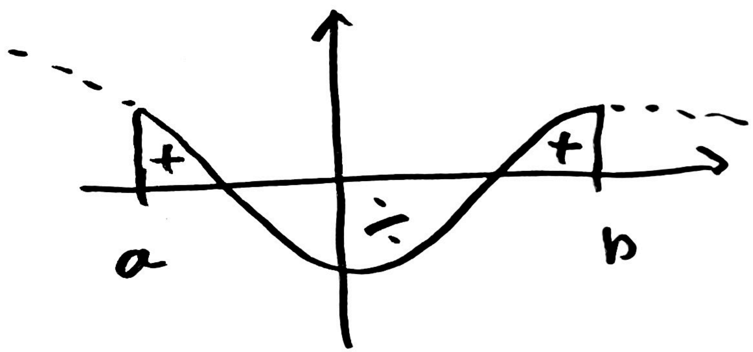
24.03.2023 16.6-7

Bestemt integral

$$\int_a^b f(x)\,dx = \text{"areal med fortegn mellom grafen til } f \text{ og } x\text{-aksen fra } x=a \text{ til } x=b$$



Fundamentalteoremet

$$\int_a^b f(x)\,dx = F(b) - F(a) = F(x)\Big|_a^b$$

$f(x)$ kontinuerlig på $[a,b]$

$F(x)$ er en antiderivert $f(x)$.

$$\int_{-\pi/4}^{\pi/4} 2\cos x \, dx = I$$

$$= 2\int_0^{\pi/4} 2\cos x \, dx$$

$$= 4\int_0^{\pi/4} \cos x \, dx$$

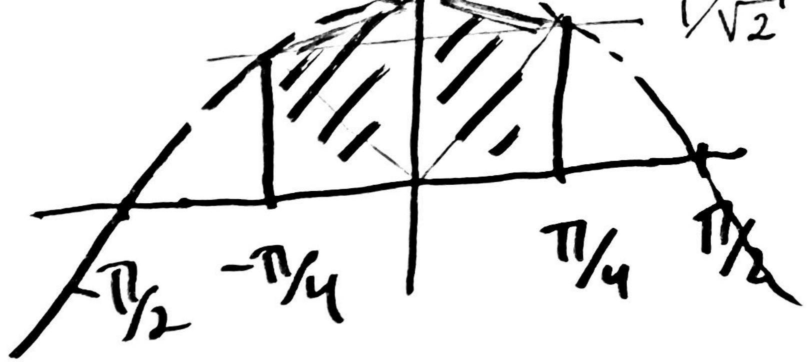
$$= 4 \sin x \Big|_0^{\pi/4}$$

$$= 4\left(\sin\left(\frac{\pi}{4}\right) - \sin 0\right)$$

$$= 4\left(\frac{1}{\sqrt{2}} - 0\right)$$

$$= \frac{4}{\sqrt{2}} = \frac{2\cdot 2}{\sqrt{2}} = 2\sqrt{2} \sim \underline{\underline{2.82}}$$

$$\int_{-\pi/4}^{\pi/4} \sin x \, dx = 0$$



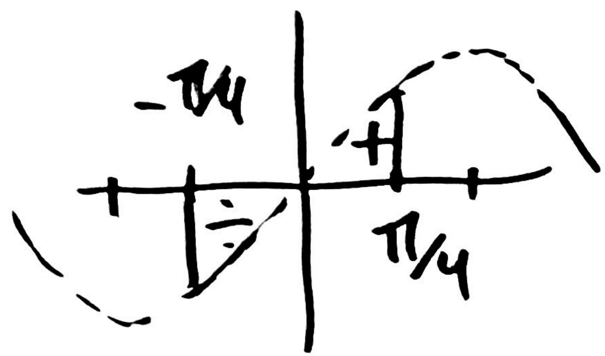
$$\frac{\pi}{\sqrt{2}} = \frac{\pi}{2}\cdot\frac{1}{\sqrt{2}}\cdot 2 \leq I \leq \frac{\pi}{2}\, 1\cdot 2 = \pi$$

$$2.22 \leq I \leq 3.14$$

= trekanten er også under grafen

Så $\frac{1}{2}\left(\frac{\pi}{\sqrt{2}} + \pi\right) \leq I$

$$2.68 \leq I < 3.14.$$



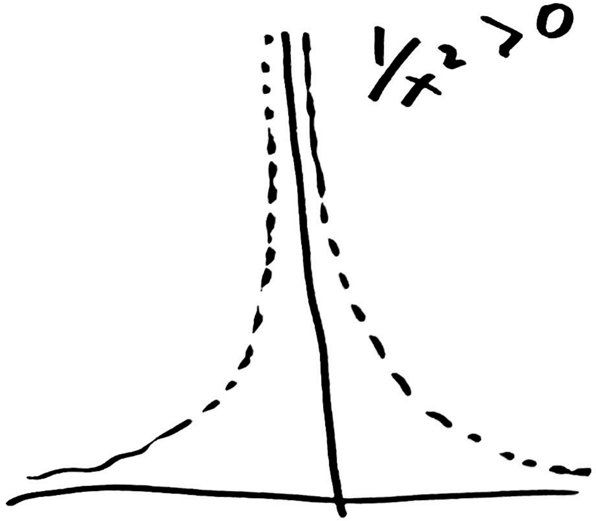
$\int_a^b f(x)\,dx$ må være ikke-negativt hvis

$a<b$ $\quad f(x) \geq 0$ i $[a,b]$

$$\int_{-1}^{1} \frac{1}{x^2}\,dx = \int_{-1}^{1} x^{-2}\,dx = \frac{x^{-1}}{-1}\Big|_{-1}^{1} = \frac{-1}{x}\Big|_{-1}^{1}$$

$$= \frac{-1}{1} - \left(\frac{-1}{-1}\right) = -1-1 = \underline{-2}$$

GALT!

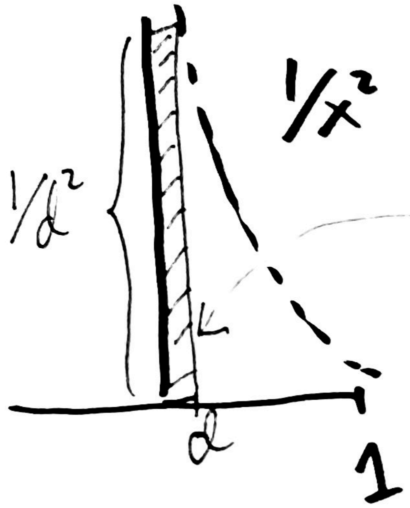


$1/x^2 > 0$

$\frac{1}{x^2}$ ikkje kontinuerlig på $[-1, 1]$

Så fundamentalteoremet er ikkje gyldig.

($\frac{1}{x^2}$ kan utvides til $\begin{cases} 1/x^2 & x \neq 0 \\ c & x=0 \end{cases}$ def $[-1,1]$ ikkje kont.)



areal boks: $d \cdot \frac{1}{d^2} = \underline{\frac{1}{d}}$

$d \to 0$ så vil $1/d \to \infty$

$$\int_d^1 \frac{1}{x^2}\,dx = \frac{-1}{x}\Big|_d^1 = \underline{-1+\frac{1}{d}}$$

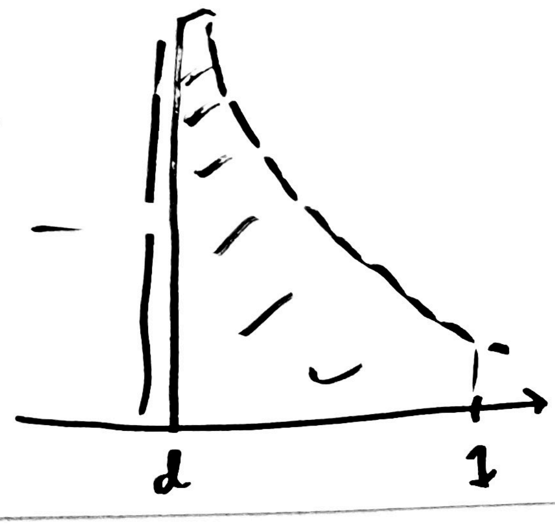
$$\int_0^1 \frac{1}{x^2}\,dx = \lim_{d\to 0^+} \int_d^1 \frac{1}{x^2}\,dx = \lim_{d\to 0^+} -1+\frac{1}{d}$$

$$= \infty$$

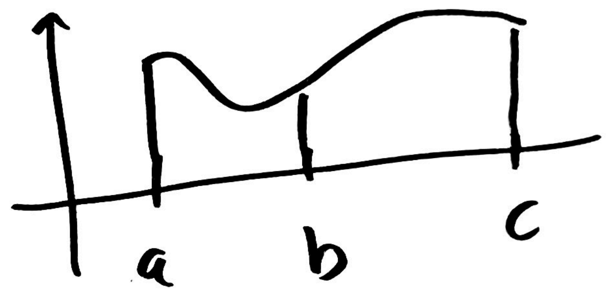
Uegentlige integral.

$$\int_0^1 \frac{1}{\sqrt{x}}\,dx = \lim_{d\to 0^+}\int_d^1 x^{-1/2}\,dx = \lim_{d\to 0^+} \frac{x^{1/2}}{1/2}\Big|_d^1$$

$$= \lim_{d\to 0^+} \left(2\sqrt{1} - 2\sqrt{d}\right) = \underline{\underline{2}}$$



$$\int_a^b f(x)\,dx + \int_b^c f(x)\,dx = \int_a^c f(x)\,dx$$



$$\int_a^b f(x)\,dx + \int_b^a f(x)\,dx$$

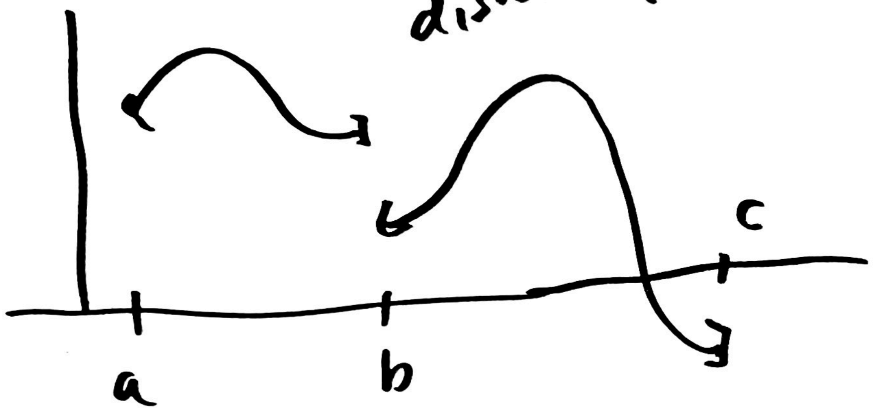
$$= \int_a^a f(x)\,dx = 0$$

$$\int_3^1 f(x)\,dx \;?$$

$$= -\int_1^3 f(x)\,dx$$

$$\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx.$$

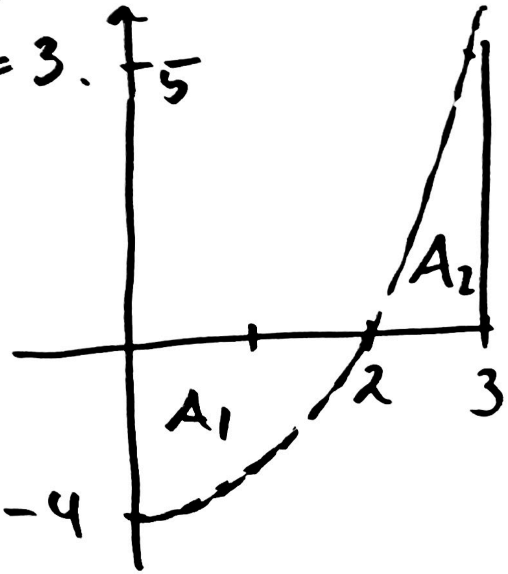
diskont.



$$\int_a^c f(x)\,dx = \int_a^b f(x)\,dx + \int_b^c f(x)\,dx$$

Stykkevis kontinuerlig funksjon.

Finn arealet A mellom grafen til $f(x) = x^2 - 4$ og x-aksen fra $x = 0$ til $x = 3$.



$$A = \int_0^3 |f(x)|\,dx = \underbrace{\int_0^2 -f(x)\,dx}_{A_1} + \underbrace{\int_2^3 f(x)\,dx}_{A_2}$$

Antiderivert til $f(x) = x^2 - 4$ er $F(x) = \frac{x^3}{3} - 4x$

$$A_1 = -\int_0^2 f(x)\,dx = -F(x)\Big|_0^2 = -\left(\frac{8}{3} - 8\right) + 0 =$$
$$\left(\int_2^0 f(x)\,dx\right) \qquad = -8\left(\frac{1}{3} - 1\right) = -8\left(\frac{-2}{3}\right)$$
$$= \underline{\frac{16}{3}}$$

$$A_2 = \int_2^3 f(x)\,dx = \frac{x^3}{3} - 4x \Big|_2^3$$
$$= \left(\frac{3^3}{3} - 4\cdot 3\right) - \left(\frac{8}{3} - 8\right)$$
$$= 9 - 12 + \frac{16}{3} = \frac{16}{3} - \frac{9}{3} = \underline{\frac{7}{3}}$$

Arealet er $A = A_1 + A_2 = \frac{16}{3} + \frac{7}{3} = \frac{23}{3} = \underline{7 + \frac{2}{3}}$

$$\left(\int_0^3 f(x)\,dx = A_2 - A_1 = \frac{7}{3} - \frac{16}{3} = \frac{-9}{3} = -3\right)$$

Oppg Finn $I = \int_{-2}^{3} |x| \, dx$

$$|x| = \begin{cases} x & x \geq 0 \\ -x & x < 0 \end{cases}$$

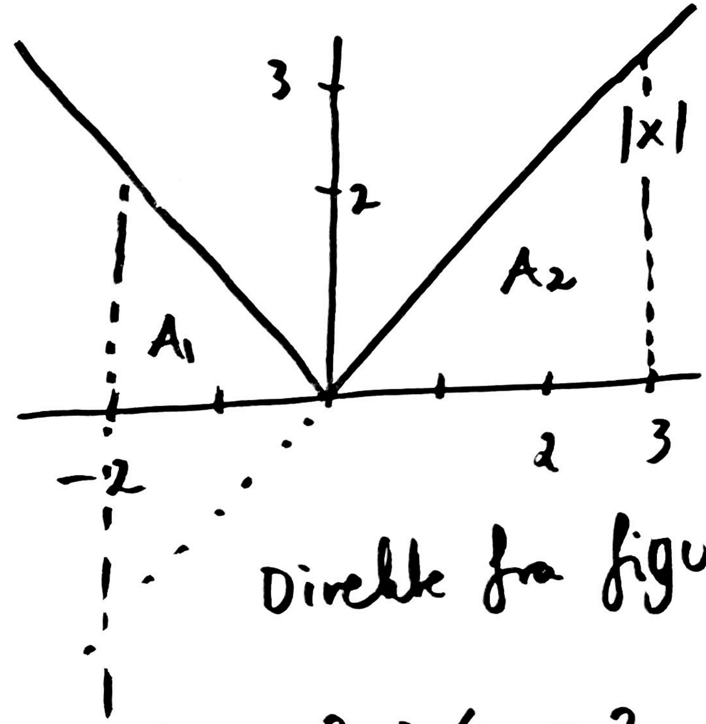
$$I = \underbrace{\int_{-2}^{0} -x \, dx}_{A_1} + \underbrace{\int_{0}^{3} x \, dx}_{A_2}$$

$$= -\frac{x^2}{2}\Big|_{-2}^{0} + \frac{x^2}{2}\Big|_{0}^{3}$$

$$= -\frac{-(-2)^2}{2} + \frac{3^2}{2}$$

$$= 2 + \frac{9}{2} = \underline{6.5}$$

En antiderivert til $|x|$ er



Direkte fra figuren

$A_1 = 2 \cdot 2/2 = 2$

$A_2 = 3 \cdot 3/2 = 9/2 = 4.5$

$A_1 + A_2 = \underline{6.5}$

$$\begin{cases} x^2/2 & x \geq 0 \\ -x^2/2 & x < 0 \end{cases}$$

(kont.)

Areal mellom to grafer

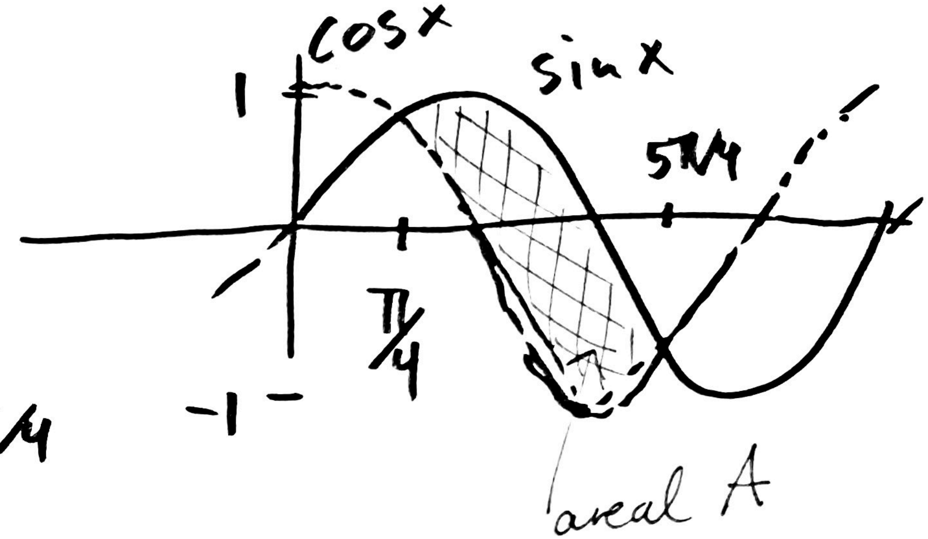
$$A = \int_{\pi/4}^{5\pi/4} \sin x - \cos x \, dx$$

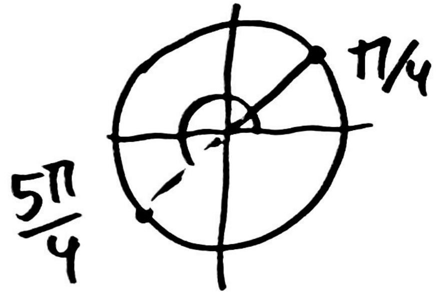
$$= -\cos x - \sin x \Big|_{\pi/4}^{5\pi/4}$$

$$= -(\cos x + \sin x) \Big|_{\pi/4}^{5\pi/4}$$

$$= -\left(2\left(-1/\sqrt{2}\right)\right) + \left(2 \cdot 1/\sqrt{2}\right)$$

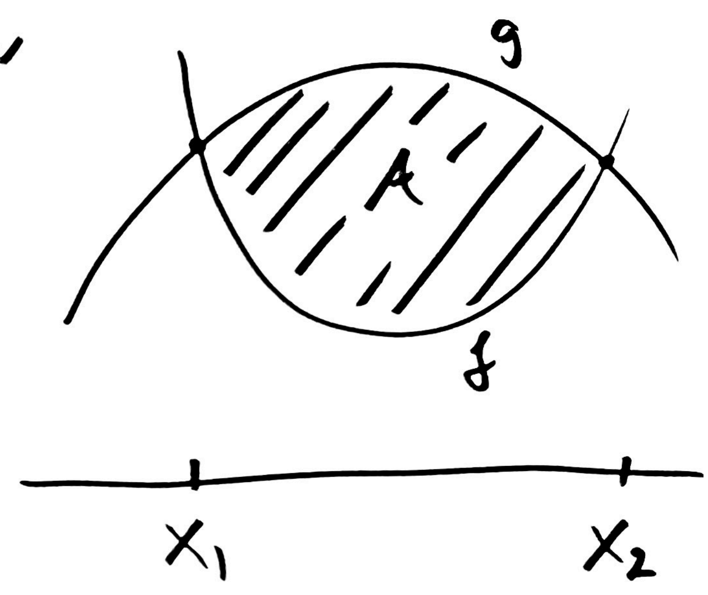
$$= 4/\sqrt{2} = \underline{2\sqrt{2}} \sim 2.82$$





Finn arealet til området avgrenset av parablene

$$f(x) = x^2 - 3x - 5$$
$$g(x) = -x^2 - x + 7$$



$$f(x) = g(x)$$
$$x^2 - 3x - 5 = -x^2 - x + 7$$
$$2x^2 - 2x - 12 = 0 \qquad \left| \cdot \tfrac{1}{2}\right.$$
$$\Leftrightarrow \quad x^2 - x - 6 = 0$$
$$(x-3)(x+2) = 0$$

$\underline{x_1 = -2}$ og $\underline{x_2 = 3}$

$$A = \int_{-2}^{3} (g - f)\, dx = \int_{-2}^{3} -2x^2 + 2x + 12 \, dx$$
$$= 2\int_{-2}^{3} -x^2 + x + 6 \, dx$$

$$2\left[-\frac{x^3}{3}+\frac{x^2}{2}+6x\right]_{-2}^{3}$$

$$=2\left[-\frac{3^3}{3}+\frac{3^2}{2}+6\cdot3-\left(-\frac{(-2)^3}{3}+\frac{(-2)^2}{2}+6(-2)\right)\right]$$

$$=2\left[-9+\frac{9}{2}+6\cdot3-\frac{8}{3}-2+6\cdot2\right]$$

$$=2\left[-9-2+6\cdot5+\frac{9}{2}-\frac{8}{3}\right]$$

$$=2\left[30-11+\frac{3\cdot9-2\cdot8}{6}\right]$$

$$=2\left[19+\frac{11}{6}\right]=2\left[20+\frac{5}{6}\right]$$

$$=40+2\cdot\frac{5}{6}=40+\frac{5}{3}=41+\frac{2}{3}$$

$$\sim \underline{41.66}$$