21.04.23.

Integrasjon med trig. funksjoner

$\int \cos x \, dx = \sin x + C$

$\int \sin x \, dx = -\cos x + C$

$\int \frac{1}{\cos^2 x} \, dx = \tan(x) + C$ siden. $(\tan x)' = \frac{1}{\cos^2 x} = \tan^2 x + 1$

$\int \frac{1}{\cos^3 x} \, dx$

$= \int \frac{1}{(1-\sin^2 x)\cos x} \, dx$

$= \int \frac{\cos x}{(1-\sin^2 x)\cos^2 x} \, dx$

$u = \sin x$

$u' = \cos x$

$du = \cos x \, dx$

$= \int \frac{1}{(1-\sin^2 x)^2} \cos x \, dx$

$= \int \frac{1}{(1-u^2)^2} \, du$

$\sin^2 x + \cos^2 x = 1$

$\cos^2 x = 1 - \sin^2 x$

$\cos(2x) = \cos^2 x - \underbrace{\sin^2 x}_{1-\cos^2 x}$

$= 2\cos^2 x - 1$

så $\cos^2 x = \frac{1}{2}(\cos 2x + 1)$

$$\int \frac{\;}{((U+1)(U-1))^2}\, dU = \int \frac{\;}{(U+1)^2 \cdot (U-1)^2}\, dU$$

$$\frac{1}{(U+1)^2(U-1)^2} = \frac{AU+B}{(U+1)^2} + \frac{CU+D}{(U-1)^2}$$

"Felles nevner"

$$1 = (AU+B)\overset{U^2-2U+1}{(U-1)^2} + (CU+D)(U+1)^2$$

$U=1$ $\quad C+D = 1/4$

$U=-1$ $\quad -A+B = 1/4$

Ganger:

$$1 = AU^3 - 2AU^2 + AU + B(U^2-2U+1)$$

$$+ CU^3 + 2CU^2 + CU \quad D(U^2+2U+1)$$

$$= U^3(A+C) + U^2(-2A+2C+B+D) + U(A+C-2B+2D)$$

$B+D=1$, $\quad A+C=0$

$A+C$ $\quad 2(-A+$ $\quad +(B$

$A = -C$

$B = D$

$B + D = 2B = 1$

så $B = 1/2 = D$

$-A + B = 1/4$

$A = B - 1/4 = 1/2 - 1/4 = 1/4.$

$A = 1/4 \qquad C = -1/4.$

$$\int \frac{1}{(1-U^2)^2}\,dU = \frac{1}{4}\left[\int \frac{U+2}{(U+1)^2} + \frac{-U+2}{(U-1)^2}\,dU\right]$$

$$= \frac{1}{4}\int \frac{U+1}{(U+1)^2}\frac{+1}{(U+1)^2} + \frac{1-U}{(U-1)^2} + \frac{1}{(U-1)^2}\,dU$$

$$= \frac{1}{4}\int \frac{1}{U+1} + \frac{1}{(U+1)^2} - \frac{1}{U-1} + \frac{1}{(U-1)^2}\,dU$$

$$\frac{1}{4}\left[\ln|U+1| - \frac{1}{(U+1)} - \ln|U-1| - \frac{1}{(U-1)}\right] + c$$

$$= \frac{1}{4}\left(\ln\left|\frac{\sin x+1}{\sin x-1}\right| - \frac{1}{(\sin x)+1} - \frac{1}{(\sin x)-1}\right) + c$$

$$\int \tan^2 x \, dx \qquad (\tan x)' = 1 + \tan^2 x$$

$$\int (1+\tan^2 x) - 1 \, dx$$

$$\int (\tan x)' - 1 \, dx = \underline{\tan(x) - x + C}$$

opg. 1) $\int \sin x \cos^3 x \, dx$ 2.) $\int \sin^3 x \cos^3 x \, dx$

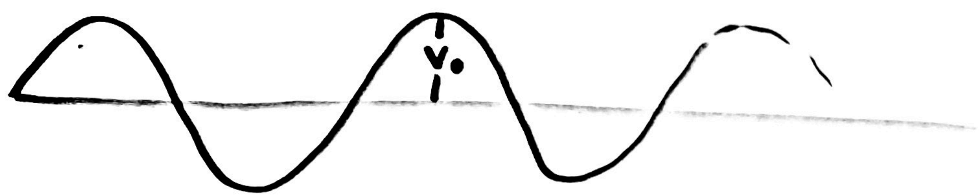
Prøv med $u = \cos x$, $u' = -\sin x$

$$\int u^3 (-du) = -\frac{u^4}{4} + C = \underline{-\frac{\cos^4 x}{4} + C}$$

2. $$\int \underbrace{\sin^2 x}_{1-u^2} \underbrace{\cos^3 x}_{u^3} (-du) = -\int u^3 - u^5 \, du$$

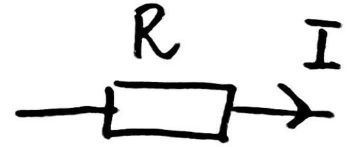
$$= \frac{u^6}{6} - \frac{u^4}{4} + C = \frac{\cos^6 x}{6} - \frac{\cos^4 x}{4} + C$$

$\int \sin^n(x) \cos^m(x) \, dx$ n eller m odde tall : tilsvarende.



220 V

50 Hz



$V(t) = V_0 \sin(100\pi t + c)$

$V^2 = V_0^2 \sin^2(100\pi t + c)$

$V = R \cdot I$

Effekt : $V \cdot I = \frac{V^2}{R}$

Gjennomsnitt til $\sin^2(u)$ over hele perioder er lik $\frac{1}{2}$.

(gjennomsnittet er gjennomsnittet til $\cos^2 u$. $\sin^2 u + \cos^2 u = 1$)

Gjennomsnittlig effekt

$\frac{V_0^2}{2} \cdot \frac{1}{R}$.

Spenningen vi oppgir er $\frac{V_0}{\sqrt{2}}$ så $V^2 = \frac{V_0^2}{2}$.

Toppspenningen (største utslag) $\sqrt{2} \cdot 220\,V = 311\,V$

$\sqrt{2} \cdot 240\,V = 339\,V$.

Vi har benyttet root mean square

$$V_{rms} = \sqrt{\text{gjennomsnitt av kvadratet } V^2}.$$

$u = x \qquad u' = 1$

$$\int \underbrace{x}_{u} \underbrace{\cos(3x-1)}_{v'} \, dx$$

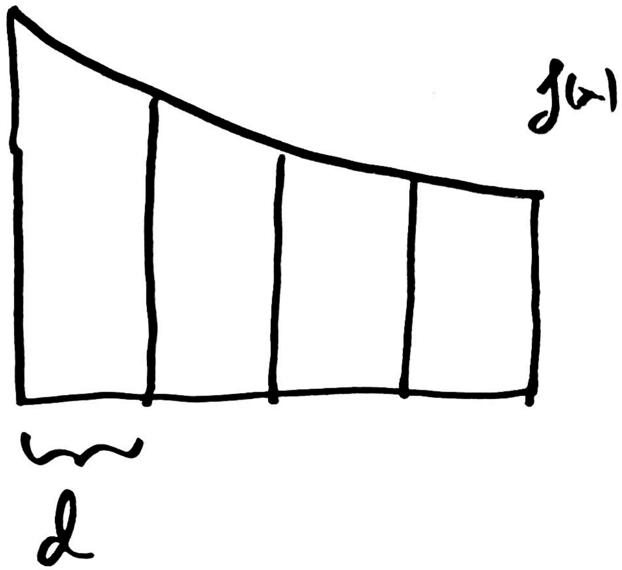
$v' = \cos(3x-1)$

$v = \frac{1}{3}\sin(3x-1)$

$$= u \cdot v - \int u' \cdot v \, dx$$

$$= \frac{x}{3}\sin(3x-1) - \int 1 \cdot \frac{1}{3}\sin(3x-1)\, dx$$

$$= \underline{\frac{x}{3}\sin(3x-1) + \frac{1}{9}\cos(3x-1)}$$



venstr

venstre sum	1	1	1	1	0	
høyre sum	0	1	1	1	1	
midtpunkt sum	0	1	0	1	0	(doble intervaller
Trapesmetoden	$\frac{1}{2}$	1	1	1	$\frac{1}{2}$	
Simpsons metode	1	4	2	4	1	

Rektangler

$$\int_0^1 \frac{1}{x^3+1}\,dx$$

($x^3+1=0$ när $x=-1$

$x^3+1 = (x+1)(x^2-x+1)$

$x^2-x+1 = (x-\frac{1}{2})^2 + \frac{3}{4}$.

$$\frac{1}{x^3+1} = \frac{A}{x+1} + \frac{Bx+C}{(x-\frac{1}{2})^2+3/4}$$

etc.)

$\frac{1}{1-x} = 1+x+x^2+\cdots$ $\quad |x|<1$

så $\frac{1}{1+x^3} = 1-x^3+x^6-x^9+x^{12}-+\cdots$

så $\int_0^1 \frac{1}{1+x^3}\,dx = \int_0^1 1-x^3+x^6-x^9+x^{12}-+\cdots\,dx$

$= 1-\frac{1}{4}+\frac{1}{7}-\frac{1}{10}+\frac{1}{13}-\frac{1}{16}+\frac{1}{19}-+\cdots$

$$I = \int_0^1 \frac{1}{x^3+1}\,dx$$

1 dobbelt intervall

Simpsons metode

$$\begin{array}{ccc} 1 & 4 & 1 \\ \vdash & + & \dashv \\ 0 & 1/2 & 1 \end{array}$$

bredde $d = 1$

$$I \sim \frac{1}{6}\left(\frac{1}{1+0} + 4\cdot\frac{1}{1+\left(\frac{1}{2}\right)^3} + 1\cdot\frac{1}{1+1}\right)\cdot d$$

$$\sim \frac{1}{6}\left(1 + \frac{4}{9/8} + \frac{1}{2}\right) \sim 0.842\ldots$$

(mer nøyaktig 0.8356...)