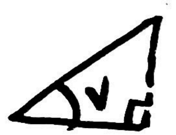
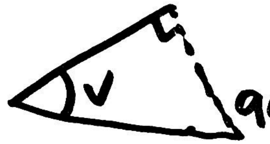
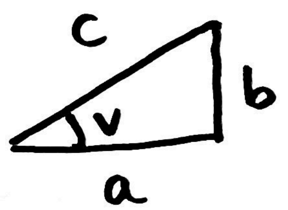
20jan 25

10C Areal setning og 10G Addisjonsformler for sin og cos

Rettvinkla trekanter med en vinkel v er bestemt opp til formlikhet



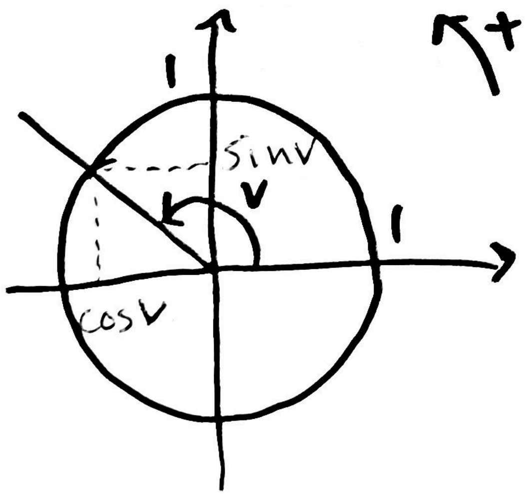


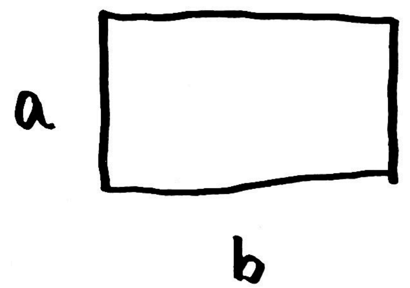


$\sin v = \frac{b}{c}$

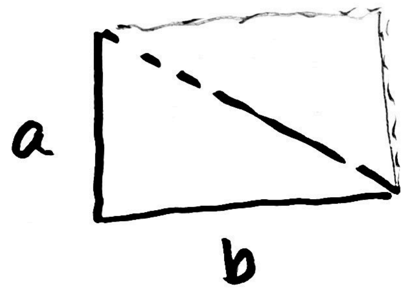
$\cos v = \frac{a}{c}$

$\tan v = \frac{b}{a}$

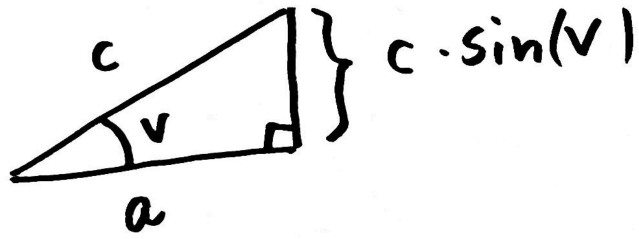




Arealet $A = a \cdot b$

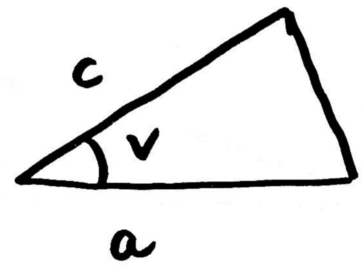


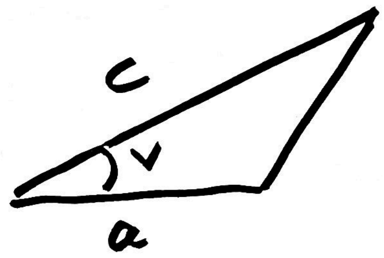
Arealet er $A = \frac{1}{2} a \cdot b$



$$A = \frac{1}{2} a \cdot (c \sin(v))$$
$$= \frac{1}{2} ac \cdot \sin(v)$$

Arealsetningen



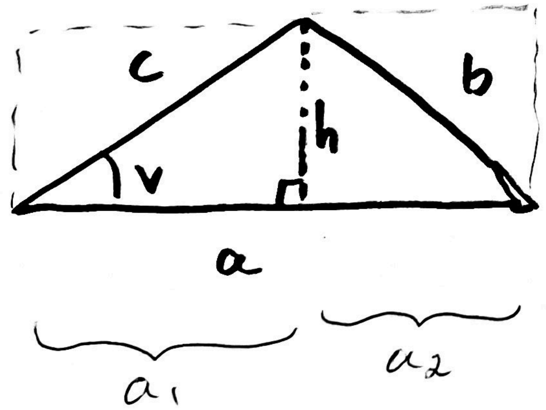


Arealet er lik $\frac{1}{2} a \cdot c \sin(v)$

v vinkelen mellom sidene med lengde a og c.

Forklaring

v spissvinkel



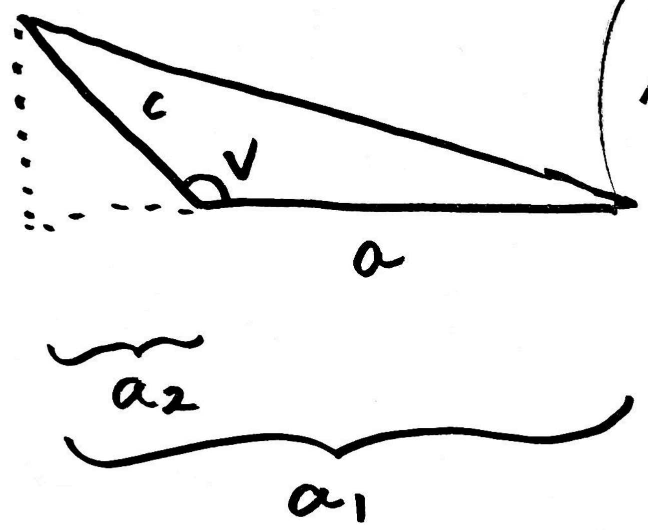
høyde $h = c \sin v$

Areal $A = \underbrace{\frac{1}{2} a \cdot h}_{\text{bredde} \cdot \text{høyde}} = \frac{1}{2} ac \sin v$

$$A = \frac{1}{2} a_1 \cdot h + \frac{1}{2} a_2 \cdot h = \frac{1}{2}(a_1 + a_2) h = \frac{1}{2} a \cdot h$$

v butt

$h = c \sin(v)$



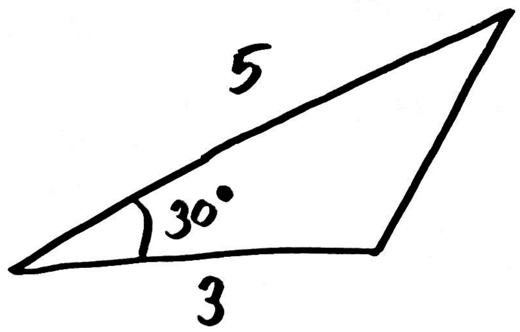
Areal = (Areal til stor rettvinkle Δ − Areal til liten rettvinkle Δ)

$a = a_1 - a_2$

$$A = \frac{1}{2} a_1 \cdot h - \frac{1}{2} a_2 \cdot h = \frac{1}{2}(a_1 - a_2) h$$

$$= \frac{1}{2} a \cdot h = \underline{\frac{1}{2} a \cdot c \sin v}$$

Eksempel



$$A = \frac{3 \cdot 5 \cdot \overbrace{\sin(30^\circ)}^{1/2}}{2}$$

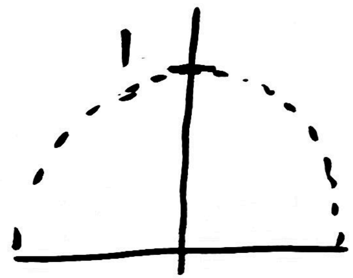
$$= \underline{\frac{15}{4}} = \frac{12+3}{4} = \underline{3.75}$$

Finn vinkelen mellom to sider med lengde 4 og 5

når arealet er lik a) 10 b) 5 c) 20.

$\frac{1}{2} \cdot 4 \cdot 5 \cdot \sin v = A$

$10 \sin(v) = A$

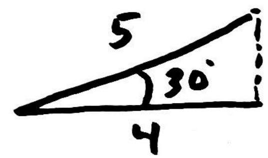


a) $10 \sin v = 10$ så $\sin v = 1$

så $v = 90°$ (vi får en rettvinklet trekant)

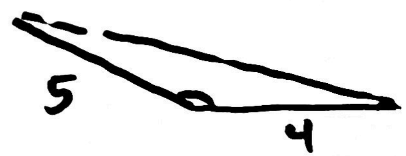
b) $A = 5 = 10 \sin v$ så $\sin v = 1/2$

$\underline{v = 30°}$ $(\arcsin(1/2))$



en annen løsning er $180° - 30° = 150°$

(reflekterer om y-aksen)



c) $A = 10\sin v = 20$ så $\sin v = 2$

<u>ingen løsning</u>

oppgave 1) Δ med sider 2 og 7

og $\angle$ mellom sidene som er lik 45°.

Finn arealet.

$$A = \frac{2 \cdot 7}{2} \cdot \sin(v) = 7 \cdot \overbrace{\sin(45^\circ)}^{1/\sqrt{2}}$$

$$= 7/\sqrt{2}.$$

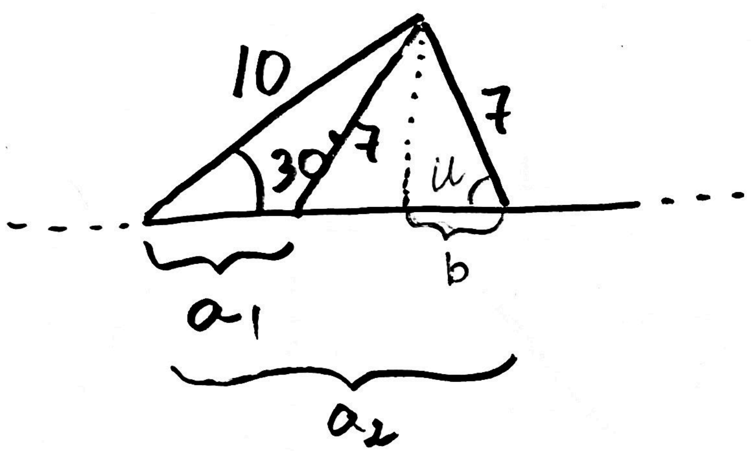
oppg. Arealet er lik $A = 7$

Hva er vinkelen mellom to sider med lengde 2 og 10?

$A = 7 = \frac{1}{2} \cdot 2 \cdot 10 \sin v$ så $\sin v = 7/10 = 0.7$. så $v = \sin^{-1} 0.7 = 44.43^\circ$

og $180^\circ - 44.4^\circ = \underline{135.6^\circ}$

En Δ har en vinkel som er $30°$. Motstående side til vinkelen har lengde 7. Én av de andre sidene har lengde 10. Hva er lengden til den tredje siden?



høyde $10 \cdot \sin 30° = 10 \cdot \frac{1}{2} = \underline{5}$

$7 \sin u = 5$

$b = 7 \cdot \cos u$

eller Pytagoras $b^2 = 7^2 - 5^2 = 49 - 25 = 24$

$b = \sqrt{24} = \sqrt{4 \cdot 6} = \underline{2\sqrt{6}}$

$c = \sqrt{10^2 - 5^2} = \sqrt{5^2(2^2-1)} = \underline{5\sqrt{3}}$

To muligheter $a_2 = c + b = \underline{5\sqrt{3} + 2\sqrt{6}}$ og $a_1 = c - b = \underline{5\sqrt{3} - 2\sqrt{6}}$

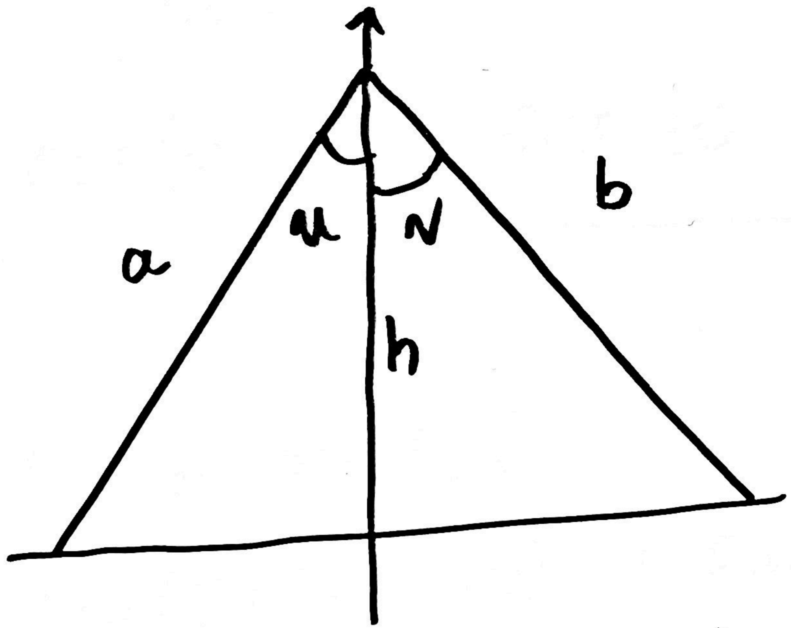
Bevis for addisjons-formlene for sin og cos.

10 G

alle u og v

$$\sin(u+v) = \sin(u)\cos(v) + \sin(v)\cos(u)$$

Reduser til vinkler i 1. kvadrant $0 \le u, v \le 90°$



Totalt areal (begge Δ-ene)

er $\underline{\frac{1}{2} a \cdot b \sin(u+v)}$

= summen av arealene til de to rettvinkla Δ-ene.

$$\frac{1}{2} a \cdot h \sin u + \frac{1}{2} b \cdot h \sin v$$

høyden $h = b\cos v = a\cos u$

$$ab\sin(u+v) = a(\underbrace{b\cos v}_{h})\sin u + (\underbrace{a\cos u}_{h}) \cdot b\sin v \quad \Big| \text{ deler med } a \cdot b$$

$$\sin(u+v) = \sin u \cdot \cos v + \sin v \cdot \cos u \quad \square$$

$$\cos(u+v) = \sin(90^\circ - (u+v))$$

$$= \sin((90^\circ - u) + (-v))$$

benytter addisjons-
formelen for sin

$$\underbrace{\sin(90^\circ - u)}_{\cos u} \underbrace{\cos(-v)}_{\cdot \cos v} + \underbrace{\sin(-v)}_{-\sin v} \underbrace{\cos(90^\circ - u)}_{\cdot \sin u}$$

$$\cos(u+v) = \underline{\cos u \cdot \cos v - \sin u \sin v}$$

u sendes til $180^\circ - u$

Refleksjon om y-aksen

$$\sin((180^\circ - u) - v) = \sin(180^\circ - u)\cos(-v) + \sin(-v)\cos(180^\circ - u)$$

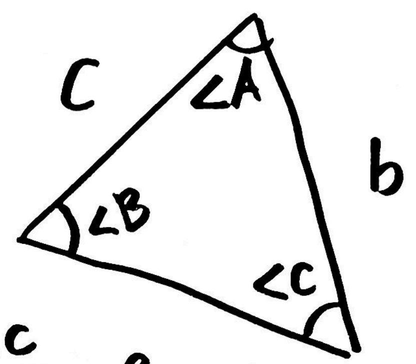
$$= \sin u \cos v - \sin(-v) \cos u$$

$$\sin(u+v) = \sin u \cos v + \sin v \cos u$$

Add formel gyldig for u og v hvis gyldig for $180^\circ - u$ og v

10 D

Sinussetning



$$\frac{\sin(\angle A)}{a} = \frac{\sin \angle B}{b} = \frac{\sin \angle C}{c}$$

bevis: Arealsetningen gir at arealet til Δ- er lik

$$A = \tfrac{1}{2} bc \sin \angle A = \tfrac{1}{2} ac \sin \angle B = \tfrac{1}{2} ab \sin \angle C$$

$$\frac{2A}{abc} = \frac{\sin \angle A}{a} = \frac{\sin \angle B}{b} = \frac{\sin \angle C}{c}$$

Tilsvarende

10.47 $\triangle ABC$ $\angle A = 25°$

$AB = 12cm$ og $AC = 7cm$

a) Finn arealet

$$A = \frac{1}{2} 12cm \cdot 7cm \sin(25°) = 42cm^2 \cdot \overbrace{\sin(25°)}^{0.4226}$$

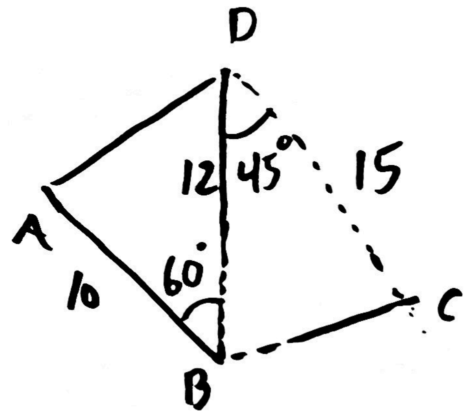
$$= \underline{17.75\ cm^2}$$

10.48 Firkant ABCD

$\angle ABD = 60°$ $\angle BDC = 45°$

$AB = 10$ $BD = 12$ $DC = 15$

Finn arealet til firkanten.



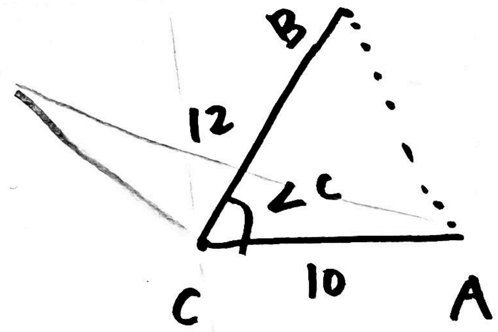
(sum av arealene til de to $\triangle$)

$$A = \frac{1}{2} 10 \cdot 12 \sin 60° + \frac{1}{2} \cdot 12 \cdot 15 \cdot \sin(45°)$$

$$60 \frac{\sqrt{3}}{2} + 6 \cdot 15 \cdot \frac{1}{\sqrt{2}} = 30\sqrt{3} + 90/\sqrt{2}$$

$$51.96 + 63.63 = \underline{115.60}$$

10.49 a)



Arealet er lik 50.

Arealsetningen : $\frac{1}{2} |AC| \cdot |CB| \cdot \sin(\angle C) = 50$

$\frac{1}{2} \cdot 10 \cdot 12 \cdot \sin(\angle C) = 50$

$\sin(\angle C) = \frac{2 \cdot 50}{10 \cdot 12} = \frac{10}{12} = \frac{5}{6}$

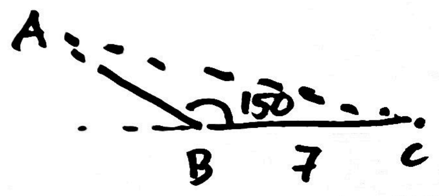
$\sin^{-1}(5/6) = \underline{56.4^\circ}$

En annen løsning er $180^\circ - \sin^{-1}(5/6)$

$\sim \underline{123.6^\circ}$

b) Finn AB i ΔABC hvor $BC = 7.00$ Areal 10.5

$B = 150^\circ$



$\frac{1}{2} AB \cdot BC \sin \angle B = \text{Areal}$

$AB = \frac{2 \cdot 10.5}{7 \sin(150^\circ)} = \frac{21}{7 \sin 30^\circ)}$

$= \frac{3}{0.5} = \underline{6.00}$

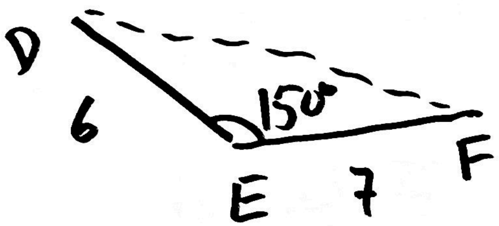
c) $\triangle DEF$

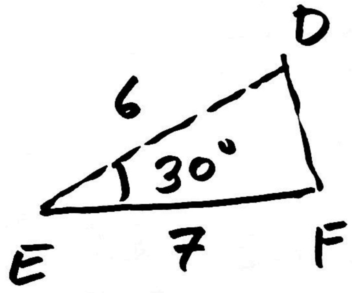
$DE = AB = 6.00$

$EF = BC = 7.00$

Finn $\angle E$

samme areal.





Det er to muligheter

$\angle E = 150°$ eller $\angle E = 30°$

($\frac{1}{2}\sin(\angle E) \cdot EF \cdot ED = 10.5$

$7 \cdot 6$

så $\sin(\angle E) = \frac{21}{3 \cdot 2 \cdot 7} = \frac{1}{2}$

...)

se på oppg 10.53 ... (blå oppgave)

10.53 a, b) muntlig

c) $A_{I} = \frac{1}{2} 3{,}2 \cdot 4.8 \cdot \sin 35^\circ$

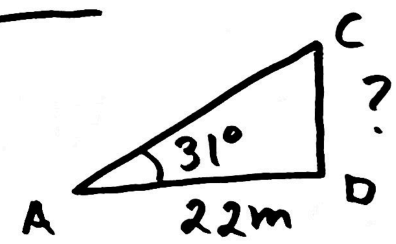
Arealet til trekant I

$= \underbrace{3.2 \cdot 2.4}_{7.68} \sin 35^\circ = \underline{4.405}$

Trekant II: $A_{II} = \underbrace{\frac{1}{2} \cdot 3.2 \cdot 4.8}_{7.68} \cdot \sin(33^\circ)$

$= \underline{4.183}$

10.54 a) $\frac{CD}{22m} = \tan(31^\circ)$



$CD = 22.0m \cdot \tan(31^\circ)$

$= \underline{13.2m}$

b) ΔADC har areal

$\frac{1}{2} 13.2m \cdot 22m = 13.2 \cdot 11 m^2$

$= (132 + 13.2) m^2 = \underline{145.2 m^2}$

c) $\frac{\#\text{tre}}{\text{areal}} = \frac{1}{9}$

Antall tre (forventet) $= \frac{1}{9} \cdot$ areal i m^2.

Arealet til $\square ABCD$ er ved arealsetningen

lik $\frac{1}{2}\, 22.0\,m \cdot 16\,m \cdot \sin(31.0° + 32.0°)$

$11 \cdot 16\,m^2 \sin(63°)$

$176\,m^2 \cdot (0.891)$

$= \underline{156\,m^2}$

Forventet antall trær er lik $156\,m^2 \cdot \frac{1}{9\,m^2}$

$\underline{17.4 \text{ trær}}$