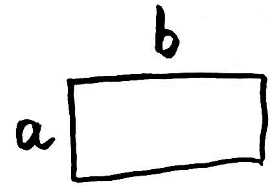
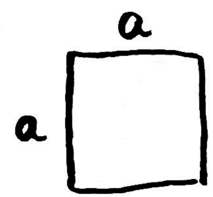
6 januar 25

Geometri

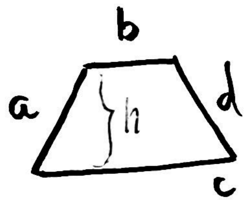


Rektangel

areal $a \cdot b$ omkrets $2a + 2b = 2(a+b)$



Kvadrat

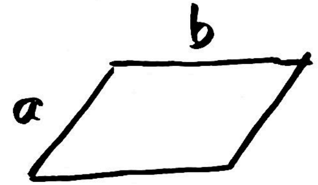


Trapes

(To motstående sider er parallelle)

Areal $\left(\frac{b+c}{2}\right) \cdot h$

omkrets $a+b+c+d$

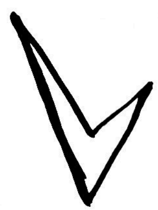


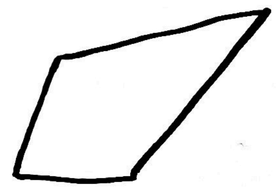
Parallellogram

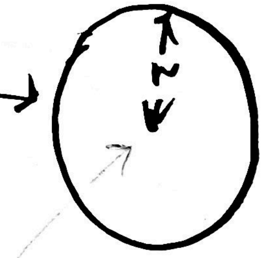
$a=b$ Rombe

Dette er firkanter

Mer generelle firkanter





Sirkel 

senter.

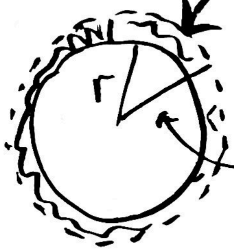
radius r

diameter $2r$

Arealet $\underline{\pi \cdot r^2}$

$\pi \sim 3.14\ldots.$

Omkrets $\underline{2\pi \cdot r}$



$\pi(r+\Delta r)^2$
$- \pi r^2$

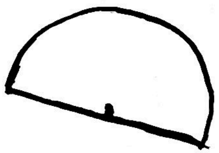
$= 2\pi r \cdot \Delta r + \pi(\Delta r)^2$

Tykkelsen er Δr

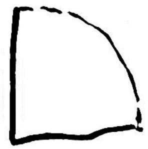
$$\frac{\text{"Areal ring"}}{\text{"tykkelse"}} = \frac{2\pi r(\Delta r) + \pi(\Delta r)^2}{\Delta r}$$

$$= 2\pi r + \pi \cdot (\Delta r)$$

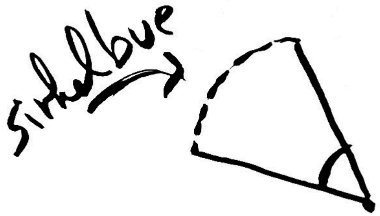
I grensen hvor $\Delta r \to 0$ får vi $\underline{2\pi r}$
som er omkretsen



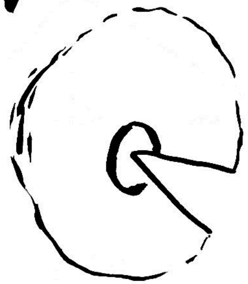
halvsirkel

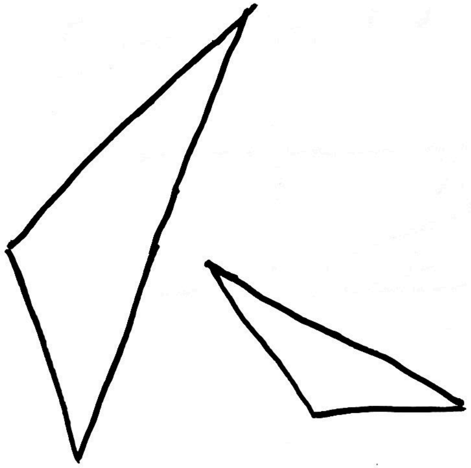


kvart sirkel

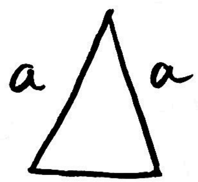


sirkelsegmenter

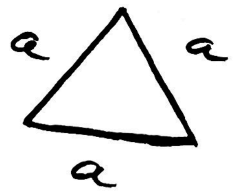




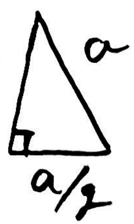
Trekanter



likebeina trekant

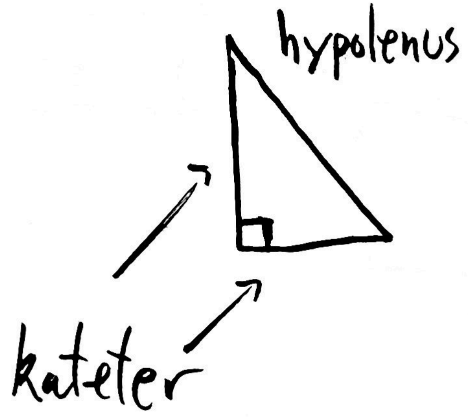


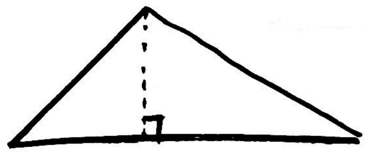
likesidet trekant



halv likesidet trekant
(30-60-90° trekant)

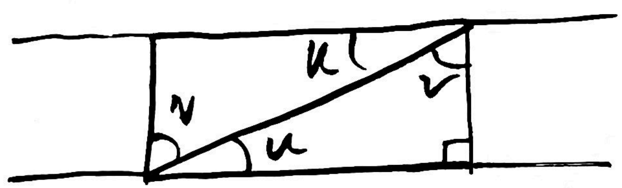
Rettvinklet trekant





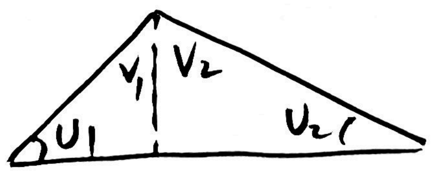
En trekant er satt sammen av to rettvinkla trekanter

Summen av vinklene i en trekant er 180°



$u + v = 90°$ (en rett vinkel)

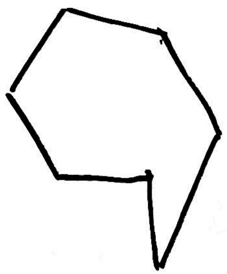
summen av de to ikke-rette vinklene i en rettvinklet trekant er 90°



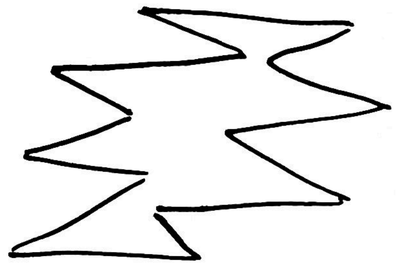
$$(u_1 + v_1) + (v_2 + u_2) = 90° + 90°$$

$$u_1 + (v_1 + v_2) + u_2 = 180°$$

n-kanter

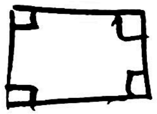


7-kant

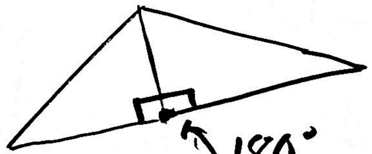


14-kant

Hva er summen av vinklene i en n-kant?

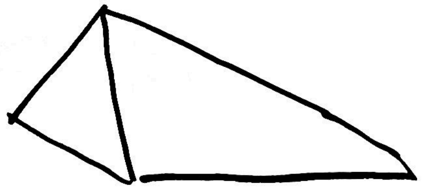


$360°$



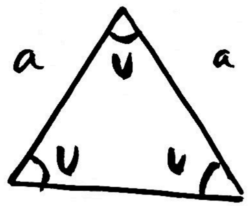
$180°$

sum av vinklen er $180°$



Øker summen av vinklene med $180°$ for hvert hjørne som innføres.

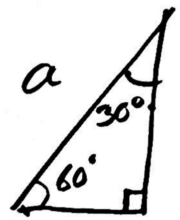
Summen av vinklene i en n-kant er lik $\underline{180°(n-2)}$



likesidet trekant

$3u = 180°$

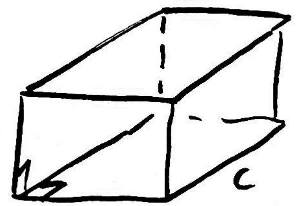
$u = 60°$



halv likesidet trekant.

3D figurer

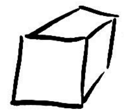
Rett prisme



Volumet er $a \cdot b \cdot c$

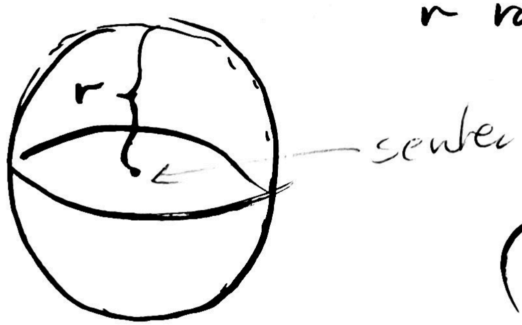
overflate: $2a \cdot b + 2b \cdot c + 2ac$

$a = b = c$



Kube

Kule



r radius

Volumet $\frac{4\pi}{3} r^3$

overflateareal $4\pi r^2$

(Merk at $\frac{d}{dr}\left(\frac{4\pi}{3} \cdot r^3\right) = \frac{4\pi}{3} 3r^2 = \underline{4\pi r^2}$)

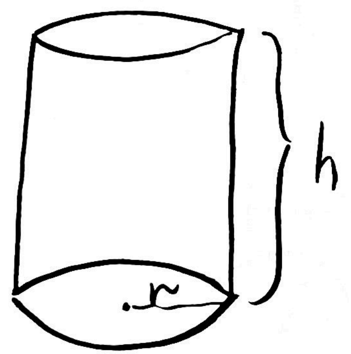
Uttrykk radien til en kule som en funksjon av volumet

$$V = \frac{4\pi}{3} r^3 \qquad (r \geq 0)$$

$$\frac{3}{4\pi} \cdot V = r^3$$

$$r = \sqrt[3]{r^3} = \underline{\sqrt[3]{\frac{3}{4\pi} V}}$$

(Rett)
Sylinder



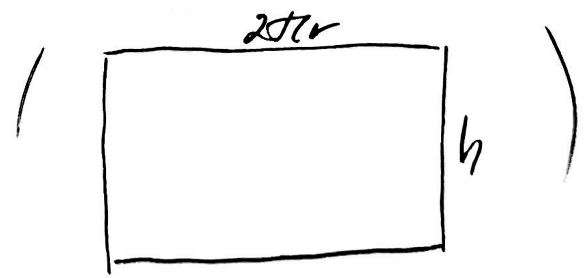
$V = \pi r^2 \cdot h$

overflateareal.

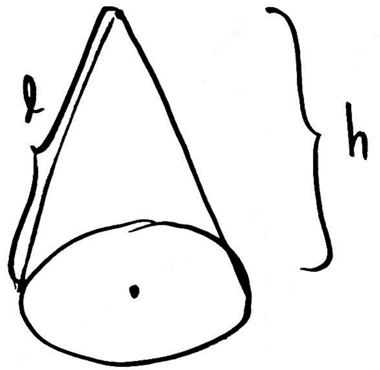
topp : πr^2

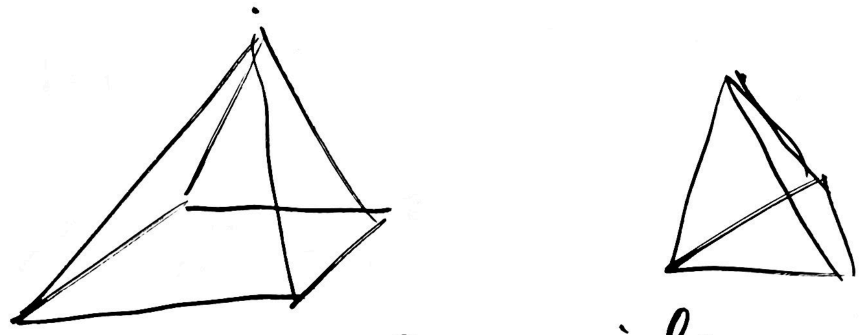
bunn πr^2

sylinderdelen : $2\pi r \cdot h$



Kjegle





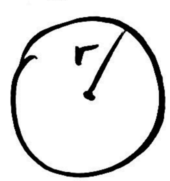
Pyramide

Volumet $V = \frac{1}{3} h \cdot \left(\begin{array}{l}\text{areal} \\ \text{til grunnflaten}\end{array}\right)$

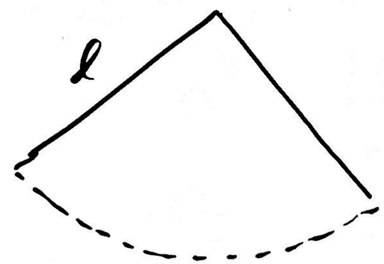
$$V_{kjegle} = \frac{1}{3} h \pi r^2 = \frac{\pi}{3} r^2 h$$

overflate

bunn

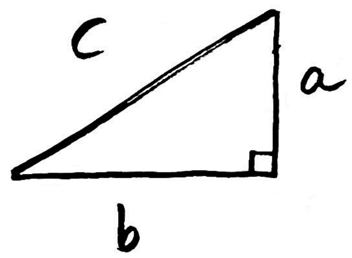


πr^2



$2\pi \cdot r$

C Pytagoras sin sats



$$a^2 + b^2 = c^2$$

To av sidene i en rettvinklet trekant har lengde 3 og 4

Hva kan lengden til den tredje siden være?

1. Katetene har lengde 3 og 4 : hypotenus $= \sqrt{3^2+4^2} = \underline{\underline{5}}$

2. Hypotenus har lengde 4 : ukjent katet a

$$a^2 + 3^2 = 4^2$$

$$a^2 = 16 - 9 = 7$$

$$\underline{a = \sqrt{7}} \quad (\text{siden } a \geq 0)$$

a, b, c heltall slik at $a^2 + b^2 = c^2$ kalles Pytagoreisk trippel

3, 4, 5.

5, 12, 13 $\quad \left(\begin{aligned} 5^2 + 12^2 &= 13^2 \\ 25 + 144 &= 169 \end{aligned} \right)$

Alle slike pyt. trippel er på formen (uendelig mange)

$$m(a^2+b^2), \quad 2ma \cdot b, \quad m(a^2-b^2)$$

siden $(a^2-b^2)^2 + (2ab)^2 = (a^2+b^2)^2$

$$a^4+b^4-2a^2b^2+4a^2b^2 = a^4+b^4+2a^2b^2$$

oppgaver 6. januar 2025

1. $f(x) = 13x - \frac{4x}{x^4} = 13x - 4 \cdot x^{-3}$

$(x^r)' = r$

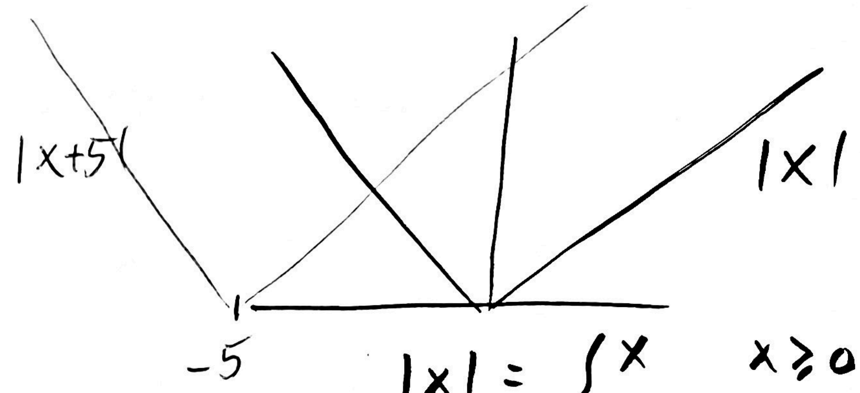
$$f'(x) = 13(x)' - 4(x^{-3})'$$
$$= 13 \cdot 1 - 4(-3)x^{-4}$$
$$= 13 + 12x^{-4} = \underline{13 + \frac{12}{x^4}}$$

2. $g(x) = \sqrt[4]{3x^2 - 1} = (3x^2-1)^{1/4}$

$$g'(x) = \frac{1}{4}(3x^2-1)^{\frac{1}{4}-1} \cdot (3x^2-1)'$$
$$= \frac{1}{4}(3x^2-1)^{-3/4}(3(2x) - 0)$$
$$= \frac{6x}{4}(3x^2-1)^{-3/4} = \frac{3x}{2}\frac{1}{((3x^2-1)^3)^{1/4}}$$
$$= \underline{\frac{3x}{2\sqrt[4]{(3x^2-1)^3}}}$$

3 $h(x) = |x+5|$

$$h'(x) = \begin{cases} -1 & x < -5 \\ 1 & x > 5. \end{cases}$$



$$|x| = \begin{cases} x & x \geq 0 \\ -x & x < 0 \end{cases}$$

$$(|x|)' = \frac{|x|}{x}$$

$$(|x|)' = \begin{cases} 1 & x > 0 \\ -1 & x < 0 \end{cases}$$

ikke deriverbar i $x = 0$.

#2 $x^3 - x$ delelig med 6 for alle heltall x.

$x(x^2-1) = x(x+1)(x-1)$

siden $x-1, x, x+1$ er tre etterfølgende heltall, så er minst én av dem er delelig med 2 og ett av tallene er delelig med 3. Produktet er derfor delelig med $\underline{2 \cdot 3 = 6}$

Når x er et oddetall er $x^3 - x$ delelig med: $2 \cdot 4 \cdot 3 = \underline{24}$

#3 $x \cdot y = 10\,cm^2$

$2(x+y) = 20\,cm$ så $x+y = 10\,cm$.

Likningssystem $x+y=10 \iff y = 10-x$

$x \cdot y = 10$

får $x(10-x) = 10$

$10x - x^2 = 10$

$\iff x^2 - 10x + 10 = 0$

$(x-5)^2 - 25 + 10 = 0$

$(x-5)^2 = 15$

$x-5 = \pm\sqrt{15}$

$x = 5+\sqrt{15}$ og $5-\sqrt{15}$.

Sidene i rektangelet har lengder $5+\sqrt{15}$ og $5-\sqrt{15}$

~ 8.73 ~ 1.27

#4

$$9^x + 3^x = 3$$

$$(3^x)^2 + 3^x = 3 \qquad \text{2.gradslikning: } 3^x = u > 0$$

$$u^2 + u - 3 = 0$$

abc-formelen: $u = \dfrac{-1 \pm \sqrt{1^2 - 4(1)(-3)}}{2 \cdot 1} = \dfrac{-1 \pm \sqrt{13}}{2}$

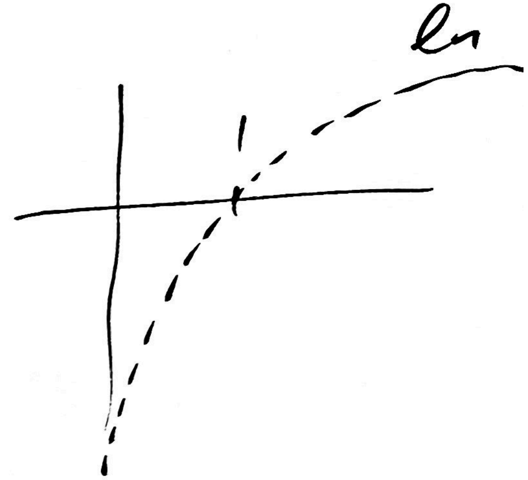
Løsning med $u > 0$ er $3^x = u = \dfrac{\sqrt{13} - 1}{2}$

$$\log 3^x = x \cdot \log 3 = \log\left(\frac{\sqrt{13} - 1}{2}\right)$$

$$\underline{x = \frac{1}{\log 3} \log\left((\sqrt{13} - 1)/2\right)}$$

#5

$$2\ln(x) \geq \ln(2x+1) \qquad x>0$$



$$\ln(x^2) - \ln(2x+1) \geq 0$$

$$\ln\left(\frac{x^2}{2x+1}\right) \geq 0$$

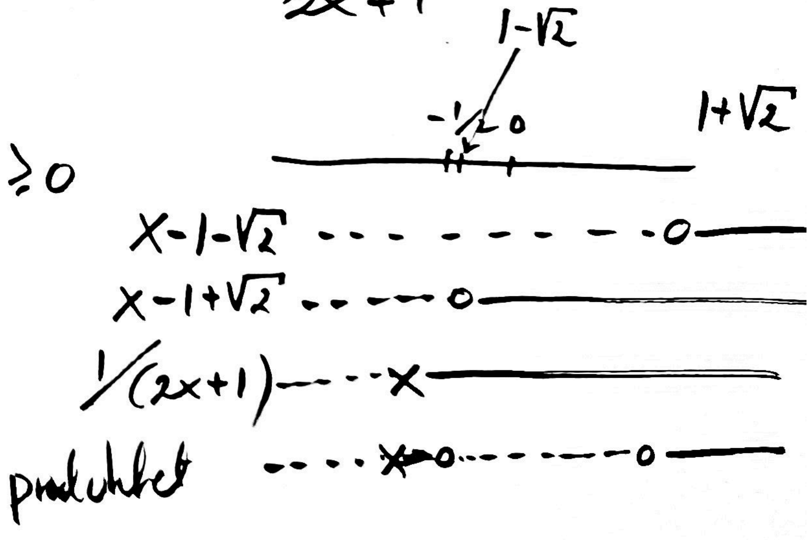
$$\Updownarrow$$

$$\frac{x^2}{2x+1} \geq 1$$

$$\frac{x^2}{2x+1} - 1 \geq 0$$

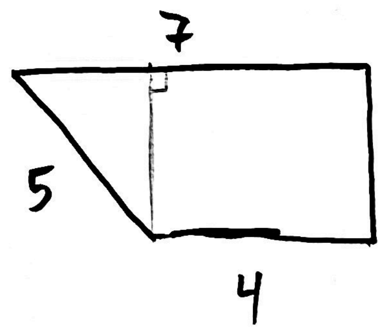
$$\frac{x^2-(2x+1)}{2x+1} \geq 0 \iff \frac{(x-1)^2 \underbrace{-1-1}_{-(\sqrt{2})^2}}{2x+1} \geq 0$$

$$\frac{(x-1-\sqrt{2})(x-1+\sqrt{2})}{2x+1} \geq 0$$



Løsningene er

$\underline{x \in [1+\sqrt{2}, \infty\rangle}$

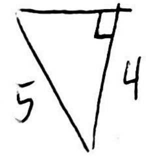


Trapes

Finn arealet

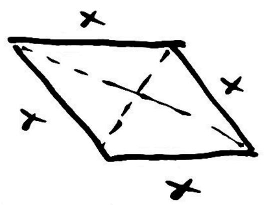
Hva er arealet?

$7-4=3$



4 ved Pytagoras.

$A = 4 \cdot 4 + \frac{3 \cdot 4}{2} = \underline{22}$

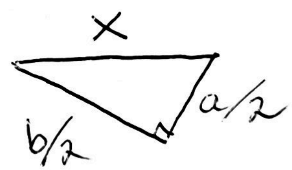


Rombe

Diagonalene har lengde a og b.
Hva er lengden på sidene x?

diagonalene
har lengde a og b.
Uttrykk x ved hjelp
av a og b.

Digonalene deler romben
i 4 like rettvinkla trekanter



Pytagoras

$x = \frac{1}{2}\sqrt{a^2+b^2}$