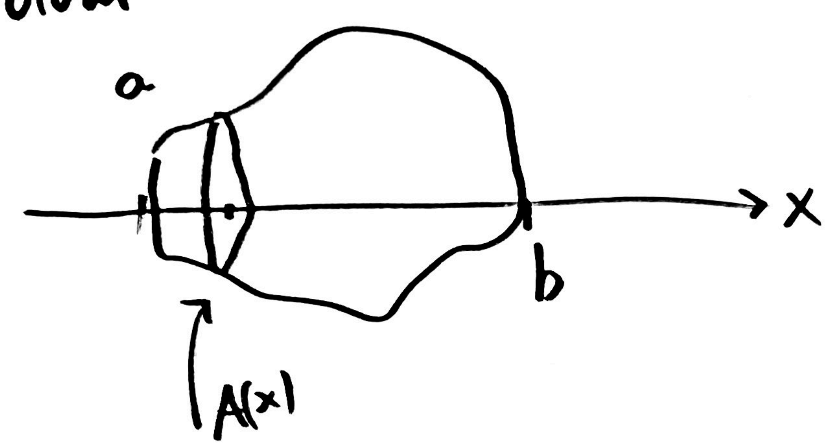
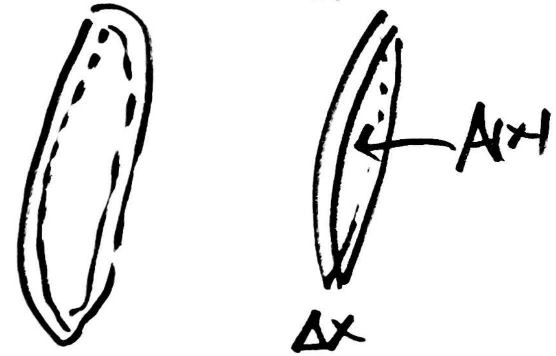
14.04.23

Volum



$\Delta V = A(x) \cdot \Delta x$



$$V \sim \sum_{i=1}^{n} A(x_i) \cdot \Delta x_i$$

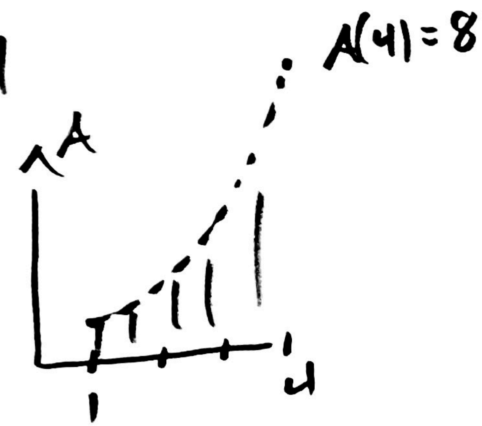
$A(x)$ tverrsnittareal.

$$V = \int_a^b A(x)\,dx$$

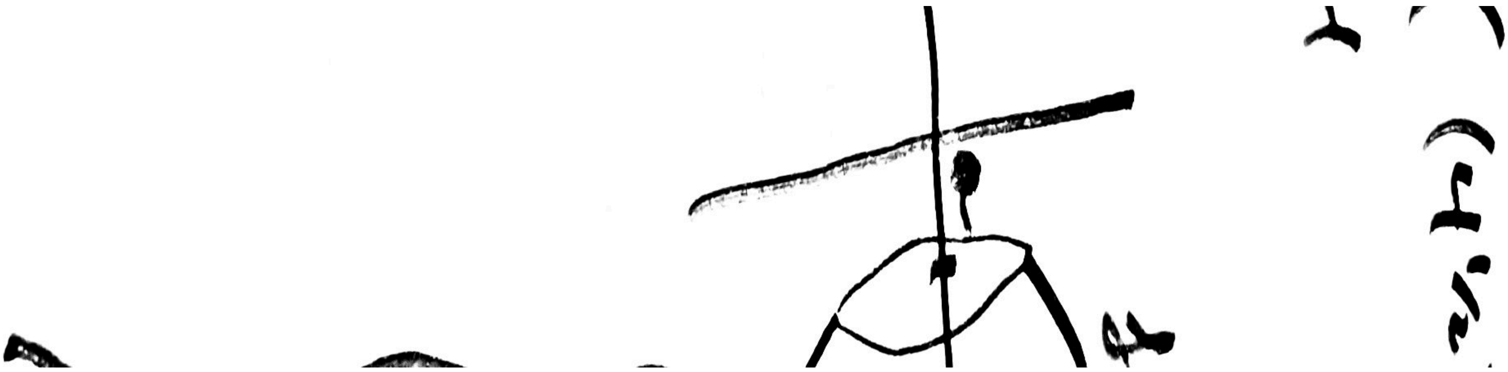
Eks

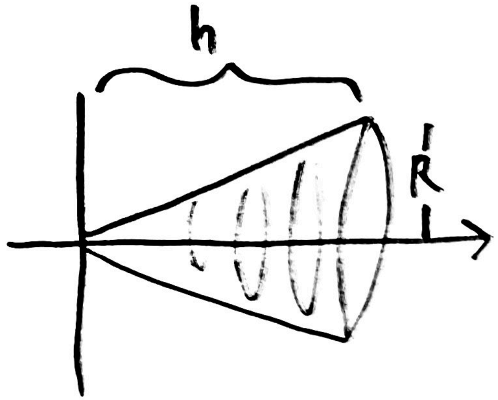
$$A(x) = \sqrt{x^3} = x^{3/2}$$

$a = 1$ til $a = 4$



$$V = \int_1^4 A(x)\,dx = \int_1^4 x^{3/2}\,dx = \frac{x^{5/2}}{5/2}\Big|_1^4$$





$f(x)$ lin funksjon

$f(0) = 0 \qquad f(h) = R$

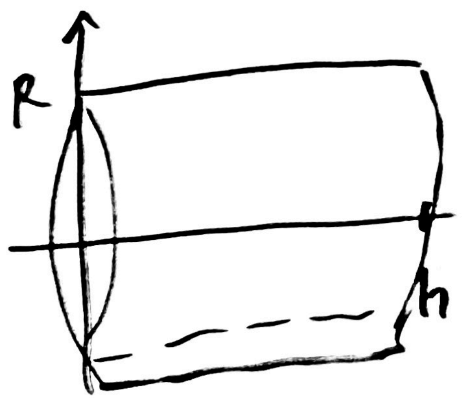
$f(x) = \frac{R}{h} \cdot x$

$$V = \pi \int_0^h \left(\frac{R}{h} x\right)^2 dx = \pi \frac{R^2}{h^2} \int_0^h x^2 dx$$

$$= \pi \frac{R^2}{h^2} \frac{x^3}{3} \Big|_0^h = \pi \frac{R^2}{h^2} \frac{h^3}{3}$$

$$V = \underline{\frac{\pi}{3} R^2 h}.$$

opg

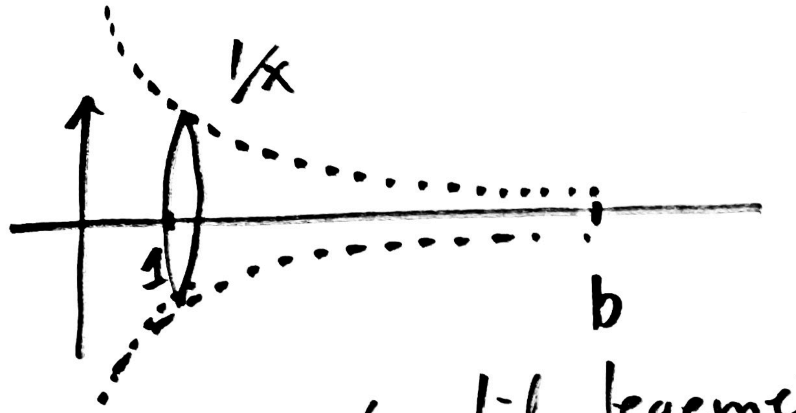


sylinder $f(x) = R$ konstant

$$V = \pi \int_0^h R^2 dx = \pi R^2 x \Big|_0^h$$

$$= \pi R^2 (h - 0) = \underline{\pi R^2 \cdot h}$$

Gabriels horn $(b \to \infty)$



Volumet til legemet som fremkommer av å rotere $f(x) = 1/x$ om x-aksen fra $x=1$ til $x=b$ er:

$$\pi \int_1^b f^2(x)\,dx = \pi \int_1^b \frac{1}{x^2}\,dx$$

$$= \pi \int_1^b x^{-2}\,dx = \pi \left. \frac{x^{-2+1}}{-1} \right|_1^b$$

$$= \pi \left(\left. \frac{-1}{x} \right|_1^b \right) = \underline{\pi \left(1 - \frac{1}{b}\right)}$$

Når $b \to \infty$ vil $V \to \pi$. (volumet er endelig)

Finn volumet til legemet som fremkommer ved å rotere $f(x) = (\sqrt{x} - x)$ om x-aksen fra $x=0$ til $x=4$.

$$V = \int_0^4 \pi f(x)^2 \, dx = \pi \int_0^4 (\sqrt{x} - x)^2 \, dx$$

$$= \pi \int_0^4 x - 2\underbrace{x\sqrt{x}}_{x^{3/2}} + x^2 \, dx = \pi \left[\frac{x^2}{2} - 2 \frac{x^{5/2}}{5/2} + \frac{x^3}{3} \right]_0^4$$

$$= \pi \left[\frac{4^2}{2} - \frac{4}{5} 4^{5/2} + \frac{4^3}{3} \right] = \pi \left[8 + \frac{64}{3} - \frac{128}{5} \right]$$

$$\sim \underline{11.72}$$

Opg. $f(x) = 1 - x^3$ Roterer om x-aksen fra $x=a$ til $x=6$

Volumet til legemet som fremkommer

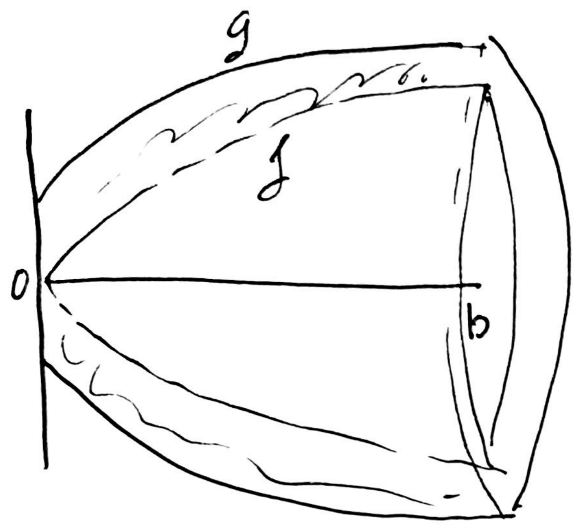
$$V = \pi \int_a^b (1 - x^3)^2 \, dx = \pi \int_a^b 1 - 2x^3 + x^6 \, dx$$

$$V = x - \frac{x^4}{2} + \frac{x^7}{7} \Big|_a^b$$

$$= (b-a) - \frac{1}{2}(b^4 - a^4) + \frac{1}{7}(b^7 - a^7)$$

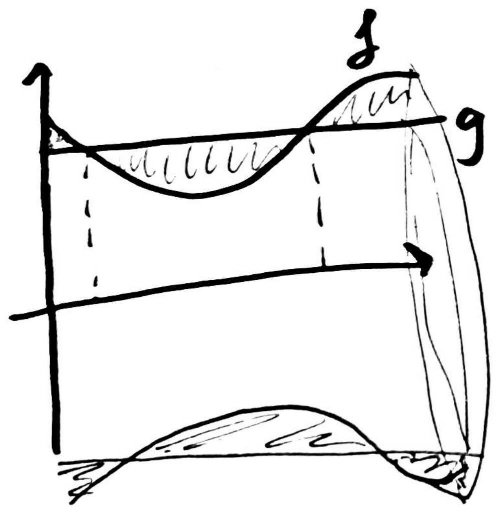
Når $a = 0$ og $b = 1$ er volumet lik

$$V = 1 - \frac{1}{2} + \frac{1}{7} = \frac{14 - 7 + 2}{14} = \underline{\frac{9}{14}}$$



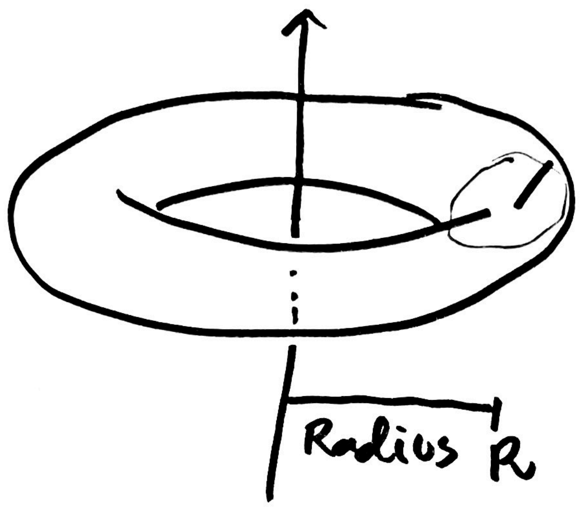
Volum til legemet som er differansen mellom legemene gitt ved å rotere kurvene gitt ved g og f

$g \geq f \quad V = \pi \int_0^b g^2(x) - f^2(x)\, dx$



$$V = \pi \int_0^b \left| g^2(x) - f^2(x) \right| dx$$

Torus

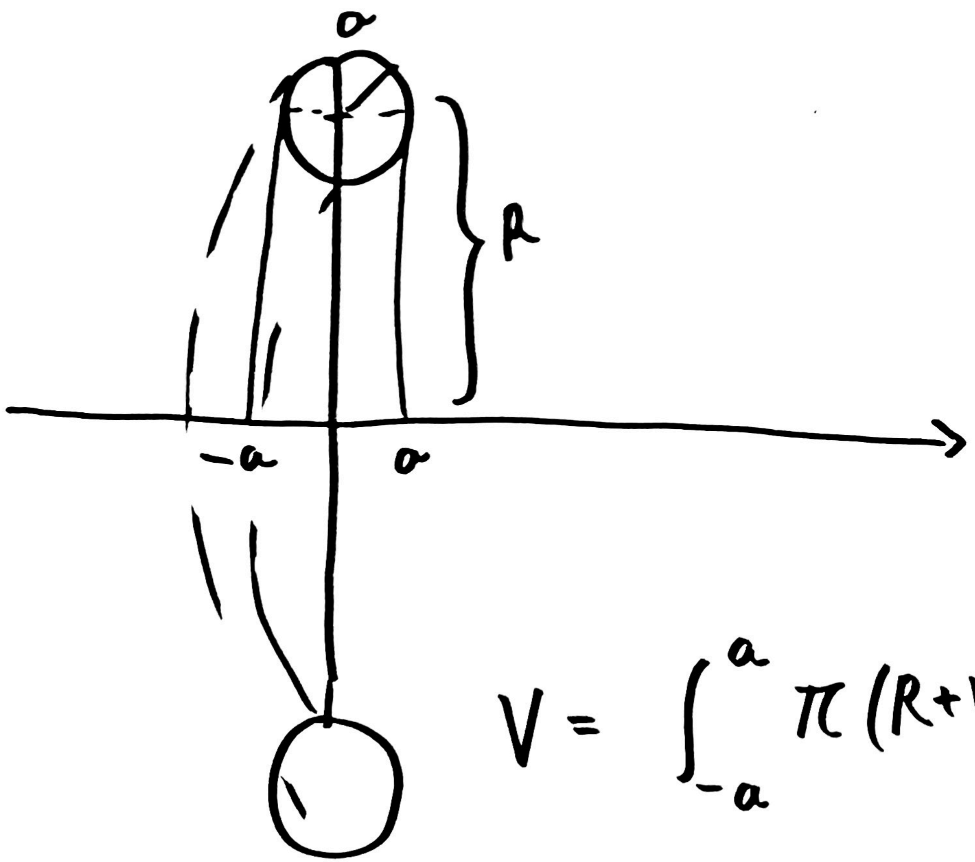


Hva er volumet?

$$V = 2\pi R \cdot \pi a^2$$

$$V = 2\pi^2 R a^2 .$$

$$R \geq a$$



øvre funksjon:

$R + \sqrt{a^2 - x^2}$

nedre funksjonen

$R - \sqrt{a^2 - x^2}$

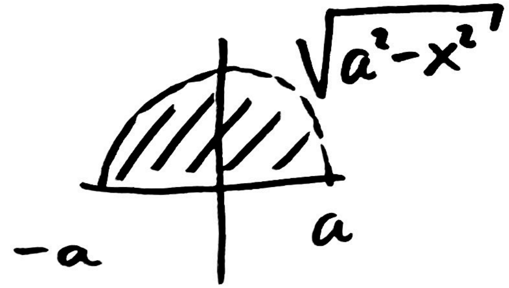
$$V = \int_{-a}^{a} \pi \left(R + \sqrt{a^2 - x^2}\right)^2 - \pi \left(R - \sqrt{a^2 - x^2}\right)^2 \, dx$$

$$V = \pi \int_{-a}^{a} 4R\sqrt{a^2-x^2}\,dx = 4R\pi \int_{-a}^{a} \sqrt{a^2-x^2}\,dx$$

$$= 4R\pi \cdot \frac{\pi a^2}{2}$$

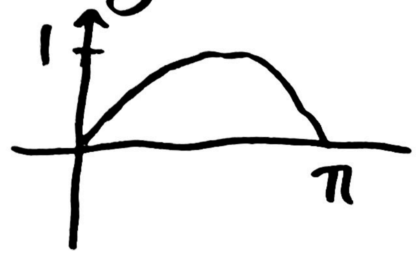
$$= 2\pi R \cdot \pi a^2$$

$$= \underline{2\pi^2 R a^2}$$



integralet er lik arealet til en halv disk med radius a:
$\pi a^2/2$

OPG $f(x) = \sin x$ $\quad x \in [0, \pi]$
Roterer $f(x)$ om x-aksen.
Hva er volumet til legemet som fremkommer.



$$V = \pi \int_0^{\pi} \sin^2 x \, dx$$

$$= \pi \int_0^{\pi} \frac{1}{2}(1-\cos(2x))\, dx$$

$$= \frac{\pi}{2}\left[x - \frac{\sin(2x)}{2}\right]\Big|_0^{\pi}$$

$$= \frac{\pi}{2}[\pi - 0 + 0] = \frac{\pi^2}{2}$$

$$\cos(2x) = \underbrace{\cos^2 x}_{1-\sin^2 x} - \sin^2 x$$

$$\sin^2 x = \frac{1}{2}(1-\cos(2x))$$

Omdreiningslegemet har volum lik halvparten av volumet til sylinderen som inneholder den.

Alternativt: $\int_0^{\pi} \sin^2 x \, dx = \int_0^{\pi} \cos^2 x \, dx$

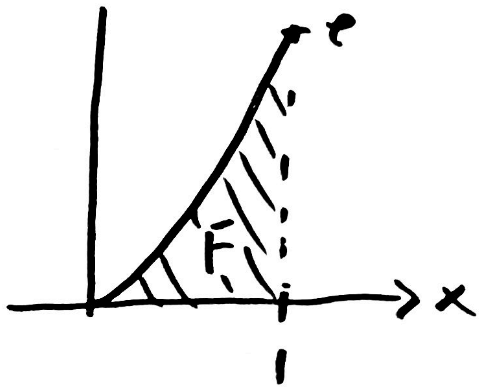
$$= \frac{1}{2}\int_0^{\pi} \underbrace{\sin^2(x) + \cos^2(x)}_{1} \, dx = \frac{1}{2}\int_0^{\pi} 1 \, dx = \frac{\pi}{2}.$$

Øving

17.81

a) Areal til F

b) Volumet til legemet som fremkommer av å rotere F om x-aksen.



$f(x) = xe^x$

a) $A = \int_0^1 xe^x\,dx \overset{\text{delvis int}}{=} xe^x\Big|_0^1 - \int_0^1 1\cdot e^x\,dx$

$= \underbrace{1\cdot e^1 - \cancel{0}\cdot e^0}_{e} - e^x\Big|_0^1 = e - (e-1) = \underline{1}$

b) $V = \int_0^1 \pi (xe^x)^2\,dx = \pi \int_0^1 x^2 e^{2x}\,dx$

$\int \underset{v}{x^2}\,\underset{u'}{e^{2x}}\,dx = x^2\frac{e^{2x}}{2} - \int 2x\cdot\frac{e^{2x}}{2}\,dx$

$u' = e^{2x}$

$u = \frac{e^{2x}}{2}$

$$= \frac{1}{2}x^2 e^{2x} - \underbrace{\int x e^{2x}\,dx}_{x\frac{e^{2x}}{2} - \int 1\cdot\frac{e^{2x}}{2}\,dx}$$

$$\int x^2 e^{2x}\,dx = \frac{1}{2}x^2 e^{2x} - \frac{1}{2}x e^{2x} + \frac{1}{4}e^{2x} + C$$

$$V = \pi\int_0^1 x^2 e^{2x}\,dx = \pi\left[\frac{1}{2}e^2 - \frac{1}{2}e^2 + \frac{1}{4}e^2 - \left(0 - 0 + \frac{1}{4}1\right)\right]$$

$$= \underline{\frac{\pi}{4}(e^2 - 1)} \sim 5.0179\ldots$$