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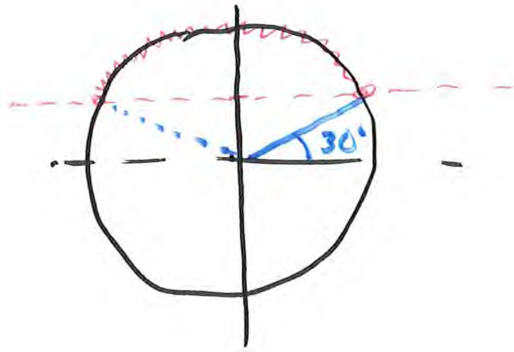
11C Trigonometriske ulikheter

$$\sin x \geq \frac{1}{2}$$

$x \in [0, 360°]$



Løser først likningen

$$\sin x = \frac{1}{2}$$

$$\arcsin\left(\frac{1}{2}\right) = \underline{30°}$$

Den andre løsningen i $[0, 2\pi]$ er $180° - 30° = \underline{150°}$

Ser av figuren at løsningene er $\underline{x \in [30°, 150°]}$

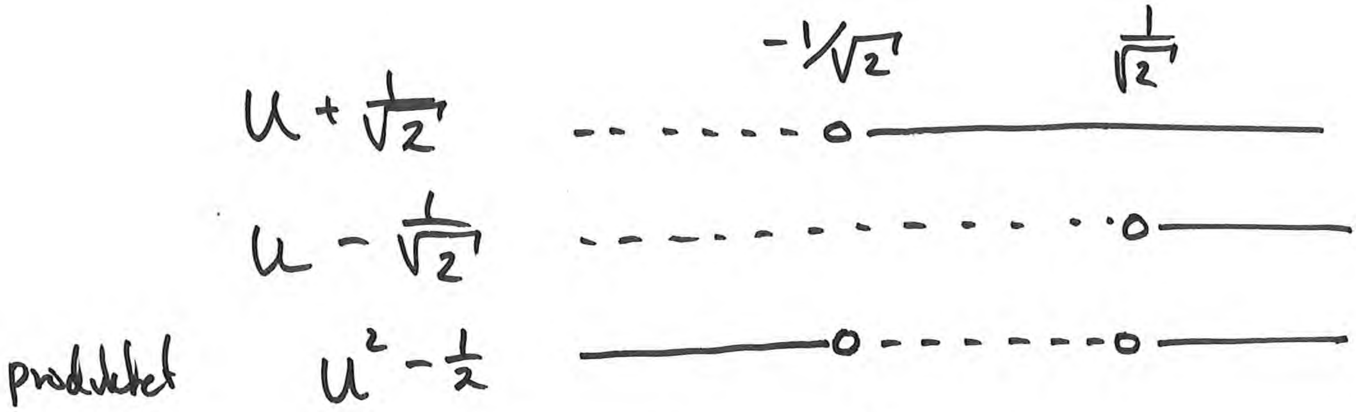
$$\sin^2 x \leq \frac{1}{2} \qquad x \in [-180°, 180°]$$

$$u^2 \leq \frac{1}{2}$$

$$u^2 - \frac{1}{2} \leq 0$$

$$u^2 - \left(\frac{1}{\sqrt{2}}\right)^2 \leq 0$$

$$\left(u + \frac{1}{\sqrt{2}}\right)\left(u - \frac{1}{\sqrt{2}}\right) \leq 0$$

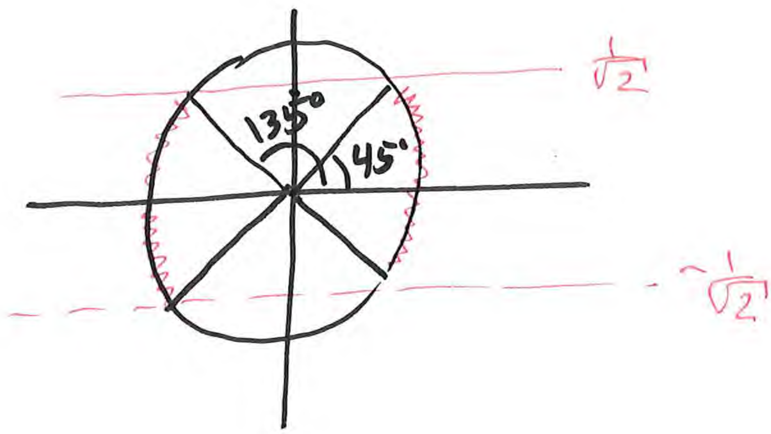


$u^2-\frac{1}{2} \leq 0$ har løsning

$$-\frac{1}{\sqrt{2}} \leq u \leq \frac{1}{\sqrt{2}}$$

$$\Leftrightarrow \quad |u| \leq \frac{1}{\sqrt{2}}.$$

$$\sin^2 x \leq \frac{1}{2} \quad \Leftrightarrow \quad -\frac{1}{\sqrt{2}} \leq \sin x \leq \frac{1}{\sqrt{2}}$$



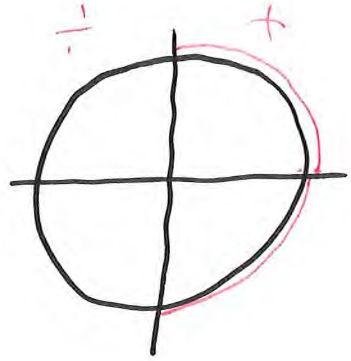
Løsningene er

$$x \in [-180°, -135°] \cup [-45°, 45°] \cup [135°, 180°]$$

$$\cos x - \underbrace{\sin 2x}_{2\sin x \cos x} \geq 0 \qquad x \in [0, 2\pi]$$

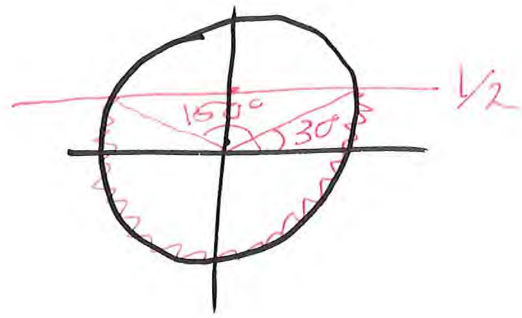
$$\cos x \,(1 - 2\sin x) \geq 0$$

Fortegn til $\cos x$

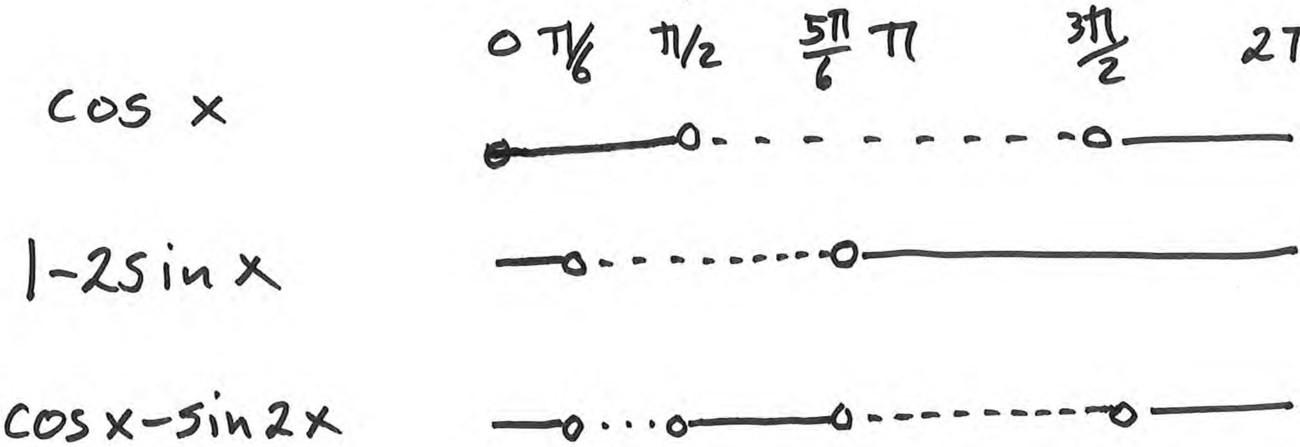


$$1 - 2\sin x \geq 0$$

$$\Leftrightarrow \quad \frac{1}{2} \geq \sin x$$



Fortegnsskjema



Løsningen til $\cos x - \sin 2x \geq 0$ i $[0, 2\pi]$ er

$$\underline{x \in [0, \pi/6] \cup [\pi/2, \tfrac{5\pi}{6}] \cup [\tfrac{3\pi}{2}, 2\pi]}$$

$\sin x + \sqrt{3}\cos x > 0$ $\qquad x \in [0, 2\pi]$.

Likningen: $\sin x + \sqrt{3}\cos x = 0$

gjør om til en tangenslikning.

Hvis $\cos x = 0$, da er $\sin x = \pm 1$.

Vi har da ingen løsning.

Vi antar $\cos x \neq 0$. Deler med $\cos x$.

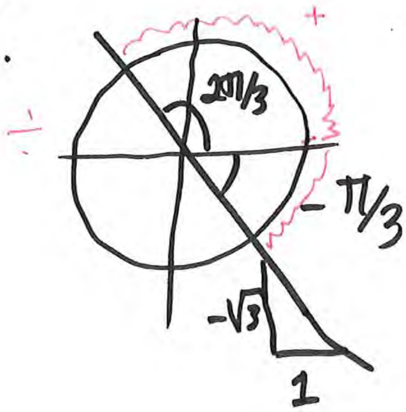
$$\frac{\sin x}{\cos x} + \sqrt{3}\,\frac{\cos x}{\cos x} = 0$$

$$\tan x + \sqrt{3} = 0$$

$$\tan x = -\sqrt{3}.$$

$$x = \frac{2\pi}{3} \text{ og } \frac{5\pi}{3}$$

$$\left(2\pi + \left(-\frac{\pi}{3}\right)\right)$$



$$\cos x \,\frac{\sin x}{\cos x} + \sqrt{3}\cos x > 0$$

$$\cos x\,(\tan x + \sqrt{3}) > 0$$

Fortegnsskjema...

Alternativt: "ser fra figuren"

$x \in [0, 2\pi/3\rangle \cup \langle \frac{5\pi}{3}, 2\pi]$

$x \in [0, 360°]$

$$2\sin^2 x - \sin x - 1 \geq 0$$

2. grads uttrykk i $\sin x$

$$2u^2 - u - 1 \geq 0$$

$2u^2 - u - 1 = 0$:

abc-formel $\quad u = \dfrac{-(-1) \pm \sqrt{1 - 4 \cdot 2(-1)}}{2 \cdot 2}$

$$= \frac{1 \pm 3}{4}$$

$$\underline{u = 1} \quad \text{og} \quad \underline{u = \tfrac{-1}{2}}$$

Så $\quad 2u^2 - u - 1 = 2(u-1)(u+\tfrac{1}{2})$

$$= \underline{(u-1)(2u+1)} \quad ✓$$

$$(\sin x - 1)(2\sin x + 1) \geq 0$$

$\sin x - 1 \leq 0$ for alle x

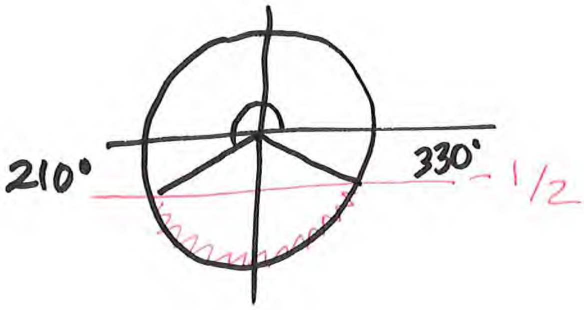
og lik 0 for $\underline{x = 90°}$

For $x \neq 90°$: $\quad \sin x - 1 < 0$

så $\quad (\sin x - 1)(2\sin x + 1) \geq 0$

$$2\sin x + 1 \leq 0$$

$$\Leftrightarrow \quad \sin x \leq \frac{-1}{2}$$



$$x \in [210°, 330°].$$

Løsningene til $2\sin^2 x - \sin x - 1 \geq 0$

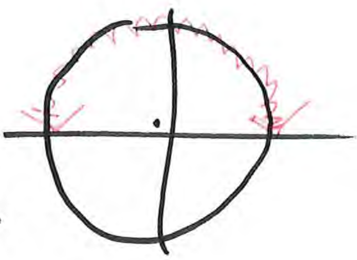
fra $x \in [0, 360°]$ er

$$\underline{x \in \{90°\} \cup [210°, 330°]}$$

* $\sin(\underbrace{2x+1}_{v}) > 0$ $\quad x \in [0, \pi]$.

$v \in [1, 2\pi+1]$

1 $\quad \sin v > 0$



$$v \in [1, \pi\rangle \cup \langle 2\pi, 2\pi+1].$$

$$v = 2x+1$$

$$x = \underline{\tfrac{1}{2}(v-1)}$$

$$\underline{x \in \left[0, \frac{\pi-1}{2}\right\rangle \cup \left\langle \frac{2\pi-1}{2}, 2\pi\right]}$$

Figurer til oblig 5

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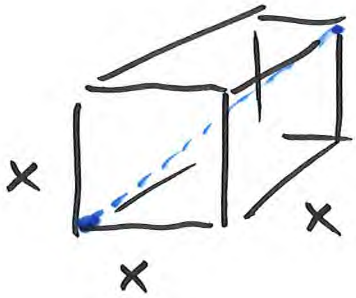

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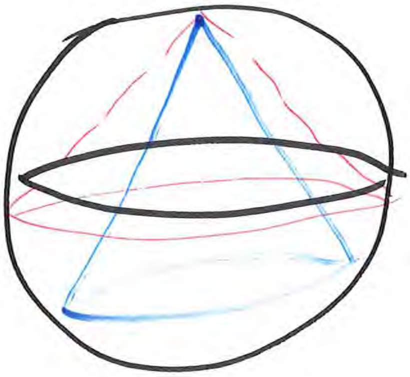

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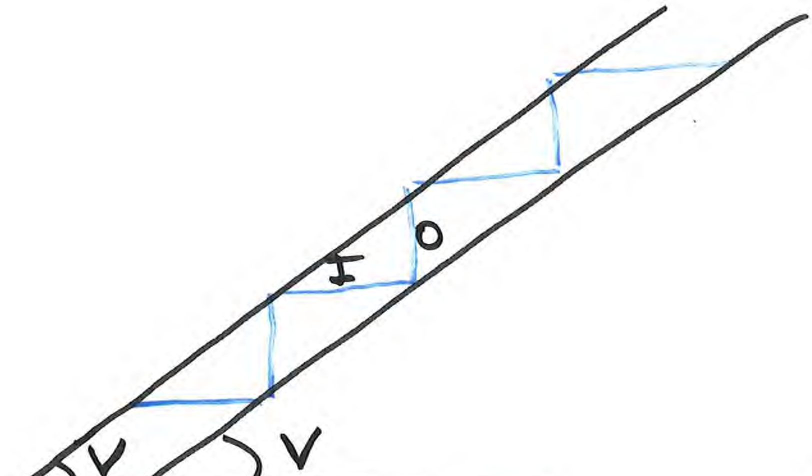

$20 + I = 62\text{cm}$