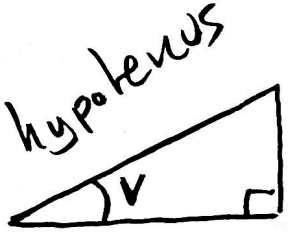
13 januar
25

Kap 10 Trigonometri



Sinus

$$\sin(v) = \frac{\text{motstående kabet}}{\text{hypotenus}}$$

cosinus

$$\cos(v) = \frac{\text{hossliggende kabet}}{\text{hypotenus}}$$

$v = 0°$

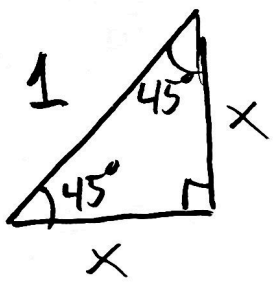
$\sin(0°) = 0$

$\cos(0°) = 1$

$v = 90°$

$\sin(90°) = 1$

$\cos(90°) = 0$



$$\sin(45^\circ) = \cos(45^\circ) = \underline{\frac{1}{\sqrt{2}} \sim 0.7071067\ldots}$$

Pytagoras: $x^2 + x^2 = 1$

$$2x^2 = 1 \iff x^2 = \frac{1}{2} \qquad x > 0$$

$$x = \sqrt{\frac{1}{2}} = \frac{1}{\sqrt{2}}$$

$\sin(15^\circ) \sim 0.2588\ldots$

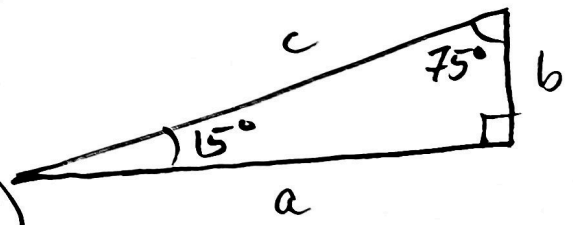
$\cos(15^\circ) \sim 0.9659\ldots$

like

like

$\sin(75^\circ) \sim 0.9659\ldots$

$\cos(75^\circ)$



Generelt $\sin(v) = \cos(90^\circ - v)$

$\cos(v) = \sin(90^\circ - v)$

Denne uken

10 A, B

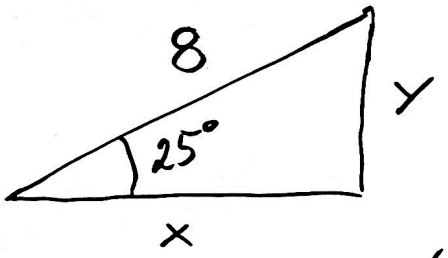
B og F (radianer)

C og G

C, D og E Sinus- og cosinussetningene.

Neste uke

oppg.



Finn

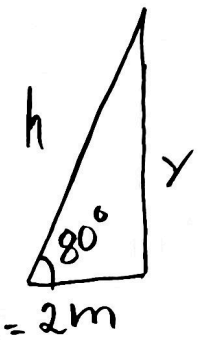
$x \sim 7.25046...$

og $y \sim 3.38$ $(8 \cdot \sin(25°))$

(Alternativt benytte Pytagoras

$y = \sqrt{8^2 - x^2}$)

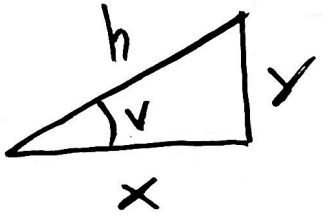
oppg



Hva er hypotenus h ? $h = 11.51754....$

Hva er y ? $y = h \cdot \sin(80°) \sim 11.34256....$

$\cos 80° = \frac{x}{h} \Leftrightarrow h = \frac{x}{\cos(80°)}$



$$\sin(v) = \frac{y}{h} \qquad \cos(v) = \frac{x}{h}$$

$$\tan(v) = \frac{\text{motstående katet}}{\text{hosliggende katet}}$$

$$= \frac{y}{x} = \frac{y/h}{x/h} \qquad (x \neq 0)$$

tangensfunksjonen: $\tan(v) = \dfrac{\sin(v)}{\cos(v)}$

$\tan(0°) = 0$

$\tan(45°) = 1$

$\tan(90°)$ ikke definert.

Fra forrige oppgave

$\tan(80°) = \dfrac{y}{2m} \sim 5.67128..$

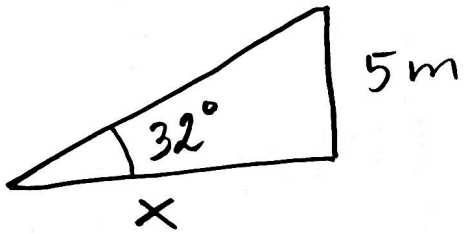
$y = 2m \tan(80°) \sim 11.3425... \text{ m}$

cotangens $\cot(v) = \dfrac{\text{hosliggende katet}}{\text{motstående katet}}$.

$\cot(0°)$ eksisterer ikke $\qquad \cot(90°) = 0$

$\tan(v) \cdot \cot(v) = 1$ (når begge eksisterer)

oppg.



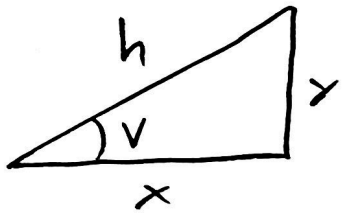
Hva er x

$\tan(32°) = \frac{5m}{x}$

$x = \frac{5m}{\tan(32°)} \sim 8.0016726...\ m$

Invers trigonometriske funksjoner

$\sin V = \frac{y}{h}$



invers sinus funksjonen

$\sin^{-1}$

arcsin

(ärkussinus)

$0 \leq x \leq 1$

$\sin^{-1} x$ vinkelen mellom $0°$ og $90°$

slik at $\sin(\sin^{-1}(x)) = x$

$\sin^{-1}(0.707) \sim 44.99134...°$

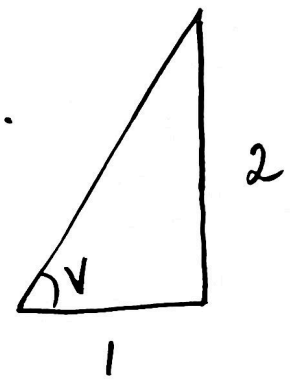
$\sin^{-1}(0.99) \sim 81.89038...°$

Tilsvarede

$\cos^{-1}$ = arccos

$\tan^{-1}$ = arctan

oppg.

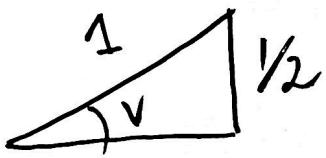


Hva er vinkelen V?

$\tan(V) = \frac{2}{1}$

$V = \tan^{-1}(2) \sim 63.43494...°$

oppg.



Hva er V?

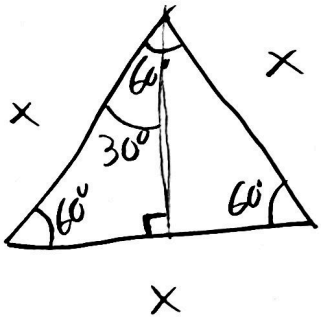
$$\sin(V) = \frac{1/2}{1} = \frac{1}{2}$$

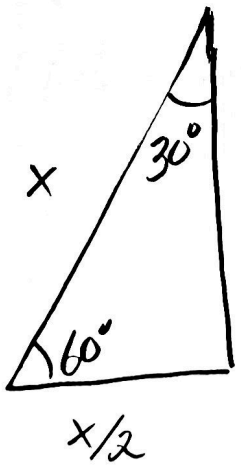
$$V = \sin^{-1}\left(\frac{1}{2}\right) = \arcsin\left(\frac{1}{2}\right) = 30°$$

$$\sin(30°) = 1/2$$

10B Eksakte verdier til trig. funksjoner

Likesidet Δ





Pythagoras

$$\sqrt{x^2-\left(\frac{x}{2}\right)^2} = x\sqrt{1-\frac{1}{4}} = x\sqrt{\frac{3}{4}} = x\cdot\frac{\sqrt{3}}{2}$$

$\sin(30°) = \cos(60°) = 1/2$

$\sin(60°) = \cos(30°) = \sqrt{3}/2 \sim \underline{0.8660254}$

$\tan(60°) = \sqrt{3} \sim 1.732$

$\tan(30°) = 1/\sqrt{3}$

$\sin(40°)$

$\sin(45°) = \frac{1}{\sqrt{2}} \sim 0.707$

$\sin(30°) = 1/2 = 0.5$

Forventer $0.71 - 0.065 \sim 0.64$?

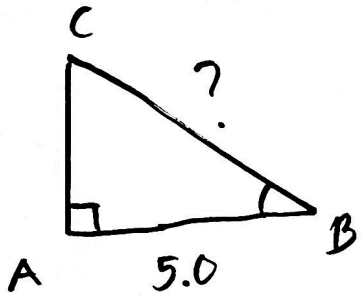
Fakktisk verdi: $\sin(40°) = 0.64278...$

10.18 Δ ABC Hva er AC?

$\angle A = 90°$

$AB = 5.0$

$\tan B = \tan(\angle B) = 0.80$



$\cos(\angle B) = \frac{AB}{BC}$

så $BC = \frac{AB}{\cos(\angle B)}$

$\angle B = \arctan(0.80)$

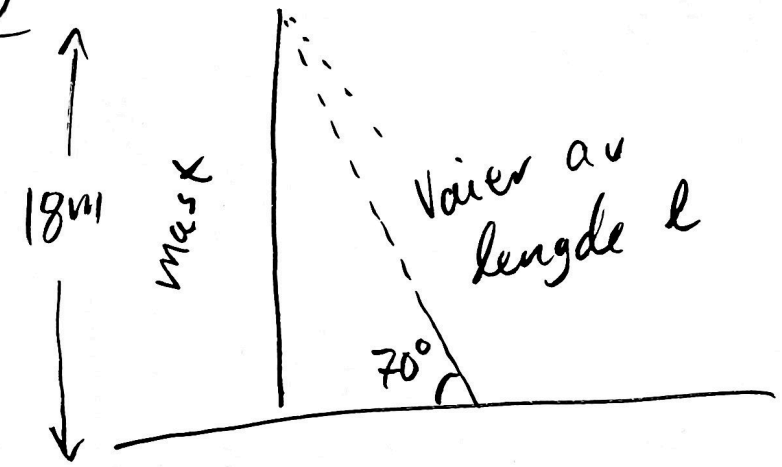
$BC \sim 6.4$

(Alternativt $BC = \sqrt{(AB)^2 + (AC)^2}$)

$\tan(\angle B) = \frac{AC}{AB}$ så $AC = AB \tan(\angle B)$

$= 5.0 \cdot 0.80 = \underline{4.0}$

10.10



$$\sin 70^\circ = \frac{18m}{l}$$

$$l = \frac{18m}{\sin(70^\circ)}$$

$$= \underline{19.2\ m}$$

10.13

ΔABC

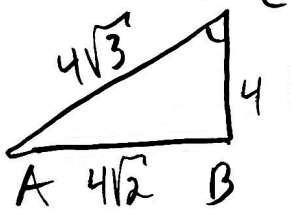
Gale forhold og Vinkler.

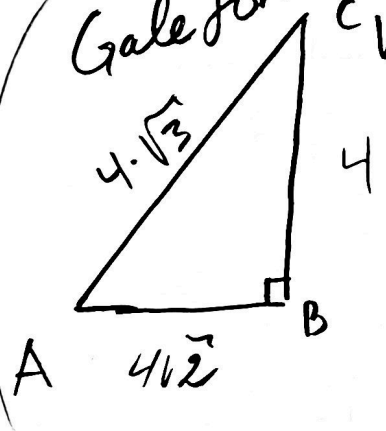
$\angle B = 90^\circ$

$AC = 4 \cdot \sqrt{3}$

$$\cos \angle C = \frac{\sqrt{3}}{3} = \frac{1}{\sqrt{3}} \sim 0.577$$

$$\angle C = \arccos\left(\frac{1}{\sqrt{3}}\right) \sim 54.73^\circ$$





a) $BC = AC \cdot \cos \angle C = 4 \cdot \sqrt{3} \cdot \frac{1}{\sqrt{3}} = \underline{4}$

$AB^2 + BC^2 = AC^2$

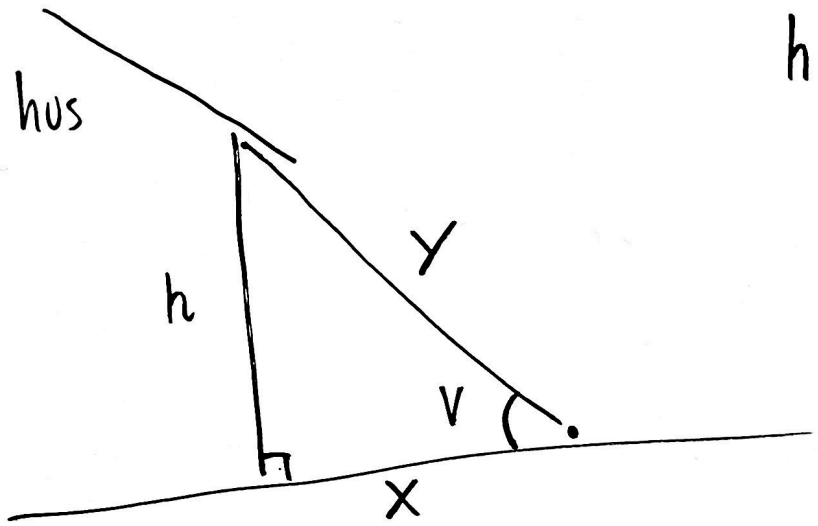
$AB^2 + 4^2 = (4 \cdot \sqrt{3})^2$ så $AB = \sqrt{4^2(3-1)}$

$= \underline{4\sqrt{2}}$

b) $\frac{AB \cdot BC}{2} = \frac{4\sqrt{2} \cdot 4}{2} = \underline{8\sqrt{2}}$

c) Omkrets $AB + BC + CA = \underline{4(\sqrt{2} + 1 + \sqrt{3})}$

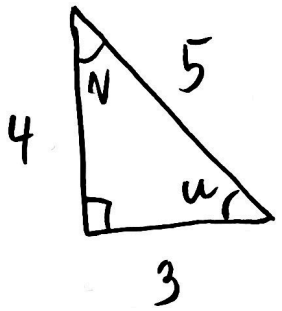
$$h = Y \cdot \sin(V)$$



$$3^2 + 4^2 = 5^2$$

Hva er u og V?

oplg



$$u = \arctan\left(\frac{4}{3}\right) = \arccos\left(\frac{3}{5}\right) = \arcsin\left(\frac{4}{5}\right)$$

$$\sim 53.13^\circ$$

$$V = 90 - u = 36.87^\circ$$