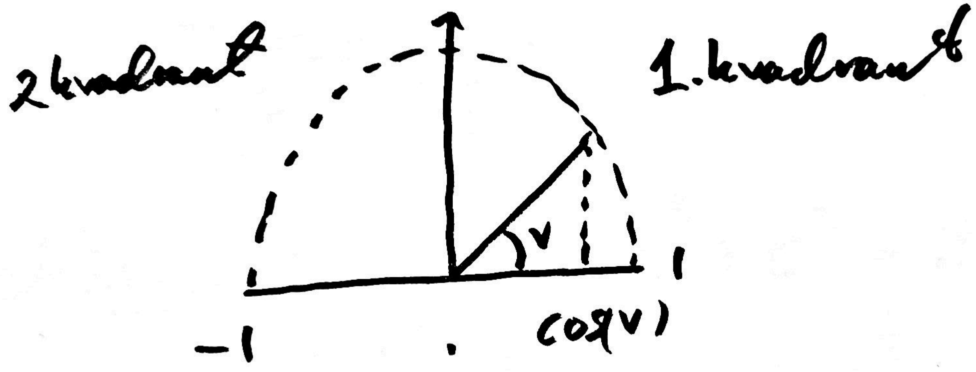
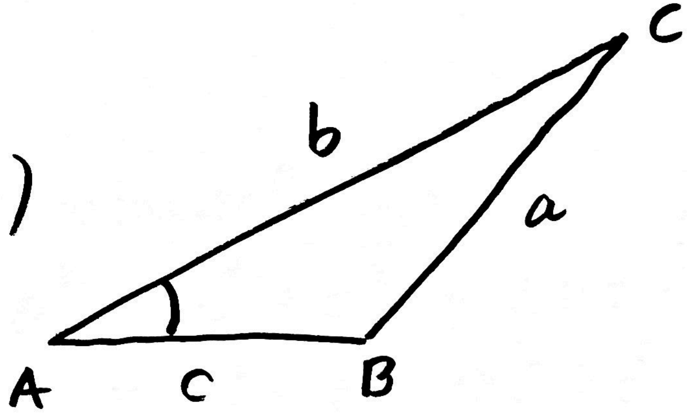
22.01.25 10E Cosinussetningen



$\cos(180^\circ - V)$
$= -\cos(V)$

$-1 \leq \cos V \leq 0$ | $1 \geq \cos(V) \geq 0$

$90^\circ \leq V \leq 180^\circ$ | $90^\circ \geq V \geq 0$

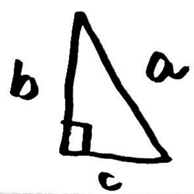


$$a^2 = b^2 + c^2 - 2bc\cos(A)$$

cosinussetningen.

tilsvarende med vinkel B og C

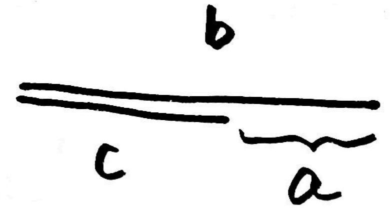
$A = 90^\circ$



$\left(\cos(90^\circ) = 0 \right)$ cosinussetningen reduseres til Pytagoras sin sats.

$$a^2 = b^2 + c^2 (+0)$$

$A = 0°$

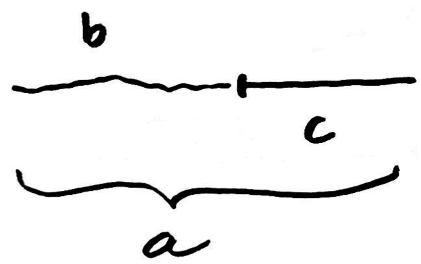


cosinussetningen: $a^2 = b^2 + c^2 - 2bc\underbrace{\cos(0°)}_{1}$

$$a^2 = (b-c)^2$$

siden $a \geq 0$ så er $\underline{a = |b-c|}$ ✓

$A = 180°$



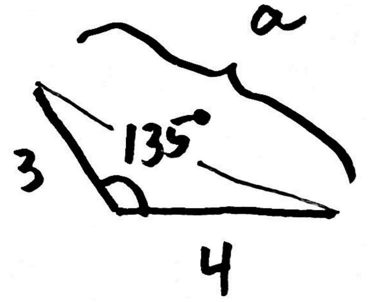
cosinussetningen

$$a^2 = b^2 + c^2 - 2bc\overbrace{\cos(180°)}^{-1}$$

$$= b^2 + c^2 + 2bc$$

$$a^2 = (b+c)^2$$

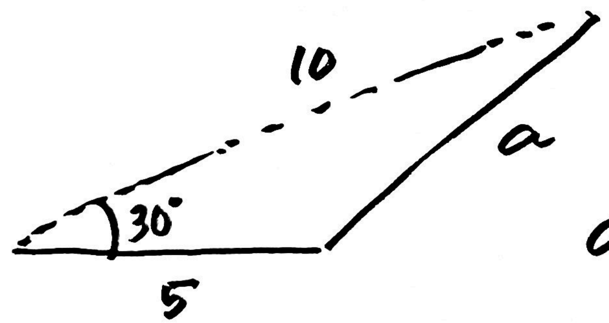
siden $\underset{b,c}{a \geq 0}$ så er $\underline{a = b+c}$ ✓



Hva er a?

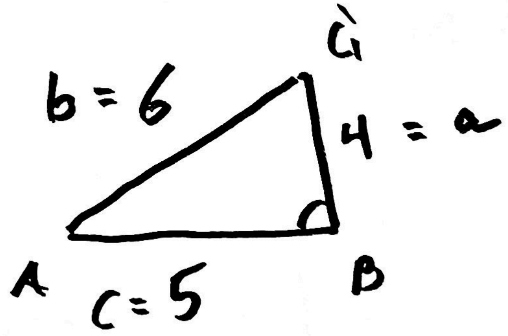
$$a^2 = 3^2 + 4^2 - 2 \cdot 3 \cdot 4 \cos(135°)$$
$$= 25 - 2 \cdot 3 \cdot 4 \left(-\frac{1}{\sqrt{2}}\right)$$
$$= 25 + 12\sqrt{2}$$
$$a = \sqrt{25 + 12\sqrt{2}} \sim \underline{6.48}$$

oppg



Finn a

$$a^2 = 5^2 + 10^2 - 2 \cdot 5 \cdot 10 \overbrace{\cos(30°)}^{\sqrt{3}/2}$$
$$= 125 - 100 \cdot \frac{\sqrt{3}}{2}$$
$$a = \sqrt{125 - 50\sqrt{3}} \sim \underline{6.19}$$



$4^2+5^2 = 16+25 = 41 > 36 = 6^2$

(Så $B < 90°$)

Hva er vinklene?

Cosinussetningen

$b^2 = a^2 + c^2 - 2ac \cos B$

$6^2 = 4^2+5^2 - 2\cdot 4\cdot 5 \underline{\cos B}$

$36 = 41 - 40 \cos B$

$40 \cos B = 41 - 36 = 5$

$\cos B = \frac{5}{40} = \underline{\frac{1}{8}}$

$B = \cos^{-1}(\frac{1}{8}) = \arccos(\frac{1}{8})$

$= \underline{82.819°}$

Benytter sinussetningen

$\frac{\sin B}{b} = \frac{\sin A}{a}$ så

$\sin A = \frac{a}{b} \sin B = \frac{4}{6}\sqrt{1-\cos^2 B}$

$$\sin A = \frac{2}{3}\sqrt{1-\frac{1}{64}} = \frac{2}{3}\frac{\sqrt{63}}{\sqrt{64}} = \frac{2}{3\cdot 8}\sqrt{63}$$

$$= \frac{\sqrt{63}}{12}$$ (litt mindre enn $\frac{8}{12} = \frac{2}{3}$)

(ser fra figuren at $A < 90°$)

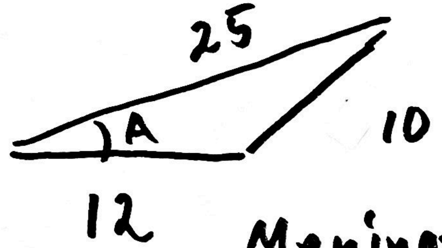
$$A = \sin^{-1}\left(\frac{\sqrt{63}}{12}\right) = \underline{41.410°}$$

$$C = 180° - (A+B) = \underline{55.771°}$$

Tips: gradetegnet i GeoGebra får deu som ALT - 0 (liten o)

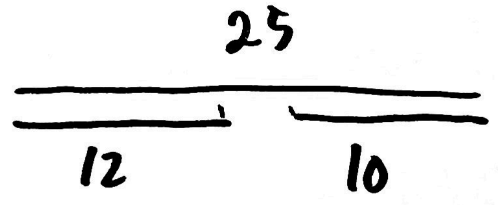
Hvis ikke gradeenheten benyttes tolkes vinkelen til å ha enhet radianer

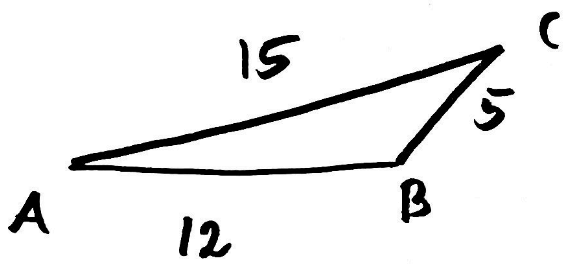
90° (grader) men 90 radianer.



$12+10=22<25$

Meningsløst! Det finnes ikke en slik trekant



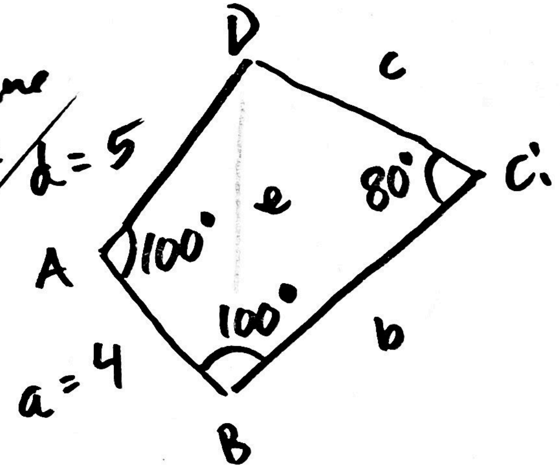


1. Finne A ved cossetningen
2. Finne B ved sinussetning.
   Vi må da benytte at vi vet at $B > 90^\circ$
3. $C = 180^\circ - A - B$.

(Detaljene er utelatt)

Alternativ til del 2: Benytt cosinussetningen.

Finn vinklene og sidene



Vi skisserer hvordan oppgaven kan løses.

1. Cosinussetningen gir oss lengden e.

2. $D = 360° - A - B - C.$

3. Ønsker å finne $\angle DBC$.

$\angle DBC = \underbrace{B}_{100°} - \angle ABD.$

$\angle ABD$ kan vi finne ved å benytte cosinus- eller sinussetningen med Δ til venstre.

$$\frac{\sin A}{BD} = \frac{\sin(\angle ABD)}{AD}$$

$(\angle ABD < 80$

5. $\frac{\sin 100°}{e} = \sin(\angle ABD)$

4 Finner c ved sinussetning

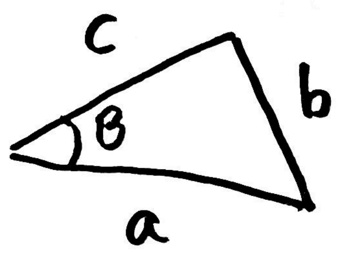
$$\frac{\sin \angle DBC}{c} = \frac{\sin(G)}{e}$$

c ← ukjent

5 $\angle BDC = D - \angle ADB$.

men $\angle ADB = 180° - A - \angle ABD$

Finner så b ved sinus- eller cosinussetningen.



Hva er arealet til en Δ med sider A, B og C?

Arealsetningen: Arealet $A = \frac{1}{2} a c \sin(B)$

Ved cosinussetningen $b^2 = a^2 + c^2 - 2ac \cdot \cos(B)$

Pythagoras $1 = \cos^2(B) + \sin^2(B)$

$0 \leq B \leq 180°$ så $\sin(B) \geq 0$

$\sin(B) = \sqrt{1 - \cos^2(B)}$

$2ac\cos(B) = a^2 + c^2 - b^2$ så $\cos B = \frac{a^2+c^2-b^2}{2ac}$

så $A = \frac{1}{2} a \cdot c \sqrt{1 - \left(\frac{a^2+c^2-b^2}{2ac}\right)^2}$

$$A = \frac{1}{2} ac \sqrt{\frac{(2ac)^2 - ((a^2+c^2)^2 - b^2)^2}{(2ac)^2}}$$

$$= \frac{1}{2} ac \cdot \frac{1}{2ac} \sqrt{4a^2c^2 - (a^2+c^2)^2 - (-b^2)^2 - 2(-b^2)(a^2+c^2)}$$

$$= \frac{1}{4} \sqrt{4a^2c^2 - (a^4+c^4+2a^2c^2) - b^4 + 2(b^2a^2 + b^2c^2)}$$

$$A = \frac{1}{4} \sqrt{2(a^2c^2 + b^2a^2 + b^2c^2) - a^4 - b^4 + c^4}$$

Dette er også lik

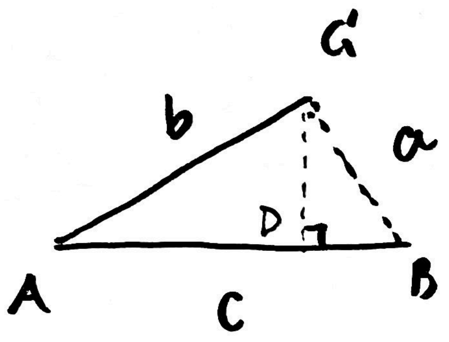
$$A = \sqrt{s(s-a)(s-b)(s-c)} \quad \text{hvor} \quad s = \frac{a+b+c}{2}$$

dette kalles Herons formel.

(vis gjerne at de er like.)

Bevis for cosinussetningen.

$A < 90^\circ$

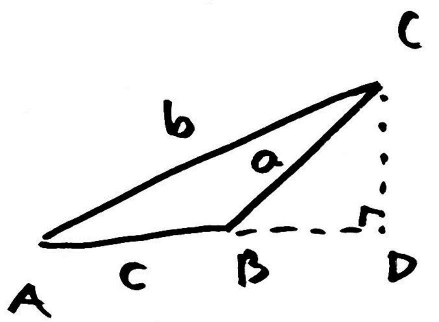


$$a^2 = (DB)^2 + (DG)^2$$

$$DG = b \sin A$$

$$AD = b \cdot \cos A$$

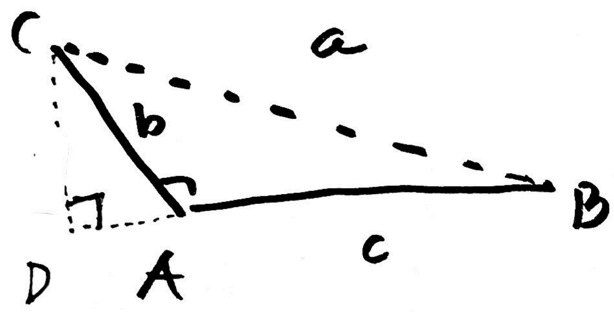
$$|DB| = |c - b\cos A|$$



så $$a^2 = (c - b\cos A)^2 + (b \sin A)^2$$
$$= c^2 - 2bc\cos A + \underbrace{b^2\cos^2 A + b^2 \sin^2 A}_{b^2(\underbrace{\cos^2 A + \sin^2 A}_{1})}$$

$$\underline{a^2 = c^2 + b^2 - 2bc\cos A}$$

$A > 90°$



$DC = b \sin A$

$DA = -b \cos A$

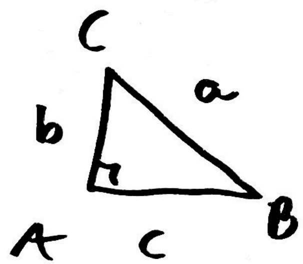
$a^2 = (DC)^2 + (DB)^2$

$DB = c + DA = c + (-b\cos A)$

$a^2 = (b \sin A)^2 + (c - b\cos A)^2$ som tidligere

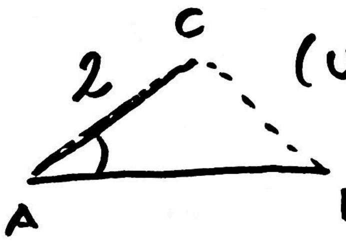
$= b^2 + c^2 - 2bc \cos A$

$A = 90°$



✓

Øving



(unøyaktig figur)

10.73 $\triangle ABC$

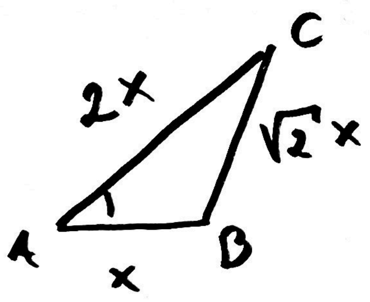
$AB = 4$ $AC = 2$

$\cos A = 3/4$

Hva er BC?

$$(BC)^2 = (AB)^2 + (AC)^2 - 2(AB)(AC)\cos A$$
$$= 4^2 + 2^2 - 2 \cdot 4 \cdot 2 \cdot \frac{3}{4}$$
$$= 4(4 + 1 - 3) = 4 \cdot 2 = 8$$
$$BC = \sqrt{4 \cdot 2} = \underline{\underline{2 \cdot \sqrt{2}}} \quad (\approx \underline{2.82})$$

10.75



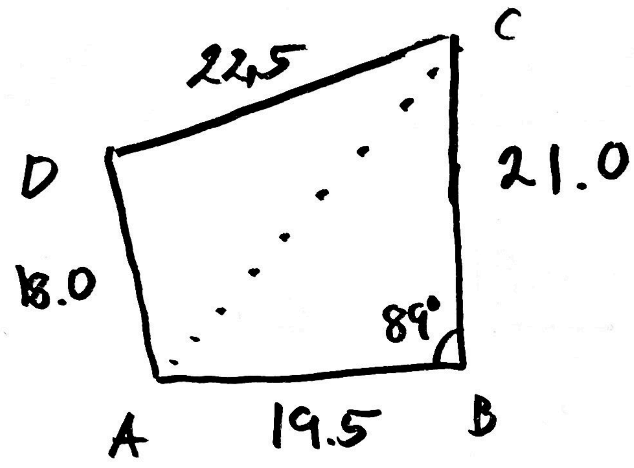
Hva er A?

cosinussetningen.

$$(\sqrt{2}x)^2 = x^2 + (2x)^2 - 2 \cdot x \cdot (2x)\cos A$$
$$2x^2 = 5x^2 - 4x^2 \cos A \quad \Big| \cdot \frac{1}{x^2}$$
$$2 = 5 - 4\cos A$$
$$4\cos A = 5 - 2 = 3 \quad \text{så} \quad \cos A = \frac{3}{4} = 0.75$$
$$A = \cos^{-1}\left(\frac{3}{4}\right) \approx \underline{41.41^\circ}$$

10.74

(og 10.78a)



Finn arealet til □ABCD.

cosinussetningen gir AC som så gir D.

Arealet til ABCD er summen av arealene til $\triangle ABC$ og $\triangle ACD$.

a) $(AC)^2 = (AB)^2 + (BC)^2 - 2(AB)(BC)\cos(B)$ gir $AC \approx 28.40$ m

b) $(AC)^2 = (AD)^2 + (CD)^2 - 2(AD)(CD)\underline{\cos D}$

gir $D = 123.16°$

$$\cos(D) = \frac{(18)^2 + (22.5)^2 - (28.46)^2}{2 \cdot 18 \cdot 22.5} \approx 0.0292$$

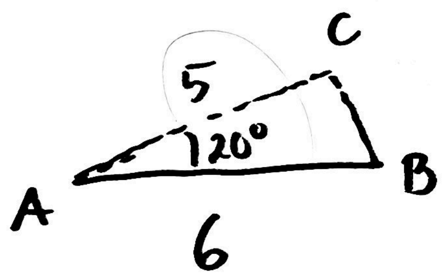
$D = \underline{88.324°}$

c) Arealet $A = \dfrac{AB \cdot BC \cdot \sin B}{2} + \dfrac{AD \cdot CD}{2} \cdot \sin D$

$= \dfrac{19.5 \cdot 21 \cdot \sin 89^\circ}{2} + \dfrac{18 \cdot 22.5}{2} \sin(88.324)$

$= \underline{407 \text{ m}^2}$

10.78 a)



$(BC)^2 = 5^2 + 6^2 - 2 \cdot 5 \cdot 6 \cos(20^\circ)$

$= \underbrace{25 + 36}_{61} - 60 \cos(20^\circ)$

$BC = \underline{2.15}$

$\sin B = 5 \cdot \dfrac{\sin(20^\circ)}{BC}$

~ 0.796 gir $B = \underline{52.7^\circ}$

$B < 90^\circ$

$C = 180^\circ - B - A = \underline{107.3^\circ}$