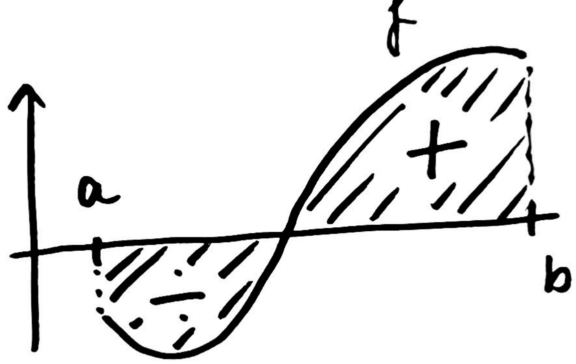
23.03 2023 # Bestemte integral

16.4-5

(1) øvre integral / nedre grense $\int_a^b f(x)\,dx$ integrand

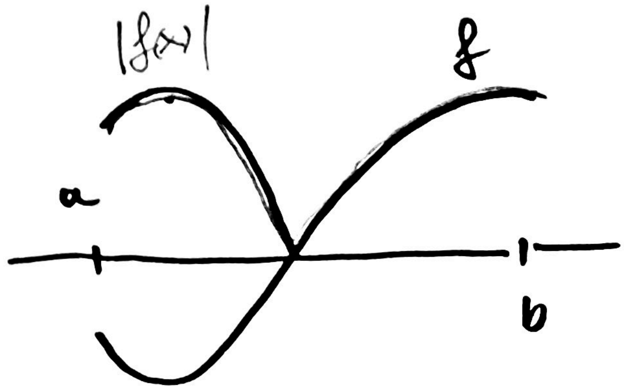
• "Det bestemte integralet av $f(x)$ fra a til b (med hensyn til x)"



$\int_a^b f(x)\,dx$ = "areal med fortegn" ~~...~~ mellom grafen til $f(x)$ og x-aksen fra $x=a$ til $x=b$.

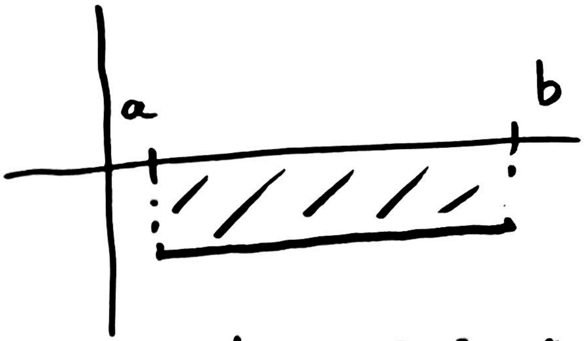
Arealet mellom grafen til $f(x)$ og x-aksen fra $x=a$ til $x=b$

$\int_a^b |f(x)|\,dx$



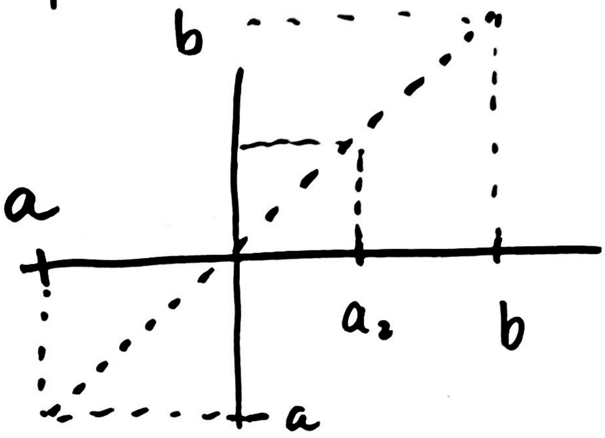
② $f(x) = k$ konstant

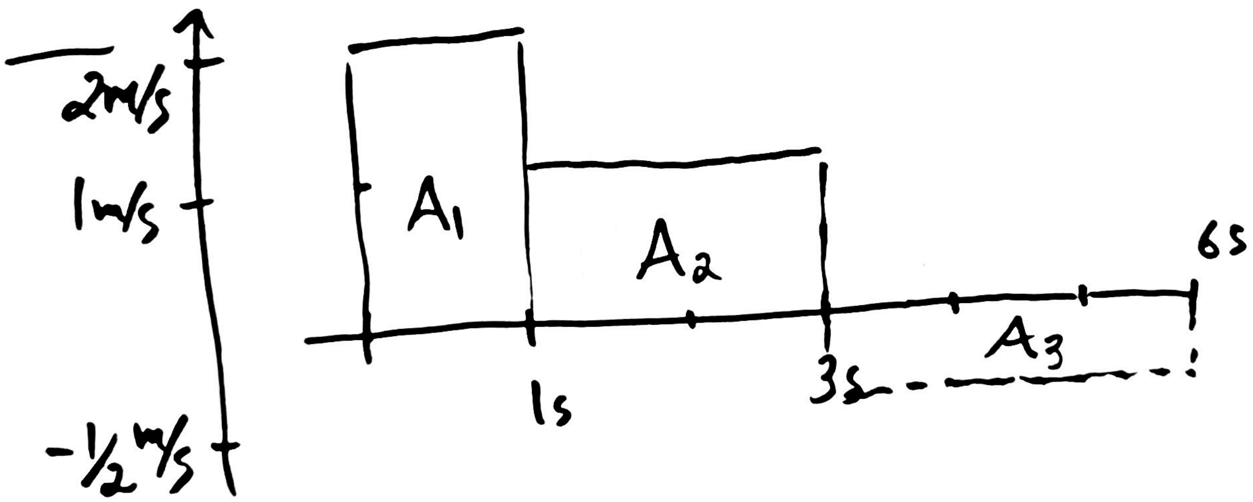
$$\int_a^b k\,dx = k(b-a)$$



$f(x) = x$

$$\int_a^b x\,dx = \underline{\frac{b^2}{2} - \frac{a^2}{2}}$$





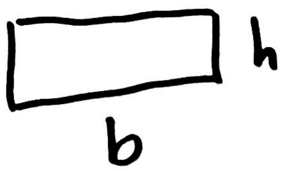
Distanse : $A_1 + A_2 - A_3$

$\left(2\frac{m}{s}\cdot 1s\right) + \left(1\frac{m}{s}\cdot 2s\right) \quad \frac{1}{2}m/s \cdot 3s$

$2m + 2m - \frac{3}{2}m$

$= \underline{2.5m}$

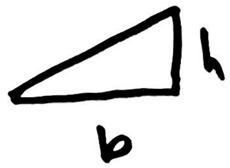
distanse forflyttet : $A_1 + A_2 + A_3$



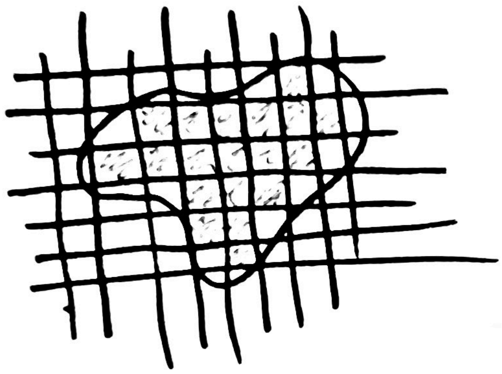
areal $b \cdot h$

(3)

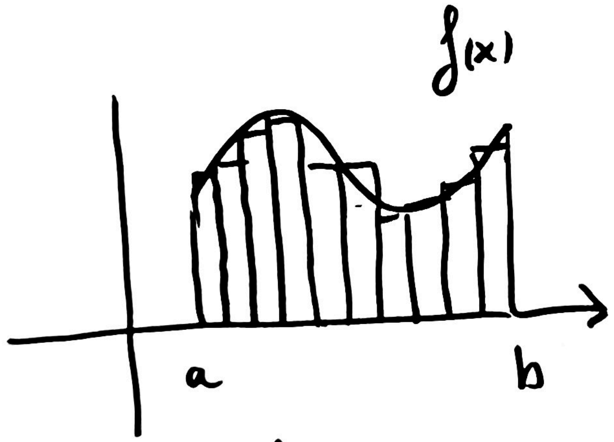
så



areal $\frac{1}{2} b \cdot h$



Hva er arealet?



N like brei intervaller

bredde $\Delta x = \frac{b-a}{N}$

Riemann integral.

$$\int_a^b f(x)\,dx = \lim_{N \to \infty} \sum_{i=1}^{N} f(x_i) \cdot \Delta x$$

Riemannsummer

x_i x-verdi

i i-te interval

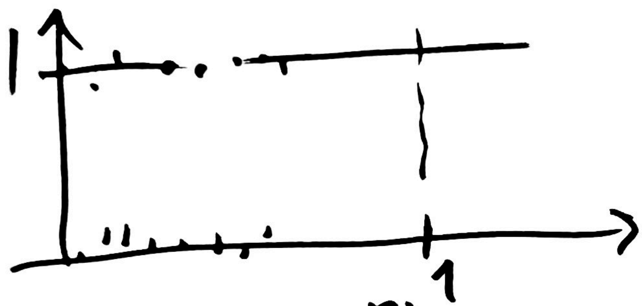
Geogebra lowersum

uppersum ...

(4) Hvis $\int_a^b f(x)\,dx$ finnes ~~ex~~ sier vi at $f(x)$ er integrerbar.

Resultat: Alle kontinuerlige funksjoner er integrerbare.

$$g(x)=\begin{cases}1 & x \text{ rasjonal}\\ 0 & x \text{ irrasjonal}\end{cases}$$



Ikke Riemann integrerbar.

deler opp i N like deler : $d=\frac{b-a}{N}$

første $[a, a+\frac{b-a}{N}]$ andre intervall

$[a+d, a+2d]$

i-te intervall $[a+d(i-1), a+di]$

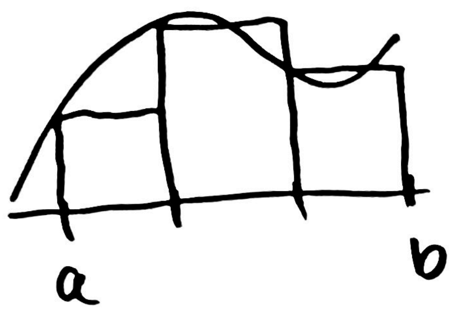
$x_i \in [a+d(i-1), a+di]$

(5)

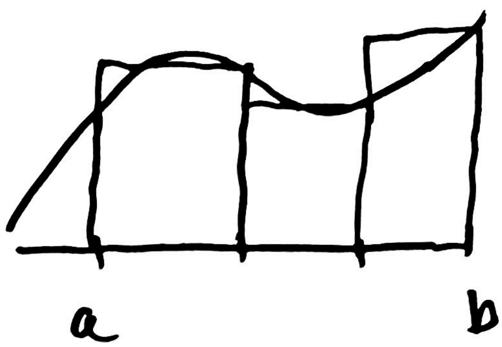
1 velge x_i: til venstre i intervallet $x_i = a + d(i-1)$

høyre i intervallet $x_i = a + di$

2. —

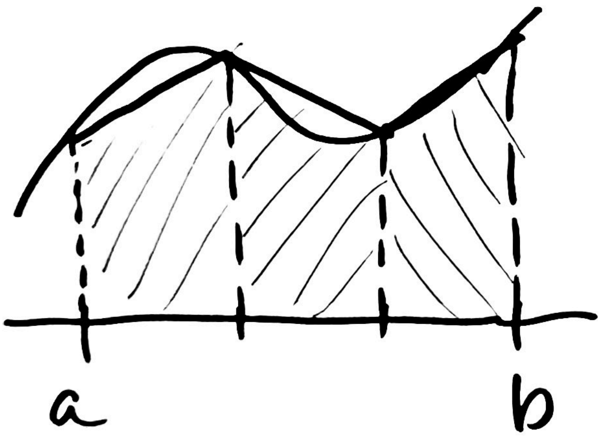


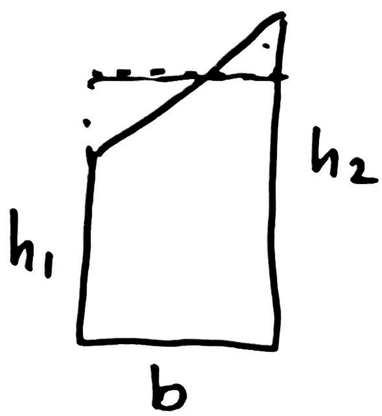
"venstre sum"



høyre sum

trapes metoden.





Tilnærming til $\int_a^b f(x)\,dx$

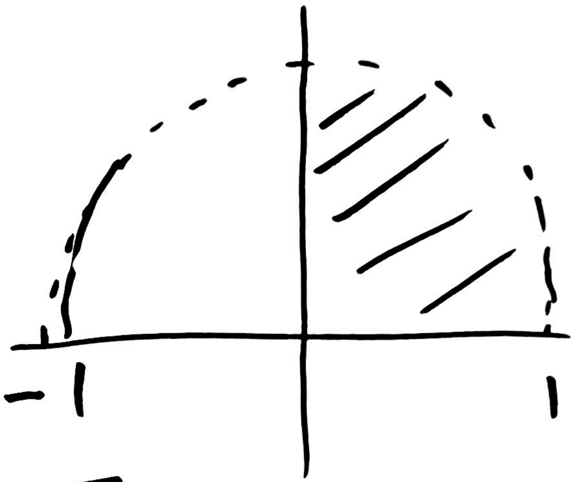
areal $\frac{h_1 + h_2}{2} b$

$$= \frac{d}{2}\left(f(a) + 2f(a+d) + \cdots 2f(b-d) + f(b)\right)$$

= gjennomsnittet av venstre og høyre sum.

(6) $x^2 + y^2 = 1$ sirkel m radius 1
senter i origo

$$y = \sqrt{1-x^2}$$



$$\int_0^1 \sqrt{1-x^2}\,dx = \frac{\pi}{4}$$

$$\pi = 4\int_0^1 \sqrt{1-x^2}\,dx$$

Fundamentalteoremet i kalkulus.

7

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

$f(x)$ kontinuerlig, F er en antiderivert

skrivemåte. $F(b) - F(a) = F(x)\Big|_a^b$

$$\int_a^b k\,dx = kx + c\Big|_a^b$$

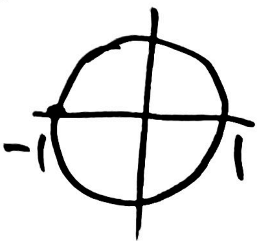
$$= (kb + c) - (k \cdot a + c)$$

$$= \underline{k(b-a)}$$

$$\int_a^b x\,dx = \frac{x^2}{2}\Big|_a^b = \frac{b^2}{2} - \frac{a^2}{2}.$$

$$\int_0^\pi \sin x\,dx = (-\cos x)\Big|_0^\pi$$

$$= -\underbrace{(-1)}_{\cos\pi} - (-\underbrace{(+1)}_{\cos 0}) = \underline{\underline{2}}$$

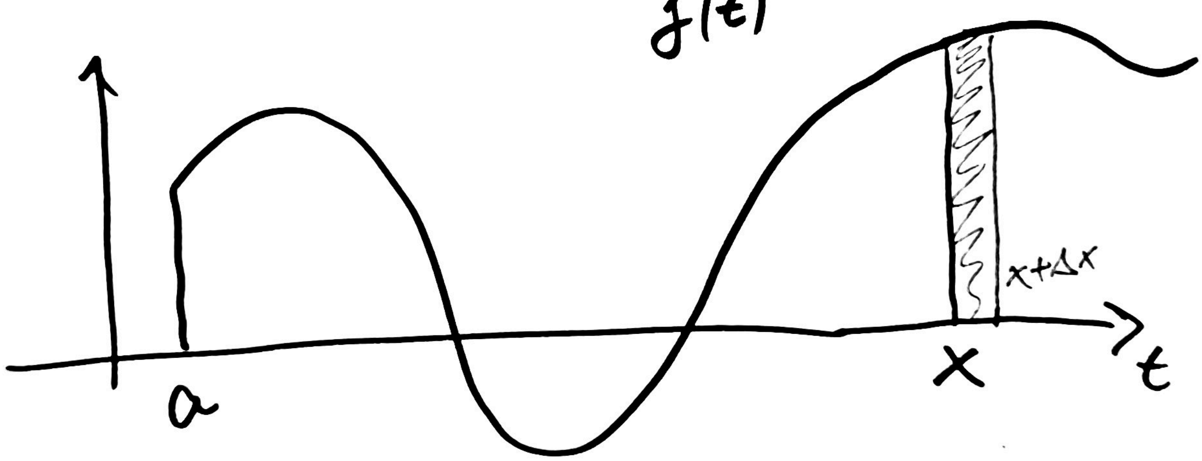


⑧ $\int_0^2 x^3 - 2x \, dx$

$$= \frac{x^4}{4} - x^2 \Big|_0^2$$

$$= \frac{2^4}{4} - 2^2 - (0 - 0)$$

$$= 4 - 4 = \underline{0}$$



$$F(x) = \int_a^x f(t)\, dt$$

$$F(x+\Delta x) - F(x) = \int_x^{x+\Delta x} f(x)\, dx$$

$$\sim \Delta x \cdot f(x)$$

$$F'(x) = \lim_{\Delta x \to 0} \frac{F(x+\Delta x) - F(x)}{\Delta x} = f(x)$$

Så $F(x)$ er en antiderivert for $f(x)$

9 La $G(x)$ være en annen antiderivert til $f(x)$

$$F(x) = G(x) + C$$

$$F(a) = \int_a^a f(x)dx = 0$$

$$0 = F(a) = G(a) + C$$

så $C = -G(a)$

$$\int_a^x f(t)dt = F(x) = G(x) + (-G(a))$$

sett $x = b$

$$\int_a^b f(t)dt = \underline{G(b) - G(a)}$$