bortsettfra i $x = \frac{\pi}{2} + \pi \cdot n$

hvor den har

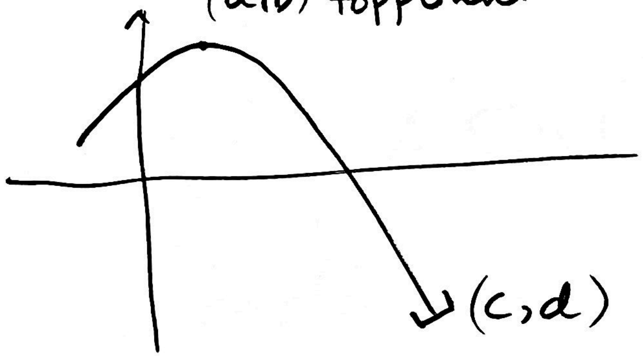
en vertikal asymptote.



$\tan x$ har periode π

$\tan(x+\pi) = \tan(x)$

(a,b) toppunkt



(c,d) bunnpunkt

a	maksimalpunkt
b	maksimalverdi
c	minimalpunkt
d	minimalverdi

Fellesbetegnelser

a, c	ekstremalpunkt
b, d	ekstremalverdier
(a,b) og (c,d)	(~~stasjonær punkt.~~ $f'(x)=0$) topp- og bunnpunkt

Harmonisk svingninger

$$A \sin(c \cdot x + \varphi) + d$$

Kan avgrenses til
$A \geq 0,\ c \geq 0$
$0 \leq \varphi < 2\pi$

$|A|$ amplituden

d likevektslinje

perioden $\underline{p = \frac{2\pi}{|c|}}$

φ "faseforskyvning".

$$\sin(cx + \varphi)$$
$$= \sin\left(c\left(x + \frac{\varphi}{c}\right)\right)$$

forskyver grafen med $\frac{\varphi}{c}$ til venstre

$$\sin(x + \pi) = -\sin x$$

$\sin x \cdot \cos x$

Harmonisk svingning

$$= \frac{1}{2}(2\sin x \cos x)$$
$$= \underline{\frac{1}{2}\sin(2x)}$$

Amplitude $\frac{1}{2}$

periode $\frac{2\pi}{2} = \underline{\pi}$

likevektslinjen: x-akse

$\varphi = 0$

$\cos^2 x$ som en sinusfunksjon

(Harmonisk svingning)

$\cos(2x) = \cos^2 x - \sin^2 x$ dobling av vinkel

Pytagoras $\sin^2 x + \cos^2 x = 1$
alle x.

$$\cos 2x = \cos^2 x - (1 - \cos^2 x)$$
$$= 2\cos^2 x - 1$$

så $\cos^2 x = \dfrac{\cos 2x + 1}{2}$

$$= \frac{1}{2}\sin\left(2x + \frac{\pi}{2}\right) + \frac{1}{2}.$$

$A = 1/2$ $\quad p = \frac{2\pi}{2} = \pi$

likevektslinjen $y = \frac{1}{2}$

$\varphi = \pi/2$.

$a \sin x + b \cos x$ er en Harmonisk svingning.

Addisjonsformelen for sinus:

$$A \sin(x+\gamma) = \underbrace{A\sin(\gamma)}_{b} \cdot \cos(x) + \underbrace{A\cos(\gamma)}_{a} \sin(x)$$

$A \geq 0.$

$$a^2 + b^2 = A^2(\sin^2\gamma + \cos^2\gamma) = A^2$$

så $\underline{A = \sqrt{a^2+b^2}}$.

$$\frac{b}{a} = \frac{A\sin(\gamma)}{A\cos(\gamma)} = \tan(\gamma) \qquad a \neq 0$$

én løsning er $\gamma = \arctan\left(\frac{b}{a}\right)$

$$(a,b) = \underset{\geq 0}{A}\,(\cos(\gamma), \sin(\gamma))$$

velg $\arctan\left(\frac{b}{a}\right)$

eller $\arctan\left(\frac{b}{a}\right) + \pi$

slik at vinkelen ligger i samme kvadrant som (a,b).

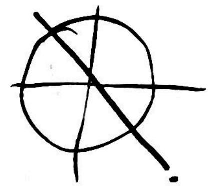
$\sin x - \cos x$

$\underbrace{1}_{a}\sin x + \underbrace{(-1)}_{b}\cos x$

$A = \sqrt{1^2 + (-1)^2} = \underline{\sqrt{2}}$.

$\arctan\left(\frac{b}{a}\right) = \arctan(-1)$

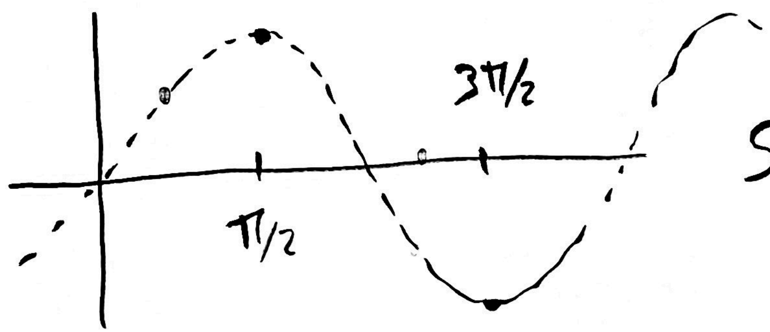
$= -\frac{\pi}{4}$



(a, b)

$= (1, -1)$

$\sin x - \cos x = \underline{\sqrt{2}\sin\left(x + \frac{-\pi}{4}\right)}$



$\sin x$ Toppunkt i $\left(\frac{\pi}{2}+2\pi\cdot n, 1\right)$

Bunnpunkt i $\left(\frac{3\pi}{2}+2\pi\cdot n, -1\right)$

eks. $\sin x$ $\left[\frac{\pi}{4}, \frac{5\pi}{4}\right]$ Finn topp/bunn punkt

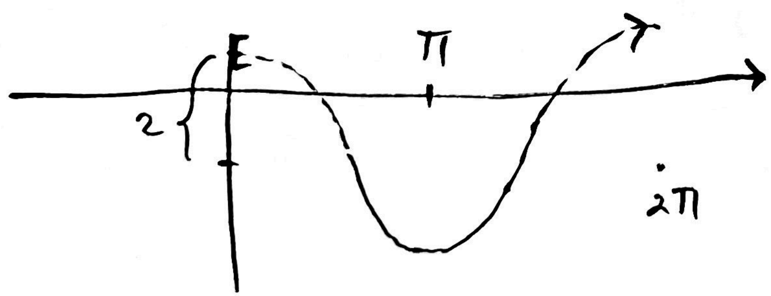
Fra grafen har vi toppunkt i $\left(\frac{\pi}{2}, 1\right)$

bunnpunkt i $\left(\frac{\pi}{4}, \frac{1}{\sqrt{2}}\right)$ lokalt

$\left(\frac{5\pi}{4}, \frac{-1}{\sqrt{2}}\right)$ globalt

$[0, 2\pi\rangle$

eks $2\cos x - \sqrt{3}$

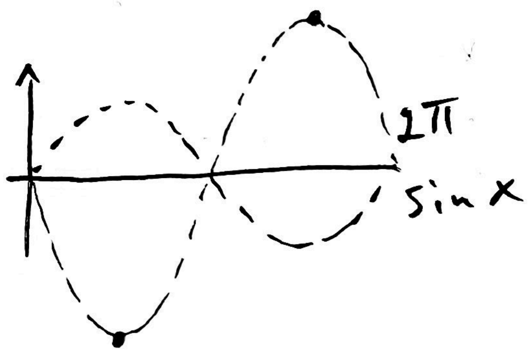


toppunkt i $(0, 2-\sqrt{3})$

bunnpunkt i $(\pi, -2-\sqrt{3})$.

Øving

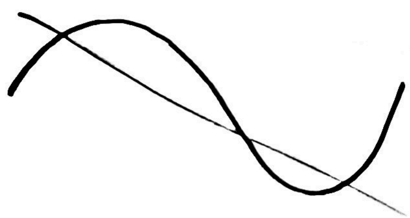
Finn ekstremalpunkt til $-2\sin x$ $\quad x \in [0, 2\pi]$.



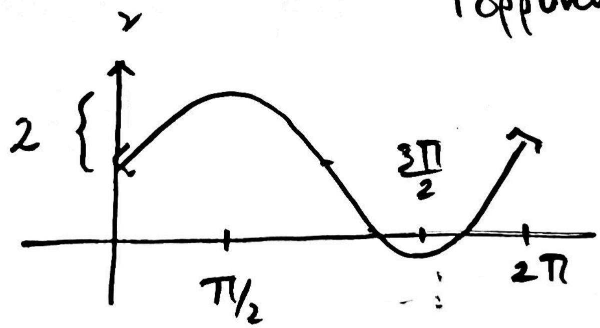
Toppunkt i $\left(\frac{3\pi}{2}, 2\right)$ og $(2\pi, 0)$

Bunnpunkt i $\left(\frac{\pi}{2}, -2\right)$ og $(0, 0)$

Finn topp, bunn og nullpunkt til $2\sin(x) + \sqrt{2}$ $\quad [0, 2\pi\rangle$

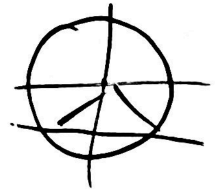


Toppunkt: $\left(\frac{\pi}{2}, 2+\sqrt{2}\right)$ og $(2\pi, \sqrt{2})$



Bunnpunkt $(0, \sqrt{2})$ og $\left(\frac{3\pi}{2}, -2+\sqrt{2}\right)$

Nullpunkt $\quad 2\sin x + \sqrt{2} = 0$



$$\sin x = \frac{-\sqrt{2}}{2} = \frac{-1}{\sqrt{2}}$$

$$\arcsin\left(\frac{-1}{\sqrt{2}}\right) = -\frac{\pi}{4}$$

$$x = \frac{-\pi}{4} + 2\pi \cdot n \qquad n \in \mathbb{Z}.$$

$$x = \pi - \left(\frac{-\pi}{4}\right) + 2\pi \cdot n$$

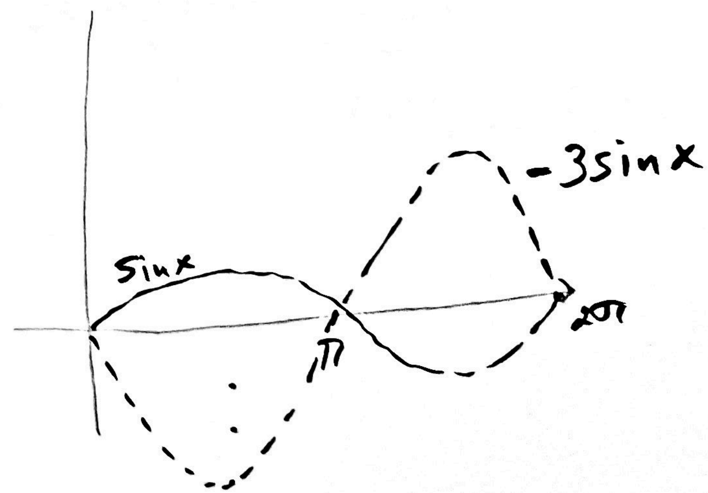
Nullpunkt i $[0, 2\pi]$ er $\frac{-\pi}{4} + 2\pi = \underline{\underline{\frac{7\pi}{4}}}$

og $\underline{\underline{\frac{5\pi}{4}}}$

* Finn topp/bunn punkt og nullpunkt til

$$-3\sin\left(x + \frac{\pi}{3}\right) + 5.$$

$$x \in [0, 2\pi]$$



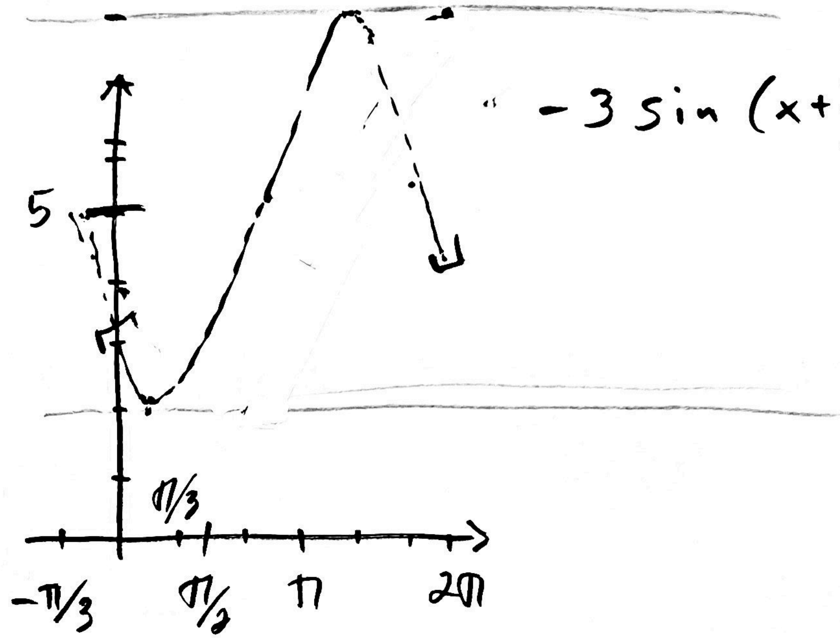
Ingen nullpunkt.

Bunnpunkt når $\sin\left(x+\frac{\pi}{3}\right) = +1$

$$x + \frac{\pi}{3} = \frac{\pi}{2} \quad (+2\pi \cdot n)$$

$$x = \frac{\pi}{2} - \frac{\pi}{3} \quad +(2\pi \cdot n)$$

maksimums-punkt $x = \frac{\pi}{6}$



" $-3\sin\left(x+\frac{\pi}{3}\right)+5$.

Toppunkt når $\sin\left(x+\frac{\pi}{3}\right) = -1$

$$x + \frac{\pi}{3} = \frac{3\pi}{2} \quad (+2\pi \cdot n)$$

maksimumspunkt $x = \frac{3\pi}{2} - \frac{\pi}{3} = \frac{9\pi}{6} - \frac{2\pi}{6} = \frac{7\pi}{6}$.

Toppunkt $\left(\frac{7\pi}{6}, 8\right)$, og $\left(0, 5 - \frac{3\sqrt{3}}{2}\right)$

Bunnpunkt $\left(\frac{\pi}{6}, 2\right)$ og $\left(2\pi, 5 - \frac{3\sqrt{3}}{2}\right)$

$$\underbrace{-\sin x + \sqrt{3}\cos x}_{\text{Harmonisk svingning.}} = 1$$

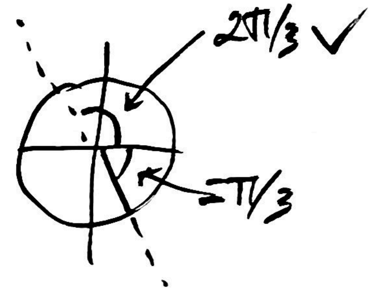
$$A\sin(x+\varphi) = \underbrace{A\cos\varphi}_{a=-1}\sin x + \underbrace{A\sin\varphi}_{b=\sqrt{3}}\cdot\cos x.$$

$$A = \sqrt{(-1)^2+(\sqrt{3})^2} = \sqrt{4} = \underline{2.}$$

$$\tan\varphi = \frac{b}{a} = \underline{-\sqrt{3}}$$

$$\varphi = -60^\circ = -\pi/3$$

$$\varphi = \frac{2\pi}{3} \; (=120^\circ)$$

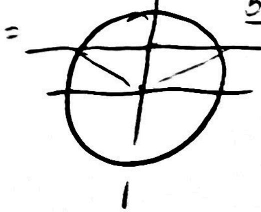


$$2\cos\varphi = -1$$

Mellomregning:

$$\frac{\pi}{6} - \frac{2\pi}{3} = \frac{\pi-4\pi}{6} = \frac{-3\pi}{6} = \frac{-\pi}{2}$$

$$\frac{5\pi}{6} - \frac{2\pi}{3} = \frac{5\pi-4\pi}{6}$$



Likningen er ekvivalent til:

$$\frac{2}{2}\sin(\underbrace{x+\tfrac{2\pi}{3}}_{v}) = \frac{1}{2}$$

$$v = x+\frac{2\pi}{3} = \begin{cases} \pi/6 + 2\pi\cdot n \\ 5\pi/6 + 2\pi\cdot n \end{cases} \quad \text{gir} \quad \underline{x = \begin{cases} -\pi/2 + 2\pi\cdot n \\ \pi/6 + 2\pi\cdot n \end{cases}} \quad n\in\mathbb{Z}$$