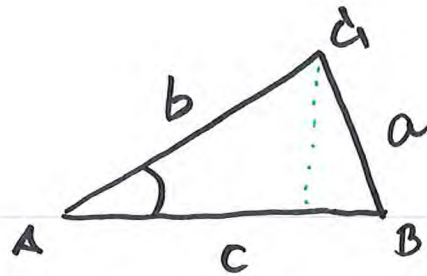


21 jan
25

Arealseringen



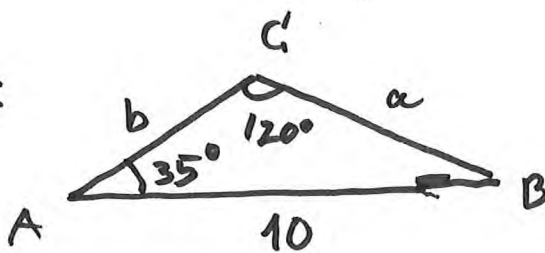
$$\begin{aligned} \text{Areal} &= \frac{1}{2} b \cdot c \cdot \sin(A) \\ &= \frac{1}{2} a \cdot c \cdot \sin(B) = \frac{1}{2} a \cdot b \cdot \sin(C) \end{aligned}$$

100 Sinussetningen

$$\frac{2 \cdot \text{Areal}}{abc} = \frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

Sinussetningen lar oss finne resterende vinkler og sidelengder til en trekant når vi kjenner en vinkel og lengden til motsatt side samt en sidelengde til eller en vinkel til.

2 vinkler:



Finn resterende vinkler og sider.

$$B = 180^\circ - A - C = 180^\circ - 120^\circ - 35^\circ = \underline{25^\circ}$$

$$\begin{aligned} \frac{\sin A}{a} &= \frac{\sin C}{c} = \frac{\sin 120^\circ}{10} \\ &= \frac{\sqrt{3}/2}{10} = \frac{\sqrt{3}}{20} \end{aligned}$$



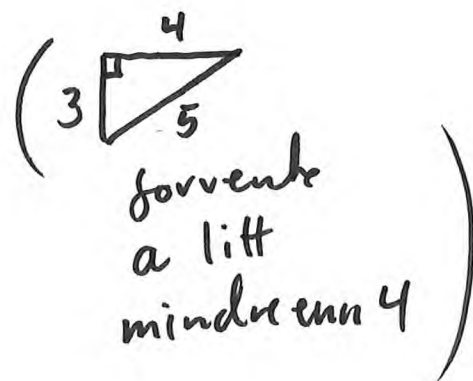
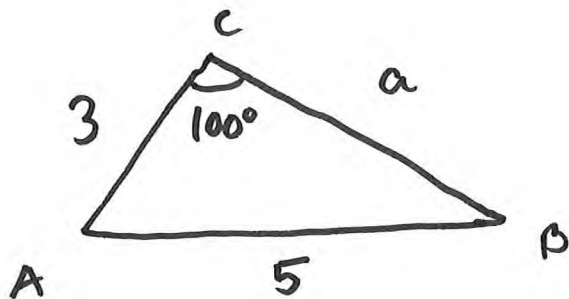
$$a = \sin A / (\sqrt{3}/20) = \sin(35^\circ) \cdot \frac{20}{\sqrt{3}} \approx \underline{6.62}$$

Tilsvarende

$$b = \sin B \cdot \frac{20}{\sqrt{3}}$$

$$= \sin(25^\circ) \cdot \frac{20}{\sqrt{3}} \approx \underline{4.88}$$

2 sider



$$\frac{\sin C}{c} = \frac{\sin 100^\circ}{5} = 0.19696\dots = f$$
$$= \frac{\sin B}{b}$$

$$\sin(B) = b \cdot f = 3 \cdot 0.19696 \sim 0.591$$

$$B = \arcsin(0.591\dots) \approx 36.22^\circ$$

$$A = 180^\circ - C - B = 180^\circ - 100^\circ - 36.22^\circ$$
$$\sim 43.78^\circ$$

$$\frac{\sin A}{a} = f \quad \text{så} \quad a = \frac{\sin A}{f}$$
$$= \underline{\underline{3.51}}$$



Sinussætningene egner seg ikke.

$\frac{\sin(\text{vinkel})}{\text{Lengde motsstående side}}$

er da ikke bestemt.

Her brukes cosinussætningene.

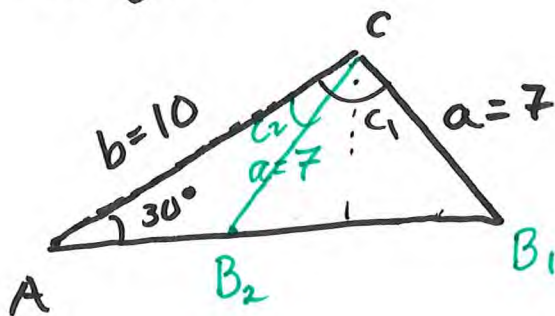
Eks

$$A = 30^\circ$$

$$b = 10$$

$$a = 7$$

Bestem mulige trekanter og finn lengden til den tredje siden.



$$\left. \begin{array}{l} \text{høyden} \\ b \sin A \\ = 10 \sin(30^\circ) = 10 \cdot \frac{1}{2} \\ = 5 \end{array} \right\}$$

$$\frac{\sin A}{a} = \frac{\sin 30^\circ}{7} = \frac{1/2}{7} = \frac{1}{14} = f$$

$$\frac{\sin B}{b} = f = \frac{1}{14}$$

$$\sin B = b \cdot \frac{1}{14} = \frac{10}{14} = \frac{1}{1.4}$$

$$B_1 = \sin^{-1}\left(\frac{10}{14}\right) \cong 45.5847$$

$$B_2 = 180^\circ - B_1 \cong 134.4153^\circ$$

$$C_1 = 180^\circ - A - B_1 = 180^\circ - B_1 - 30^\circ \cong \underline{104.415^\circ}$$

$$C_2 = 180^\circ - \cancel{30^\circ} - B_2 = 180^\circ - (180^\circ - B_1) - 30^\circ$$

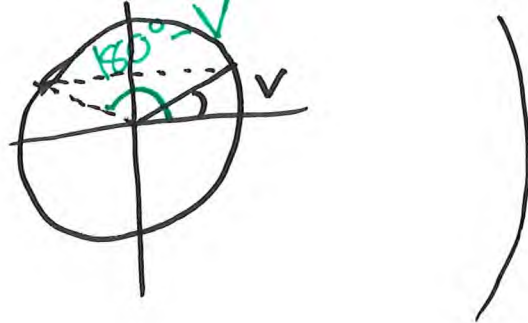
$$C_2 = B_1 - 30^\circ \sim 15.5847^\circ$$

$$\frac{\sin C}{c} = f = \frac{1}{14} \quad \text{så} \quad c = 14 \sin C$$

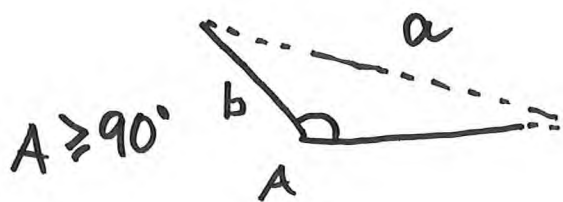
$$c_1 = 14 \cdot \sin C_1 = 14 \sin(104.415^\circ) \cong \underline{13.56}$$

$$c_2 = 14 \sin(C_2) = 14 \sin(15.5847^\circ) \cong \underline{3.76}$$

Refleksjon om γ -aksen
 $\sin V = \sin(180^\circ - V)$

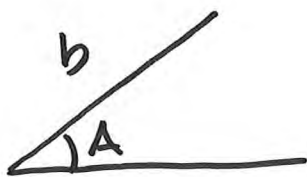


2 kjente sider og
 én vinkel (motstående en av sidene)



én løsning når $a > b$

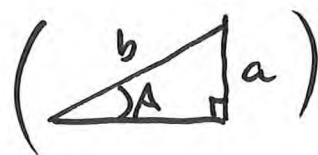
$A < 90^\circ$



} høyde
 $b \sin A$

$a < b \sin A$

ingen løsning.



$a = b \sin A$

én løsning

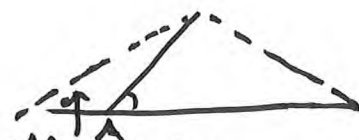
$b \sin A < a < b$

to løsninger



$a \geq b$

én løsning



ikke en løsning
 (vinkel A endres)
 til $180^\circ - A$

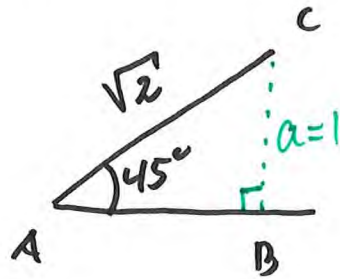
oppg.

$$\angle A = 45^\circ$$

$$a = 1$$

$$1) \quad b = \sqrt{2}$$

$$\frac{\sin B}{\sqrt{2}} = \frac{\sin A}{a}$$



$$\left. \begin{array}{l} \text{høyde} \\ \sqrt{2} \sin(45^\circ) \\ = 1 \end{array} \right\}$$

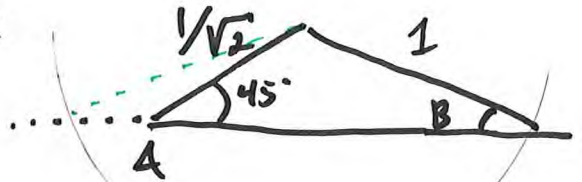
$$\sin B = \sqrt{2} \cdot \frac{\sin(45^\circ)}{1} = \sqrt{2} \cdot \frac{1}{\sqrt{2}} = 1.$$

$$\text{så} \quad B = 90^\circ$$



$$2) \quad b = \frac{1}{\sqrt{2}} \sim 0.707$$

én trekant



$$\frac{\sin B}{b} = \frac{\sin A}{a} = \frac{\sin(45^\circ)}{1} = \frac{1}{\sqrt{2}}$$

$$\sin B = b \cdot \frac{1}{\sqrt{2}} = \left(\frac{1}{\sqrt{2}}\right) \cdot \frac{1}{\sqrt{2}} = \frac{1}{2}.$$

$$B = 30^\circ$$

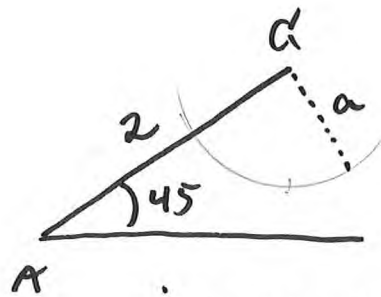
($180 - 30 = 150^\circ$
ikke engyldig
løsning)

$$\begin{aligned} C' &= 180^\circ - A - B = 180^\circ - 30^\circ - 45^\circ \\ &= \underline{105^\circ} (= 90^\circ + 15^\circ) \end{aligned}$$

$$\frac{\sin C'}{c} = \frac{\sin A}{a} = \frac{1}{\sqrt{2}}$$

$$c = \sqrt{2} \cdot \sin(C') = \underline{1.366}$$

3 $b = 2$
 $a = 1$



$$\left. \begin{aligned} &2 \sin 45^\circ \\ &= 2 \cdot \frac{1}{\sqrt{2}} \\ &= \sqrt{2} \end{aligned} \right\}$$

ingen mulig
trekant.

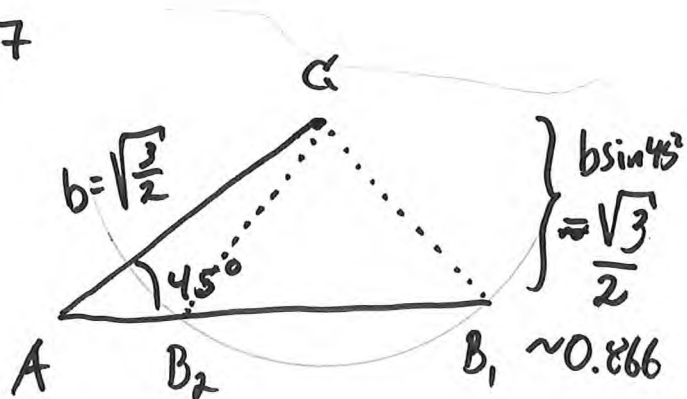
$$\frac{\sin A}{a} = \frac{\sin 45^\circ}{1} = \frac{1}{\sqrt{2}}$$

$$= \frac{\sin B}{b}$$

så $\sin B = b \cdot \frac{1}{\sqrt{2}} = 2 \cdot \frac{1}{\sqrt{2}} = \sqrt{2}$
 $\sim 1.41\dots$

ingen løsning.

4. $b = \sqrt{\frac{3}{2}} \sim 1.2247$



$$\left. \begin{aligned} &b \sin 45^\circ \\ &= \frac{\sqrt{3}}{2} \\ &\sim 0.866 \end{aligned} \right\}$$

$$\frac{\sin A}{a} = \frac{1}{\sqrt{2}} = \frac{\sin B}{b}$$

så $\sin B = b \cdot \frac{1}{\sqrt{2}}$
 $= \sqrt{\frac{3}{2}} \cdot \frac{1}{\sqrt{2}} = \frac{\sqrt{3}}{2}$

(kjent verdi)

$$B_1 = 60^\circ$$

$$C_1 = 180^\circ - 45^\circ - 60^\circ = 75^\circ$$

$$B_2 = 180^\circ - 60^\circ = 120^\circ$$

$$C_2 = 180^\circ - 120^\circ - 45^\circ = 15^\circ$$