9.12.2022

H Fausk

LF oblig 4

1 a) $x^4 + 5x^3 + 6x^2 = 0$

$\Leftrightarrow \quad x^2(x^2 + 5x + 6) = 0$

$x^2(x+2)(x+3) = 0$

$\Leftrightarrow \quad x^2 = 0$ eller $x + 2 = 0$ eller $x + 3 = 0$

Løsningene er $\underline{x = -3, -2 \text{ og } 0}$.

b) $100^x = 25$ anvender Log

$\Leftrightarrow \quad x \operatorname{Log} 100 = \operatorname{Log} 100^x = \operatorname{Log} 25$

Så $x = \dfrac{\operatorname{Log}(25)}{\operatorname{Log}(100)} = \dfrac{\operatorname{Log}(5^2)}{\operatorname{Log}(10^2)} = \dfrac{2 \operatorname{Log} 5}{2 \operatorname{Log} 10}$

$\underline{x = \operatorname{Log} 5} \sim 0.7$

1 c) $\ln|x+3| = 2$

$\Leftrightarrow \quad |x+3| = e^{\ln|x+3|} = e^2$

$\Leftrightarrow \quad x+3 = e^2$ eller $x+3 = -e^2$

Løsningene er $x = \underline{-3+e^2}$

og $x = \underline{-3-e^2}$.

d) $x + \sqrt{2x+9} = 5 \quad \Leftrightarrow \quad \sqrt{2x+9} = 5-x$

$\Rightarrow \quad 2x+9 = (5-x)^2 = x^2 - 10x + 25$

$x^2 - 12x + 16 = 0$

Røtter: $\dfrac{12 \pm \sqrt{12^2 - 4\cdot 16}}{2} = 6 \pm \sqrt{6^2 - 16} = 6 \pm \sqrt{36-16}$

$= 6 \pm \sqrt{20} = 6 \pm \sqrt{2^2}\sqrt{5} = \underline{6 \pm 2\sqrt{5}}$

sjekker for falske løsninger. ($6+2\sqrt{5}$ falsk) Løsningen er $\underline{6-2\sqrt{5}}$

1e) $\log_5(4x+3) = \log_5(2x-3) + 1$

$\log_5\left(\frac{4x+3}{2x-3}\right) = 1$ opphøyer begge sider

$\Leftrightarrow \frac{4x+3}{2x-3} = 5^{\log \sim} = 5^1 = 5$

$\Leftrightarrow 4x+3 = 5(2x-3) = 10x - 15$

$3 + 15 = 10x - 4x$

$\Leftrightarrow 6x = 18 \Leftrightarrow \underline{x = 3}$

(test: $\log_5 15 = \log_5(5 \cdot 3) = \log_5 3 + \overbrace{\log_5 5}^{1}$

$\log_5(3) + 1$ så like)

2.

$$\frac{200^{1000}\ 6^{998}\cdot 25^{1001}}{30^{999}\ 1000^{1000}} = \frac{200^{1000}\ 6^{1000}\ 25^{1000}\cdot 25/6^2}{30^{1000}\,1000^{1000}\cdot 1/30}$$

$$= \frac{(200\cdot 6\cdot 25)^{1000}}{(30\,000)^{1000}}\cdot 30\cdot\frac{25}{36} = \frac{5\cdot 25}{6}$$

$$= \underline{\frac{125}{6}} = \underline{20+\frac{5}{6}}$$

3

$$P_0(1.2)(1.4)(\underbrace{1-0.3}_{0.7})$$

$$P_0\,(1.68)\,(0.7) = (1.176)\,P_0 = P_0 + 17.6\%\,P_0$$

Prisøkningen er på $\underline{17.6\%}$

4 $p(x) = ax^3 + 4x - 1$ kan deles med

$x + 1 = x - (-1)$

hvis og bare hvis $p(-1) = 0$

$p(-1) = a(-1)^3 + 4(-1) - 1 = 0$

$-a - 5 = 0$

$\underline{a = -5}$

$$-5x^3 \quad 4x - 1 : x + 1 = -5x^2 + 5x - 1$$

$$\begin{array}{l}
-5x^3 \quad 4x - 1 \\
\underline{-5x^3 - 5x^2} \\
\quad 5x^2 + 4x - 1 \\
\quad \underline{5x^2 + 5x} \\
\qquad -x - 1 \\
\qquad \underline{-x - 1} \\
\qquad\quad 0
\end{array}$$

$$-5x^2+5x-1=0$$

Rqllter $\dfrac{-5\pm\sqrt{5^2-4\cdot 5}}{2(-5)} = \dfrac{1}{2} \pm \dfrac{\sqrt{5}}{(-2\cdot 5)}$

$$= \frac{1}{2} \pm \frac{\sqrt{5}}{10}$$

$$-5x^2+5x-1 = -5\left(x-\frac{1}{2}-\frac{\sqrt{5}}{10}\right)\left(x-\frac{1}{2}+\frac{\sqrt{5}}{10}\right)$$

$$p(x) = \underline{-5(x+1)\left(x-\frac{1}{2}-\frac{\sqrt{5}}{10}\right)\left(x-\frac{1}{2}+\frac{\sqrt{5}}{10}\right)}$$

5 $\quad y = ax^2 + bx + c$ går gjennom $(-1,9)$, $(1,3)$, $(2,6)$

når $\quad y(-1) = a - b + c = 9$

$y(1) = a + b + c = 3$

$y(2) = 4a + 2b + c = 6$

L2 − L1 gir $\quad 2b = 3 - 9 = -6 \qquad \underline{b = -3}$

$\left.\begin{aligned} a + c &= 3 - b = 6 \\ 4a + c &= 6 - 2b = 12 \end{aligned}\right\} \quad 3a = 12 - 6 = 6$

$\underline{a = 2}$

$c = 6 - a = 4$

$y(x) = \underline{2x^2 - 3x + 4}$

6 a) $f(x) = \dfrac{-3x+2}{(e^4)^x} = (-3x+2)e^{-4x}$ Def. mengde $\mathbb{R}$

$$f'(x) = (-3x+2)' e^{-4x} + (-3x+2)(e^{-4x})'$$
$$= -3e^{-4x} \cancel{e^{4x}} + (-3x+2)e^{-4x} \cdot (-4x)'$$
$$= (-3 - 4(-3x+2))e^{-4x}$$
$$= \underline{(12x - 11)e^{-4x}}$$

b) $g(x) = \ln\sqrt{|\ln|3x+2||} = \frac{1}{2}\ln|\ln|3x+2||$

$\ln|3x+2|$ def når $3x+2 \neq 0$. $x \neq -2/3$.

$\ln|\ln|3x+2||$ def når $\ln|3x+2| \neq 0$

$\ln|3x+2| = 0 \Leftrightarrow |3x+2| = 1$

$\Leftrightarrow 3x+2 = 1$ eller $3x+2 = -1$

$x = -1/3$ eller $x = -1$

$$D_g = \underline{\mathbb{R} \setminus \left\{ -\tfrac{2}{3}, -\tfrac{1}{3}, -1 \right\}}$$

$$g'(x) = \frac{1}{2}\,\frac{1}{\ln|3x+2|}\cdot\underbrace{(\ln|3x+2|)'}_{\frac{1}{3x+2}\cdot\underbrace{(3x+2)'}_{3}}$$

$$= \underline{\frac{3}{2(3x+2)\ln|3x+2|}}$$

7. $h(x) = x^3 - 2x^2 + x - 4$

$h'(x) = 3x^2 - 4x + 1$

$h'(1) = 3 - 4 + 1 = 0$ tangentlinje : $\underline{y = -4}$

$h'(2) = 3\cdot4 - 4\cdot2 + 1 = \underline{5}$ —"—

$(y-(-2)) = 5(x-2)$

$y = -2 + 5(x-2)$

$\underline{y = 5x - 12}$

kysser $y = -4 = 5x - 12$

$8 = 12 - 4 = 5x$ så $x = \underline{\underline{8/5 = 1.6}}$

snittpunkt $\underline{\left(\frac{8}{5}, -4\right)}$

8 $\quad f(x) = \dfrac{x^2}{x-3} = \dfrac{x^2-9+9}{x-3} = \dfrac{(x-3)(x+3)+9}{x-3}$

$= x+3 + \dfrac{9}{x-3}$

1 $\quad$ skrå asymptote $\quad y = x+3$

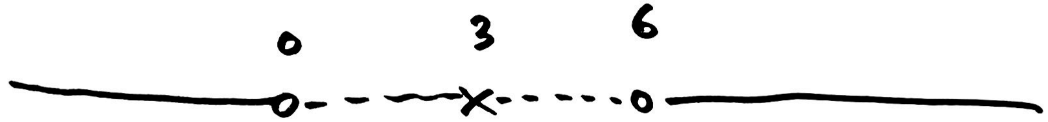
vertikal asymptote $\quad x = 3$.

2 $\quad f'(x) = (x+3)' + 9\left((x-3)^{-1}\right)' = 1 + 9\dfrac{-1}{(x-3)^2}\underbrace{(x-3)'}_{1}$

$= 1 - \dfrac{9}{(x-3)^2}$

$= \dfrac{x^2-6x+9-9}{(x-3)^2} = \dfrac{x^2-6x}{(x-3)^2} = \dfrac{x(x-6)}{(x-3)^2}$

Fortegn til $f'(x)$



$f(x)$ stigende $\langle -\infty, 0\rangle$ og i $\langle 6, \infty\rangle$

$f(x)$ avtagende i $\langle 0, 3\rangle$ og i $\langle 3, 6\rangle$.

Toppunkt: $(0, f(0)) = \underline{(0,0)}$

Bunnpunkt i $(6, f(6)) = \underline{(6,12)}$

3. $f''(x) = \left(1 - \frac{9}{(x-3)^2}\right)' = 0 - 9\left((x-3)^{-2}\right)'$

$= -9(-2)(x-3)^{-3}(x-3)' = \frac{18}{(x-3)^3}.$

$f''(x) < 0$ når $\underline{x < 3}$ konkav ned

$f''(x) > 0$ når $\underline{x > 3}$ konkav opp

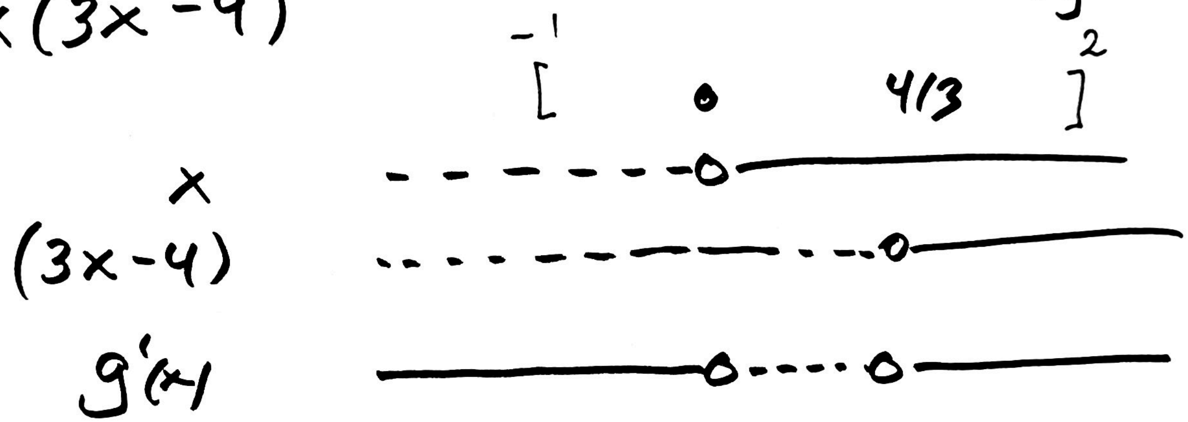
$f(x)$ har ikke vendepunkt (ikke def i $x=3$).

9 $g(x) = x^3 - 2x^2$ $D_g = [-1, 2]$

$g'(x) = 3x^2 - 4x$ $g'(x) = 0$ når $x = 0$

$= x(3x - 4)$ og $x = 4/3$

Fortegn:



Kritiske punkt

$x = -1$, $g(-1) = -3$

bunnpunkt $(-1, -3)$

$x = 0$ $g(0) = 0$

toppunkt (0.0)

$x = 4/3$ $g(4/3) = \left(\frac{4}{3}\right)^2 \big(\underbrace{\tfrac{4}{3} - 2}_{-2/3}\big) = \frac{-32}{27}$

bunnpunkt $\left(\frac{4}{3}, -32/27\right)$

$x = 2$ $g(2) = 0$ toppunkt $(2,0)$

Globalt bunnpunkt $\underline{(-1,-3)}$

globale toppunkt $\underline{(0,0) \text{ og } (2,0).}$

10. a) $3-x^2 < 2x \leq x^2$

$\Leftrightarrow \quad 3-x^2 < 2x \quad \text{og} \quad 2x \leq x^2$

$3-x^2 < 2x \Leftrightarrow x^2+2x-3 > 0 \Leftrightarrow (x-1)(x+3) > 0$

$x \in \langle -\infty, -3\rangle \cup \langle 1, \infty\rangle$

$2x \leq x^2 \Leftrightarrow x(x-2) \geq 0$

$x \in \langle -\infty, 0] \cup [2, \infty\rangle.$

Løsningsmengden til den doble ulikheten er da

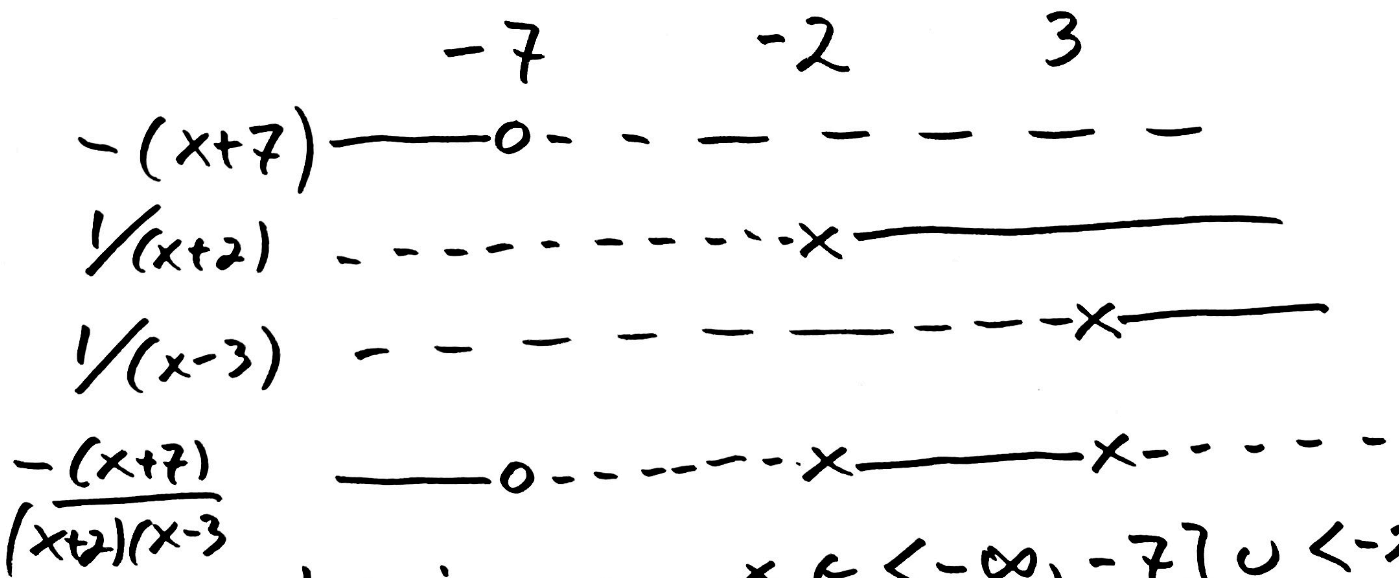
$\underline{\langle -\infty, -3\rangle \cup [2, \infty\rangle}$

10 b) $\frac{1}{x+2} \geq \frac{2}{x-3}$

$$\frac{1}{x+2} - \frac{2}{x-3} \geq 0$$

$$\frac{(x-3) - 2(x+2)}{(x+2)(x-3)} \geq 0$$

$$\frac{-(x+7)}{(x+2)(x-3)} \geq 0$$



Løsningene $\underline{x \in \langle -\infty, -7] \cup \langle -2, 3\rangle}$

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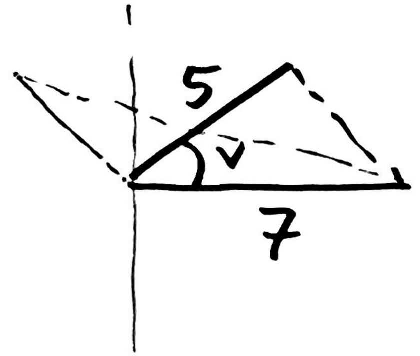


Areal $A = 12$

$\frac{1}{2} 5 \cdot 7 \sin V = 12$

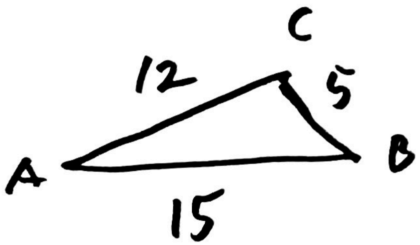
$\sin V = \frac{24}{35} \approx 0.68$

$V_1 = \arcsin\left(\frac{24}{35}\right) = 43.3^\circ$

Den andre muligheten $V_2 = 180^\circ - 43.3^\circ$

$= \underline{136.7^\circ}$

12



$\angle C = 117.82^\circ$

$\angle A = 17.146^\circ$

$\angle B = 45.03^\circ$

Kosinussetning $15^2 = 5^2 + 12^2 - 2 \cdot 5 \cdot 12 \cos(\angle C)$

$\frac{225 - 144 - 25}{-120} = \cos(\angle C)$

$\cos(\angle C) = \frac{-56}{120} = \frac{-8 \cdot 7}{8 \cdot 15} = \underline{\frac{-7}{15}}$

Sinussetning $\sin \angle A = \frac{5}{15} \sin(\angle C)$

$\sin \angle B = \frac{12}{15} \sin(\angle C)$