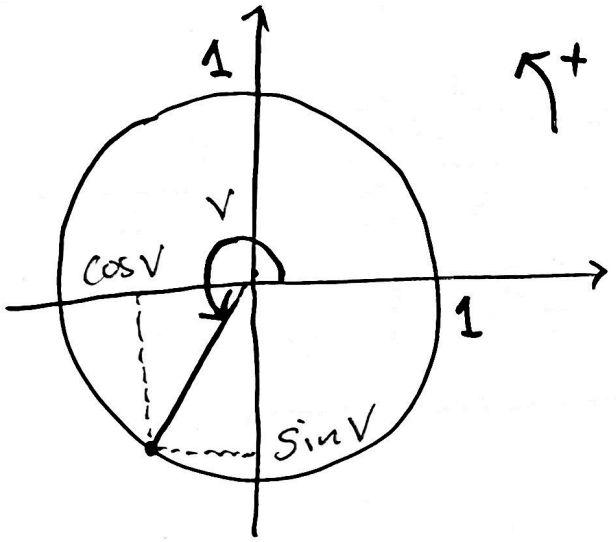
15 januar
25

10 F og G
Radianer
Trigonometriske identiteter.



↑+

For alle V

Pytagoras

$|\cos V|^2 + |\sin V|^2 = 1$

$\cos^2 V + \sin^2 V = 1$

Minner om $\sin^2 V = (\sin V)^2$

$\sin^5 V = (\sin V)^5$

men $\sin^{-1} V$ er ikke $(\sin V)^{-1}$!!

↑ invers sinus funksjon $\arcsin(V)$

Skriv heller $(\sin V)^{-1} = \frac{1}{\sin V}$.

Sin v bestemmer cos v opp til fortegn

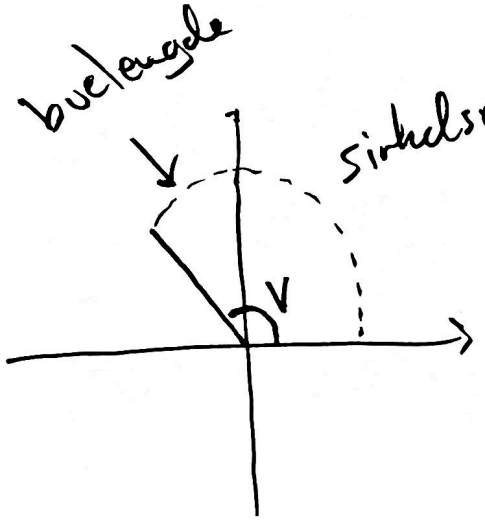
$$\cos^2 v = 1 - \sin^2 v$$

$$\cos v = \pm\sqrt{1 - \sin^2 v}$$

én av dem.

$$\frac{\cos^2 v}{\cos^2 v} + \frac{\sin^2 v}{\cos^2 v} = \frac{1}{\cos^2 v}$$

$$1 + \tan^2 v = \frac{1}{\cos^2 v}$$



vinkel i radian

$$v_{rad} = \frac{\text{buelengde}}{\text{radius}} = \frac{b}{r}$$

på enhetssirkelen er v_{rad} buelengden

$$\frac{\pi \text{ rad} = 180^\circ}{}$$

$$v_{rad} = \frac{\pi}{180^\circ} \cdot v_{deg}$$

$60^\circ = \frac{\pi}{3}$ rad

$30^\circ = \frac{\pi}{6}$ rad

$45^\circ = \frac{\pi}{4}$ rad

$90^\circ = \frac{\pi}{2}$ rad etc

$$\pi^\circ = \frac{\pi^2}{180} \text{ rad}$$

$$10 \text{ rad} = \frac{180^\circ}{\pi \text{ rad}} \cdot 10 \text{ rad}$$

$$= \frac{1800}{\pi}{}^\circ$$

oppg

Endre enhet:

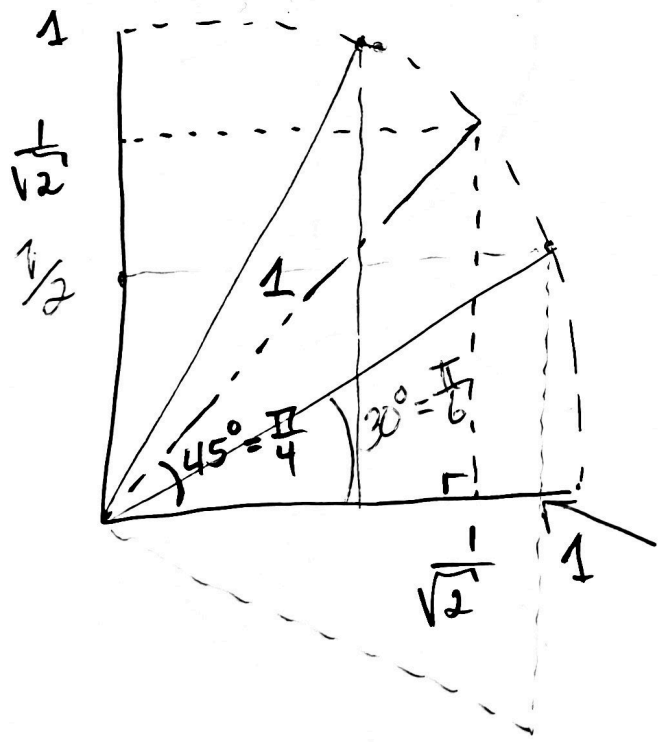
$$1\,\text{rad} = 1\,\text{rad} \cdot \frac{180^\circ}{\pi\,\text{rad}} = \frac{180^\circ}{\pi} \sim 57.3^\circ$$

$$1^\circ = 1^\circ \cdot \frac{\pi\,\text{rad}}{180^\circ} = \frac{\pi}{180}\,\text{rad} \sim 0.01745\,\text{rad}$$

$$60^\circ = 60^\circ \cdot \frac{\pi}{180^\circ} = \frac{\pi}{3} \sim 1.05\,\text{rad}.$$

$$\left(\frac{\pi\,\text{rad}}{180^\circ} = \right.$$

$$\left(\frac{1\,\text{m}}{100\,\text{cm}} = 1\right.$$



$$\sin\left(\frac{\pi}{4}\right) = \cos\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}} \sim 0.707$$

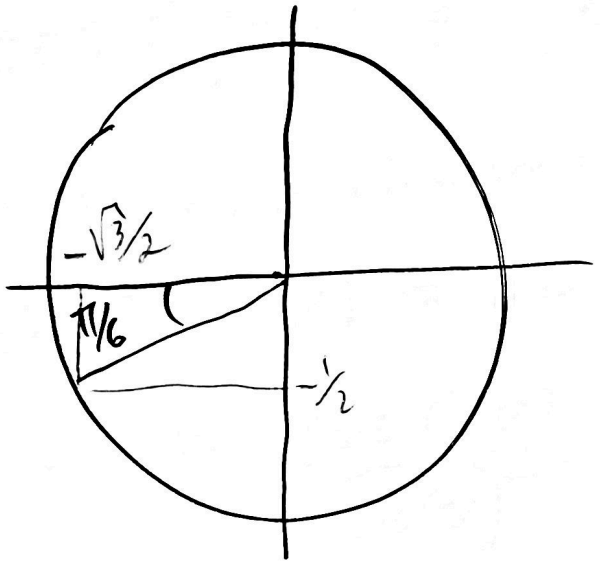
$$\sin\left(\frac{\pi}{6}\right) = \frac{1}{2} \qquad \cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$$

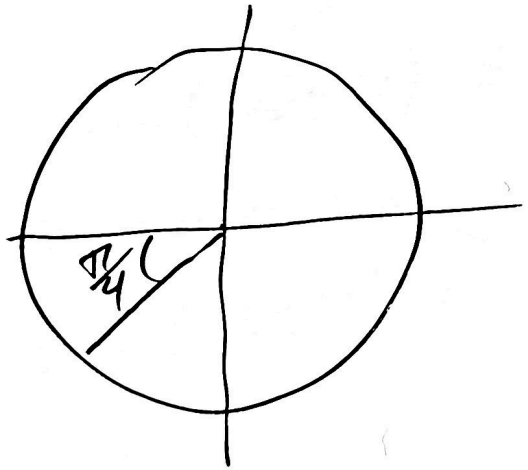
$$\sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2} \qquad \cos\left(\frac{\pi}{3}\right) = \frac{1}{2}$$

$$\sqrt{1 - \left(\frac{1}{2}\right)^2} = \frac{\sqrt{3}}{2} \sim 0.866$$

Hva er sin og cos til vinkelen $\frac{7\pi}{6}$?

$\frac{-3\pi}{4}$?





$$\frac{7\pi}{6} = \pi + \frac{\pi}{6}$$

$$\cos\left(\frac{7\pi}{6}\right) = -\frac{\sqrt{3}}{2}$$

$$\sin\left(\frac{7\pi}{6}\right) = \frac{-1}{2}$$

$$-\frac{3\pi}{4} = -\pi + \frac{\pi}{4}$$

$$\sin\left(\frac{-3\pi}{4}\right) = \cos\left(\frac{-3\pi}{4}\right) = \frac{-1}{\sqrt{2}}.$$

10G Addisjonsformlene for sin og cos.

$$\sin(u+v) = \sin u \cdot \cos v + \sin v \cdot \cos u$$

$$\cos(u+v) = \cos u \cdot \cos v - \sin u \cdot \sin v$$

differanse av vinkler:

$\sin(u-v) = \sin(u + (-v))$ setter inn i addisjons-formelen

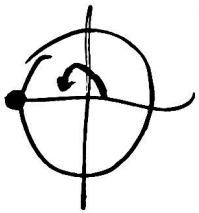
$$\sin u \cdot \underbrace{\cos(-v)}_{\cos(v)} + \underbrace{\sin(-v)}_{-\sin(v)} \cos u$$

så $\sin(u-v) = \sin u \cdot \cos v - \sin v \cos u$

Dette kan oppsummeres som

$$\sin(u \pm v) = \sin u \cos v \pm \sin v \cos u.$$

$$\sin(180^\circ - v) = \underbrace{\sin(180^\circ)}_{0}\cos(-v) + \sin(-v)\underbrace{\cos(180^\circ)}_{-1}$$



$$= -\sin(-v) = -(-\sin(v))$$

$$\sin(180^\circ - v) = \sin(v) \quad \checkmark$$

Tilsvarende $\sin(90^\circ - v) = 1 \cdot \cos(-v) + 0$

$$= \cos(v) \quad \checkmark$$

$$\sin\left(\frac{\pi}{4} + v\right) = \underbrace{\sin\left(\frac{\pi}{4}\right)}_{1/\sqrt{2}}\cos v + \sin v \cdot \underbrace{\cos\left(\frac{\pi}{4}\right)}_{1/\sqrt{2}}$$

$$= \frac{1}{\sqrt{2}}(\cos v + \sin v)$$

(Så $\cos v + \sin v = \sqrt{2}\sin\left(v + \frac{\pi}{4}\right)$)

$$\tan(u+v) = \frac{\sin(u+v)}{\cos(u+v)} = \frac{\sin u \cdot \cos v + \sin v \cdot \cos u}{\cos u \cos v - \sin u \cdot \sin v}$$

deler teller og nevner med $\cos u \cdot \cos v$

$$\tan(u+v) = \frac{\tan u + \tan v}{1 - \tan u \cdot \tan v}$$

$$\tan(90^\circ) = \tan(45^\circ + 45^\circ) = \frac{1+1}{1-1\cdot 1} = \frac{2}{0}$$

eksisterer ikke

oppg 10.99 Eksakt verdier til sin og cos av 75° og 15°.

$75^\circ = 30^\circ + 45^\circ$ $\qquad$ $15^\circ = 45^\circ - 30^\circ$

$$\sin(75^\circ) = \underset{1/2}{\sin(30^\circ)}\ \underset{1/\sqrt{2}}{\cos(45^\circ)} + \underset{1/\sqrt{2}}{\sin(45^\circ)}\ \underset{\sqrt{3}/2}{\cos(30^\circ)} = \frac{1+\sqrt{3}}{2\sqrt{2}}$$

$$\sin(15^\circ) = \underset{-1/2}{\sin(-30^\circ)}\ \underset{1/\sqrt{2}}{\cos(45^\circ)} + \underset{1/\sqrt{2}}{\sin(45^\circ)}\ \underset{\sqrt{3}/2}{\cos(-30^\circ)} = \frac{-1+\sqrt{3}}{2\sqrt{2}}$$

$$\cos(15^\circ) = \sin(75^\circ) = \frac{\sqrt{3}+1}{2\sqrt{2}} \sim 0.966$$

$$\cos(75^\circ) = \sin(15^\circ) = \frac{\sqrt{3}-1}{2\sqrt{2}} \sim 0.2588$$

Dobling av vinkel

$u = v$

$$\sin(2u) = 2\sin u \cos u$$

$$\cos(2u) = \cos^2 u - \sin^2 u$$

kombineres med Pythagoras $\cos^2 u + \sin^2 u = 1$

$$\cos(2u) = 1 - 2\sin^2 u \quad (= 1 - 2(\sin u)^2)$$

$$= 2\cos^2 u - 1 \quad (= 2\cdot(\cos u)^2 - 1)$$

5a)

$$\cos^2 u = \frac{1}{2}(1 + \cos(2u)) \qquad \text{alle } u.$$

$$\sin^2 u = \frac{1}{2}(1 - \cos(2u))$$

tilsvarende $\tan(2u) = \dfrac{2\tan u}{1-\tan^2 u}$.

Eksakte verdier av sin og cos til $22.5°$ og $67.5°$.
$(\frac{1}{2}\cdot 45°)$ $(90°-22.5°)$

$$\sin^2(22.5°) = \frac{1}{2}\left(1-\cos(\overbrace{2\cdot 22.5°}^{45°})\right) = \frac{1}{2}\left(1-\frac{1}{\sqrt{2}}\right)$$

$$\sin(22.5°) = \sqrt{\frac{(1-1/\sqrt{2})\cdot 2}{2\cdot 2}} = \frac{\sqrt{2-\sqrt{2}}}{2}$$

positiv

$$\cos^2(22.5°) = \frac{1}{2}\left(1+\cos(45°)\right), \text{ så } \cos(22.5°) = \frac{\sqrt{2+\sqrt{2}}}{2}$$

$$\sin(22.5°) = \cos(67.5°) = \frac{\sqrt{2-\sqrt{2}}}{2} \sim 0.3827$$

$$\sin(67.5°) = \cos(22.5°) = \frac{\sqrt{2+\sqrt{2}}}{2} \sim 0.9239$$

$\sin V = 0.6 = \frac{3}{5}$

V ligger i 2 kvadrant.

1) Hva er $\cos V$

2) $\sin(2V)$

3) $\tan(2V)$

Pytagoras

$\cos^2 V = 1 - \sin^2 V$

$= 1 - (\frac{3}{5})^2 = \frac{25-9}{5^2}$

$= (\frac{4}{5})^2$

$\cos V < 0$

siden 2kvadrant

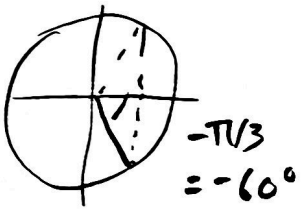
1) $\cos V = -0.8 = -4/5$.

Gjennomgang 10:45

2) $\sin(2V) = 2\sin V \cos V = 2 \cdot \frac{3}{5} \cdot (\frac{-4}{5}) = -\frac{24}{25} = -0.96$

3) $\tan(2V) = \frac{2\tan V}{1-\tan^2 V} = \frac{2(-3/4)}{1-(-3/4)^2} = \frac{-3/2}{1-9/16} \cdot \frac{16}{16} = \frac{-24}{7}$

$(\tan V = \frac{\sin V}{\cos V} = \frac{3/5}{-4/5} = \frac{-3}{4})$



Se gjerne på oppg. 10.100, 102, 107 og 108

Gjennomgang 11:45.

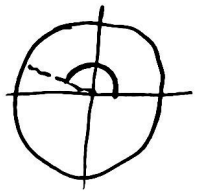
10.100 a) Tidligere

b) $-2\sin(x-\frac{\pi}{3}) = -2(\sin x \underbrace{\cos(\frac{-\pi}{3})}_{1/2} + \sin(\frac{-\pi}{3}) \cos x)$ $\quad -\sqrt{3}/2$

$= -\sin x + \sqrt{3}\cos x$

10.102 $\sin x = \frac{1}{3}$ $\quad x \in [90°, 180°]$

Finn $\cos x$ $\quad \sin 2x$ og $\tan x$



$\cos x < 0$

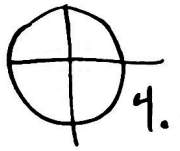
Pytagoras $\quad \cos^2 x = 1 - \sin^2 x = 1 - \frac{1}{9} = \frac{8}{9}$

$\cos x < 0$ gir $\quad \underline{\cos x = -\frac{2\sqrt{2}}{3}}$

$$\sin(2x) = 2\sin x \cos x = 2 \cdot \frac{1}{3} \cdot \left(-\frac{2\sqrt{2}}{3}\right) = \underline{-\frac{4\sqrt{2}}{3}}$$

$$\tan x = \frac{\sin x}{\cos x} = \frac{1/3}{-2\sqrt{2}/3} = \underline{\frac{-1}{2\sqrt{2}}}$$

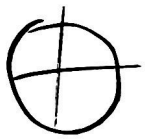
10.107 Bestem $\sin v$ og $\sin 2v$ når $\quad$ a) v i 4kvadrant og $\cos v = \frac{3}{4}$.



Pyt: $\quad \sin v = -\sqrt{1 - \cos^2 v} = -\sqrt{1 - (3/4)^2} = \underline{-\frac{\sqrt{7}}{4}}$

$$\sin(2v) = 2\sin v \cos v = \underline{-\frac{3\sqrt{7}}{8}}$$

b) v i 3. kvadrant $\quad \tan v = 2$.

3. 

$$\frac{\cos^2 v + \sin^2 v}{\sin^2 v} = \frac{1}{\sin^2 v}$$

$$\frac{1}{\tan^2 v} + 1 = \frac{1}{\sin^2 v}$$

Så $\frac{1}{\sin^2 v} = 1 + \frac{1}{4} = \frac{5}{4} \quad \Leftrightarrow \quad \sin^2 v = \frac{4}{5}$

$\sin v = \underline{-\frac{2}{\sqrt{5}}}$ $\qquad \cos v = -\sqrt{1 - \frac{4}{5}} = -\sqrt{\frac{1}{5}}$

$$\sin(2v) = 2 \cdot \sin v \cos v$$

$$= 2\left(\frac{-2}{\sqrt{5}}\right) \cdot \left(\frac{-1}{\sqrt{5}}\right) = \underline{\frac{4}{5}}$$

10.108 $f(v) = 1 + 5\cos^2 v + 3\sin^2 v$

a) Største og minste verdi til $f(x)$

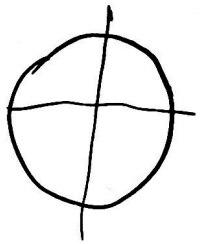
$$f(v) = 1 + 2\cos^2 v + 3\underbrace{(\cos^2 v + \sin^2 v)}_{1}$$

$$= 4 + 2\cos^2 v$$

$$0 \le \cos^2 v \le 1$$

Minste verdi: for $f(v)$ er 4 ($v = \frac{\pi}{2} = 90° \ldots$)

Største verdi: — er 6 ($v = 0, \pi \ldots$)

b) 

største $\cos^2 v = 1 \iff \cos v = \pm 1$

1 omløp $v = 0$ og π radian

minst $\cos^2 v = 0 \iff \cos v = 0$

$v = \frac{\pi}{2}$ og $\frac{3\pi}{2}$.