20.03.23

16. Antiderivert og ubestemte integral.

$(x^2)' = 2x$ $\quad$ x^2 er en antiderivert til $2x$.

$S'(t) = V(t)$

posisjon $\quad$ hastighet

$V(t) = at + V_0$, $V_0 = V(0)$

$S(t) = a\frac{t^2}{2} + V_0 t + S_0$

En antiderivert $F(x)$ til en funksjon $f(x)$ har egenskapen

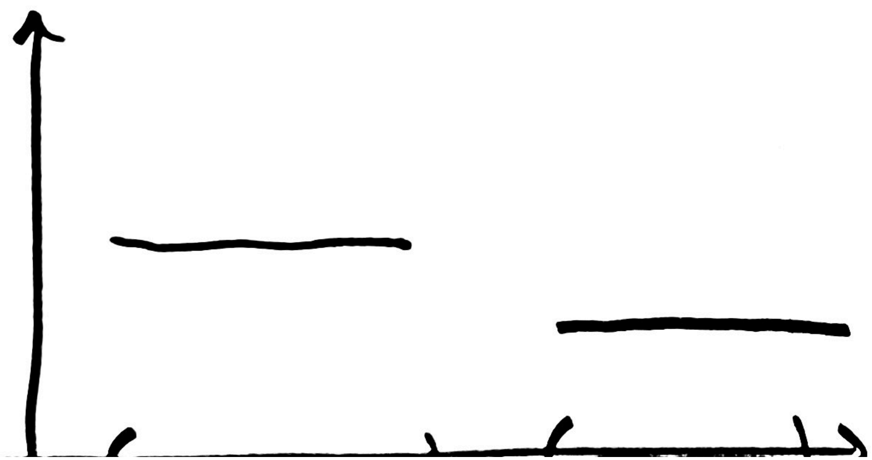
$$F'(x) = f(x)$$

$2x$ har antideriverte x^2, $x^2 - 13$, $x^2 + 2$ etc.

Det ubestemte integralet $\int f(x)\,dx$ er samlingen av alle antideriverte til $f(x)$

integraltegn $\quad$ integrand $\quad$ variabelen vi integrer med hensyn til.

på hver komponent av
det. mengden.



konstant

$$F(x) = E(x) + C$$

$$\int f(x)\,dx = F(x) + C$$

en antideivert

$$\int 2x\,dx = x^2 + C$$

$$\int x^4\,dx = \frac{1}{5}x^5 + C$$

$$\boxed{\int x^r\,dx = \frac{x^{r+1}}{r+1} + C \qquad r \neq -1}$$

$$(x^5)' = 5x^{5-1}$$

$$= 5x^4$$

$$\left(\frac{1}{5}x^5\right)' = x^4$$

$$(x^r)' = rx^{r-1}$$

$$\left(\frac{1}{r}x^r\right)' = x^{r-1}$$

Så $\left(\frac{x^{r+1}}{r+1}\right)' = x^r$

$$\int x^7\,dx = \frac{x^8}{8} + C$$

$$\int \frac{1}{x^3}\,dx = \int x^{-3}\,dx = \frac{x^{-2}}{-2} + C$$

$$= \frac{-1}{2x^2} + C$$

(strengt talt $\int \frac{1}{x^3}\,dx = \begin{cases} \frac{-1}{2x^2} + C_1 & x<0 \\ \frac{-1}{2x^2} + C_2 & x>0 \end{cases}$

$$\left(\frac{1}{3}+1=\frac{4}{3}\right)$$

$$\int \sqrt[3]{x}\,dx = \int x^{1/3}\,dx = \frac{x^{\frac{1}{3}+1}}{4/3} + C$$

$$= \frac{3x\sqrt[3]{x}}{4} + C$$

$$\int x^9\,dx = \frac{x^{10}}{10} + C$$

$$\int \frac{1}{x^5}\,dx = \int x^{-5}\,dx = \frac{x^{-4}}{-4} + C = \underline{\frac{-1}{4x^4} + C}$$

$$\int \frac{1}{\sqrt{x}}\,dx = \int x^{-1/2}\,dx = \frac{x^{1/2}}{1/2} + C = \underline{2\sqrt{x} + C}$$

$$\int \frac{2x^2-3}{x^2}\,dx = \int 2 - 3x^{-2}\,dx$$

$$= 2x - 3\left(\frac{x^{-1}}{-1}\right) + C$$

$$= \underline{2x + \frac{3}{x} + C}$$

$(F)' = f$ og $(G)' = g$

$$\left(a \cdot F(x) + b\,G(x)\right)' = a\,F'(x) + b \cdot G'(x)$$
$$= a\,f(x) + b\,g(x)$$

$$\int a\,f(x) + b\,g(x)\,dx = a\,F(x) + b\,G(x) + C$$

$$\underline{\int a\,f(x) + b\,g(x)\,dx = a\int f(x)\,dx + b\int g(x)\,dx}$$

lineært

Merk: $\int 0 \cdot f(x)\,dx = 0 \cdot \int f(x)\,dx = 0(F(x) + C)$

$= 0$ (legger til $+ C$)

C

$$\int x(3-5x)\,dx = \int 3x - 5x^2\,dx$$

$$= 3\int x\,dx - 5\int x^2\,dx$$

$$= 3\left(\frac{x^2}{2}\right) - 5\frac{x^3}{3} + C$$

$$\underset{x>0}{(\ln(x))'} = \frac{1}{x}$$

$$\underset{x<0}{(\ln(-x))'} = \frac{1}{-x}(-x)' = \frac{1}{x}$$

$$\underset{x\neq 0}{(\ln|x|)'} = \frac{1}{x}$$

$$\int \frac{1}{x}\,dx = \underline{\ln|x| + C} \quad \left(\begin{cases} \ln|x| + C_1 & x>0 \\ \ln|x| + C_2 & x<0 \end{cases} \right)$$

$$(e^x)' = e^x$$

$e = 2.718\ldots$
Euler tallet

$$(a^x)' = (\ln a)\, a^x$$

$$\left(a^x = (e^{\ln a})^x = e^{\ln a \cdot x} \;\ldots \right)$$

$$\int e^x \, dx = e^x + C$$

opg

$$\int 2e^x - \frac{7}{x} + \frac{3}{x^2} \, dx$$

$$= 2\int e^x dx - 7\int \frac{1}{x} dx + 3\int \frac{1}{x^2} dx$$

$$= 2e^x - 7\ln|x| + 3\,\frac{x^{-1}}{-1} + C$$

$$= \underline{2e^x - 7\ln|x| - \frac{3}{x} + C}$$

$$\int (1+x)^2\,dx = \int 1 + 2x + x^2\,dx$$
$$= x + x^2 + \frac{x^3}{3} + C$$
$$= \frac{1}{3}(1+x)^3 + C$$

$$((1+x)^3)'$$
$$= 3(1+x)^2(1+x)'$$
$$= 3(1+x)^2$$

$$\int (1+x)^7\,dx = \frac{(1+x)^8}{8} + C$$

Linear substitusjon

$F'(x) = f(x)$ da er $(F(ax+b))' = F'(ax+b)\cdot(ax+b)'$

$$= a\,f(ax+b)$$

$$\left(\frac{1}{a}F(ax+b)\right)' = f(ax+b)$$

$$\int f(ax+b)\,dx = \frac{1}{a}\int f(u)\,du$$ hvor $u = ax+b$

$$\int (3-4x)^{13}\,dx$$

$u = 3-4x$

$u' = -4$

$$= \frac{1}{-4}\int u^{13}\,du$$

$$= \frac{-1}{4}\left(\frac{u^{14}}{14}\right) + C$$

$$= \underline{\frac{-1}{56}(3-4x)^{14} + C}$$

$$\int e^{-x+3}\,dx = \frac{1}{-1}\int e^{u}\,du$$

$$= -e^{u} + C$$

$$= \underline{-e^{-x+3} + C}$$

$u = -x+3$

$u' = -1$

$$\int \frac{3}{2x-5}\,dx$$

$$= 3\int \frac{1}{u}\cdot\frac{1}{2}\,du$$

$$= \frac{3}{2}\ln|u| + C$$

$$= \underline{\frac{3}{2}\ln|2x-5| + C}$$

$$\left(\begin{aligned} u &= 2x-5 \\ u' &= 2 \\ \frac{du}{dx} &= 2 \\ du &= 2dx \quad \Big| \cdot \frac{1}{2} \\ \frac{1}{2}du &= dx \end{aligned}\right)$$

$$\left((2+x^3)^5\right)' = 5(2+x^3)^4 \cdot (2+x^3)'$$

$$= 5 \cdot 3x^2 (2+x^3)^4$$

$$(u^5)' = 5u' u^4$$

$$\int (5u^4) \underbrace{u' dx}_{du} = \int 5u^4 du$$

$$15 \int x^2 (2+x^3)^4 dx = (2+x^3)^5 + C$$

$$5 \int \underbrace{(2+x^3)^4}_{u^4} \underbrace{(3x^2) dx}_{\substack{u' \\ du}}$$

Substitusjon.

$$\int \frac{5}{(2-3x)^4}\,dx$$

$$= 5\int (2-3x)^{-4}\,dx$$

$$= 5\int u^{-4}\frac{1}{-3}\,du$$

$$= \frac{-5}{3}\int u^{-4}\,du$$

$$= \frac{-5}{3}\left(\frac{u^{-3}}{-3}\right) + c$$

$$= \frac{5}{9(2-3x)^3} + C$$

$u = 2-3x$

$u' = -3$

$du = -3\,dx$

$dx = \frac{1}{-3}\,du$

Øving

16.11 $f(x) = 8x^3 + 12x^2 + 10x + 3$

F antiderivert

$F(-1) = 2$

$$\int f(x)\,dx = 8\int x^3\,dx + 12\int x^2\,dx + 10\int x\,dx + 3\int 1\,dx$$

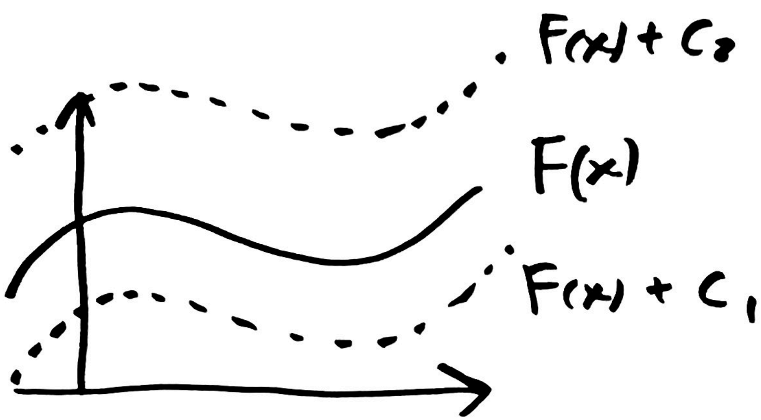
$$F(x) = 8\frac{x^4}{4} + 12\frac{x^3}{3} + 10\frac{x^2}{2} + 3x + C$$

$$F(x) = 2x^4 + 4x^3 + 5x^2 + 3x + C$$

$$F(-1) = 2 + -4 + 5 - 3 + C = 2$$

$$\underline{C = 2}$$

$$F(x) = \underline{2x^4 + 4x^3 + 5x^2 + 3x + 2}$$



16.10 d) $\int 4x^3 - 6x + 1 \, dx$

$$= 4\int x^3 dx - 6\int x \, dx + \int 1 \, dx$$

$$= 4\frac{x^4}{4} - 6\frac{x^2}{2} + \frac{x^1}{1} + c$$

$$= \underline{x^4 - 3x^2 + x + C} \qquad C \in \mathbb{R}$$

16.13 c) $\int \left(2t - 5\sqrt[3]{t^2}\right) dt$

$$= 2\int t\,dt - 5\int t^{2/3}\,dt$$

$$= 2\,\frac{t^2}{2} - 5\,\frac{t^{5/3}}{5/3} + C$$

$$= t^2 - 3t^{5/3} + C$$

$$= \underline{t^2 - 3t\sqrt[3]{t^2} + C}$$

$$\sqrt[3]{t^2} = (t^2)^{1/3} = t^{2/3}$$

$$\frac{-5}{5/3} \,\Big|\, \cdot \frac{3}{3}$$

$$= \frac{-5\cdot 3}{5} = -3.$$

Prøv på 16.15.

16.13 d) $\int \frac{5}{2\sqrt{x}}\,dx = \frac{5}{2}\int x^{-1/2}\,dx$

$= \frac{5}{2} \cdot \frac{x^{-\frac{1}{2}+1}}{1/2} + C$

$\left(\frac{1}{\sqrt{x}} = \left(\sqrt{x}\right)^{-1} = \left(x^{1/2}\right)^{-1} = x^{-1/2} \right)$

$= 5x^{1/2} + C$

$= \underline{5\sqrt{x} + C}$