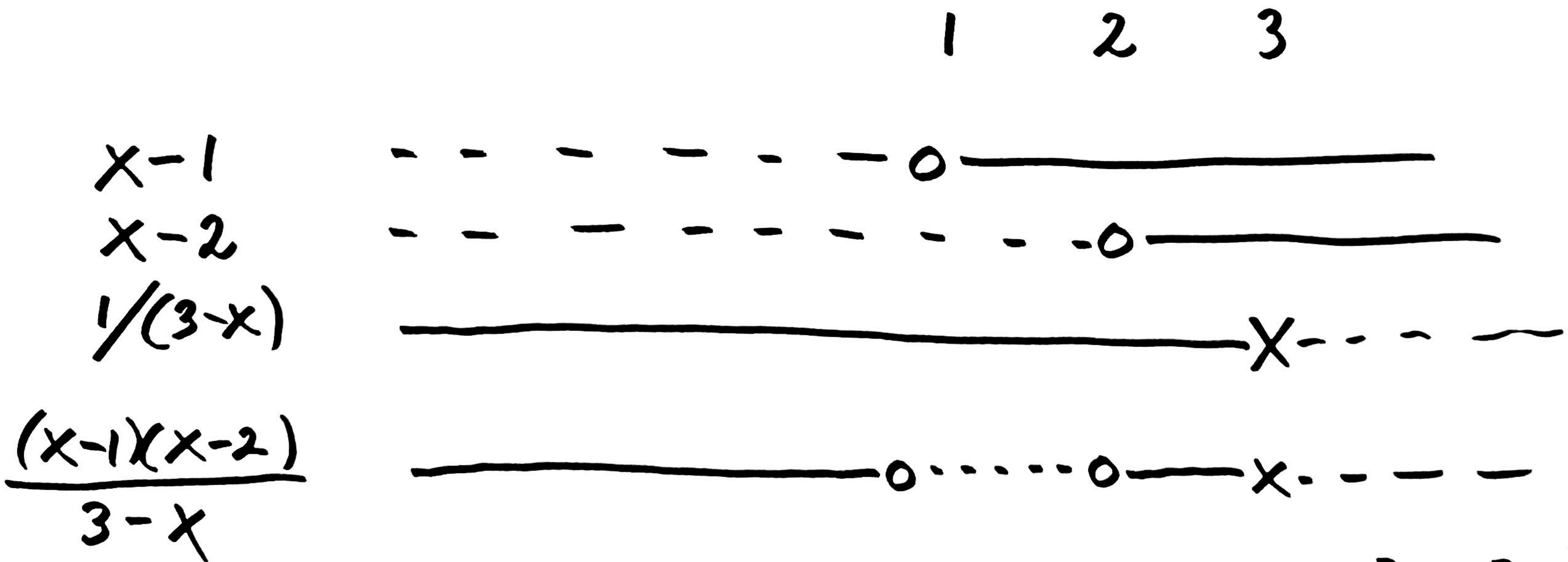
Øving. LF til test

1 $\dfrac{2}{3-x} \geq x \iff \dfrac{2}{3-x} - x \geq 0$

$\iff \dfrac{2 - x(3-x)}{3-x} \geq 0 \iff \dfrac{x^2 - 3x + 2}{3-x} \geq 0$

$\dfrac{(x-1)(x-2)}{3-x} \geq 0$



Løsningsmengden $\underline{x \in \langle -\infty, 1] \cup [2, 3\rangle}$

b) $x^2+x^3+\dots+x^n+\dots = -x^3$

$1+x+x^2+\dots+x^n = \frac{1-x^{n+1}}{1-x}$

$x^2(1+x+x^2+\dots)$ $\qquad |x|<1$

$x^2\left(\frac{1}{1-x}\right) = \frac{x^2}{1-x} = -x^3$

$\Leftrightarrow x^2 = -x^3(1-x) = x^4 - x^3 \quad (x \neq 1)$

$x^2(x^2-x-1) = 0$

$\Leftrightarrow x=0$ og $x^2-x-1=0$

abc formel: $x = \frac{1\pm\sqrt{1+4}}{2} = \frac{1\pm\sqrt{5}}{2}.$

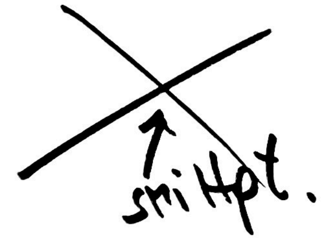
$x = \frac{1+\sqrt{5}}{2} = 1.618\dots$ (gylne snitt)

$= \frac{1-\sqrt{5}}{2} = -0.618\dots$

Løsningene er

$x=0$ og

3 $x = 1+3t$, $y = 2+4t$ $\qquad$ $x = -4+s$, $y = 2-2s$



Snitter når x og y-verdiene er like

$$\begin{aligned} 1+3t &= -4+s \\ 2+4t &= 2-2s \end{aligned} \iff \begin{aligned} 3t - s &= -5 \\ 4t + 2s &= 0 \end{aligned}$$

$2t+s=0$ så $s = -2t$.

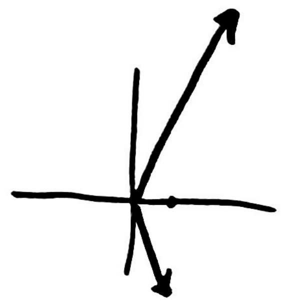
Setter inn i L1: $3t-(-2t) = -5$

$5t = -5$ så $t = -1$

$s = 2$.

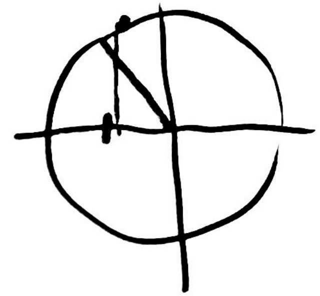
$\underline{(-2,-2)}$ er snittpunktet

Retningsvektorer: $[3,4]$ og $[1,-2]$



$[x,y] = [1,2] + [3,4]\cdot t$

Vinkelen mellom vektoren:



$$\cos(u) = \frac{[3,4] \cdot [1,-2]}{|[3,4]| \cdot |[1,-2]|}$$

$$= \frac{3-8}{5 \cdot \sqrt{5}} = \frac{-1}{\sqrt{5}} \approx -0.4472\ldots$$

$$u \approx 116.565^\circ$$

Så vinkelen mellom linjene er

$$180^\circ - 116.565^\circ = \underline{63.435^\circ}$$

$$u = 4 - x^2$$
$$u' = -2x$$
$$-\frac{1}{2} du = x\,dx$$

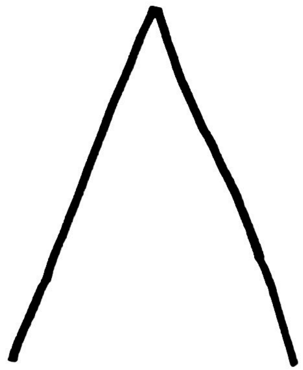
#4. $\int_0^2 x\sqrt{4-x^2}\,dx$

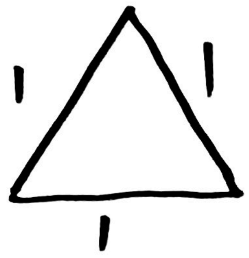
$$= \int_4^0 -\frac{1}{2}\sqrt{u}\,du$$

$$= \frac{1}{2}\int_0^4 \sqrt{u}\,du = \frac{1}{2}\int_0^4 u^{1/2}\,du = \frac{1}{2} \frac{u^{3/2}}{3/2}\Big|_0^4$$

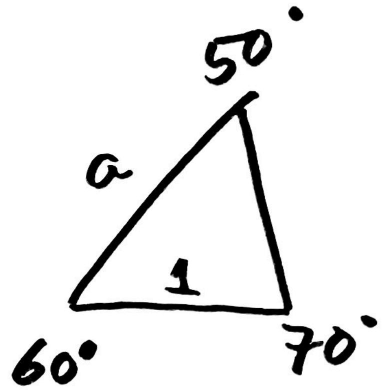
$$= \frac{1}{3}\left(4^{3/2} - 0\right) = \underline{\frac{8}{3}} \approx \underline{2 + \frac{2}{3}} \approx 2{,}66.$$

5

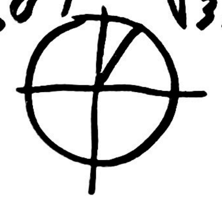




$$\left(A \approx \frac{1}{2} \sin 60^\circ \approx \frac{\sqrt{3}}{4} \sim 0.433\right)$$



$\sin(60^\circ) = \sqrt{3}/2$



$$\frac{\sin V}{\text{motstående side}} = \text{konstant}$$

$$\frac{\sin 70^\circ}{a} = \frac{\sin 50^\circ}{1} \quad \text{så} \quad a = \frac{\sin 70^\circ}{\sin 50^\circ}.$$

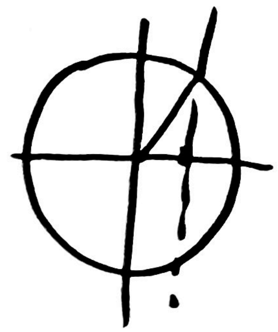
Arealsetning

$$A = \frac{1}{2} 1 \cdot a \cdot \sin 60^\circ$$
$$= \frac{1}{2} \frac{\sin 70^\circ \cdot \sin 60^\circ}{\sin 50^\circ}$$
$$= \frac{\sqrt{3}}{4} \frac{\sin 70^\circ}{\sin 50^\circ} \approx \underline{0.5311}$$

6 $f(x) = \cos x - \cos^2 x$ $\quad x \in [0, 3\pi/2]$

1) Ekstremalpunkt

2) globale Ekstremalverdier.



$f(x)$ kontinuerlig på $[0, 3\pi/2]$

$$f'(x) = -\sin x - 2\cos(x)(-\sin x)$$
$$= \sin x\,(2\cos x - 1).$$

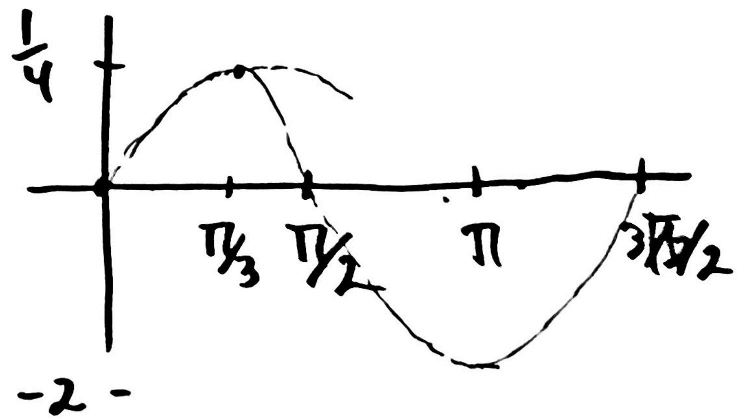
$f'(x) = 0$: $\sin x = 0$ og $\cos x = \frac{1}{2}$

$x = 0, \pi$ $\qquad x = \pi/3.$

1. toppunkt $\left(\frac{\pi}{3}, \frac{1}{4}\right)$ og $\left(\frac{3\pi}{2}, 0\right)$

bunnpunkt $(0,0)$ og $(\pi, -2)$

2. Ekstremalverdier: $\frac{1}{4}$ og -2.



7 $I = \int_0^4 \sqrt{3+x^2}\, dx$

$\overbrace{}^{2}$

	0	1	2	3	4
Vekting	1	4	2	4	1

$$I \sim \frac{1}{6} \cdot \underset{\text{bredde}}{2} \left(1\sqrt{3} + 4\sqrt{4} + 2\sqrt{3+4} + 4\sqrt{3+3^2} + 1\sqrt{3+4^2} \right)$$

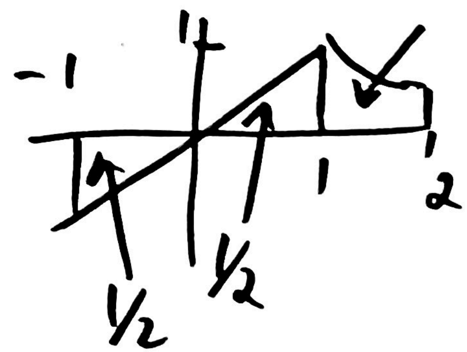
$$\frac{1}{3}\left(\sqrt{3} + 8 + 2\sqrt{7} + 8\sqrt{3} + \sqrt{19}\right) \sim \underline{11.0796}$$

(mer nøyaktig 11.0788)

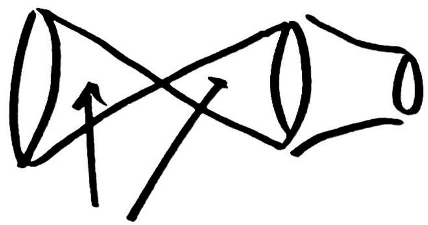
8 $f(x) = \begin{cases} x & x \leq 1 \\ 1/x & x \geq 1 \end{cases}$

$A = \frac{1}{2} + \frac{1}{2} + \int_1^2 \frac{1}{x}\, dx = 1 + \ln|x| \Big|_1^2$

$A = \underline{1 + \ln 2}$



Volum



kjegler

$$V = \left(\frac{1}{3}\pi 1^2 \cdot 1\right) \cdot 2 + \int_1^2 \left(\frac{1}{x}\right)^2 \cdot \pi \, dx$$

$$= \frac{2}{3}\pi + \pi \int_1^2 x^{-2} \, dx$$

$$= \frac{2}{3}\pi + \pi \left(\frac{x^{-1}}{-1}\right)\Big|_1^2$$

$$= \frac{2}{3}\pi + \pi \left(\frac{-1}{2} - \left(\frac{-1}{1}\right)\right)$$

$$= \frac{2}{3}\pi + \frac{\pi}{2} = \underline{\frac{7}{6}\pi}$$