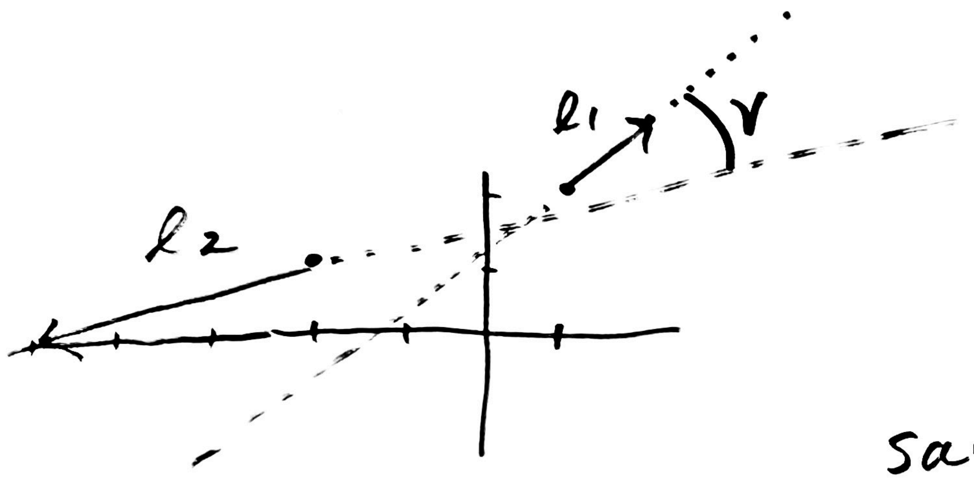
10.02.23 Gitt to linjer: $\ell_1 \begin{cases} x = 1 + t \\ y = 2 + t \end{cases}$ $t \in \mathbb{R}$ $\qquad \ell_2 \begin{cases} x = -2 - 3s \\ y = 1 - s \end{cases}$ $s \in \mathbb{R}$



Finn snittpunktet til ℓ_1 og ℓ_2

samme x og y-koordinater

x: $1 + t = -2 - 3s$ likningssystem

y: $2 + t = 1 - s$

$t = -2 + 1 - s = \underline{-1} - s$

Likning for y gir:

setter inn i likning for x:

$1 + \underbrace{(-1 - s)}_{t} = -2 - 3s$

$-s = -2 - 3s$

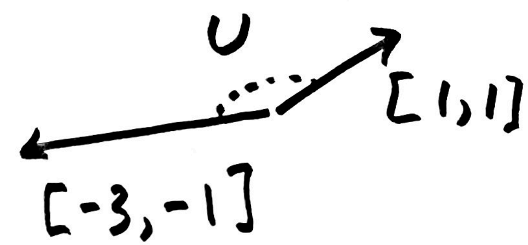
$3s - s = -2$

$2s = -2$ så $\underline{s = -1}$

Skjæringspunktet er $(-2-3(-1),\ 1-(-1))$

$$\underline{(1,2)}$$

Hva er vinkelen mellom de to linjene?



$$\vec{a}\cdot\vec{b} = |\vec{a}|\cdot|\vec{b}|\cos(u)$$

$$\cos(u) = \frac{[1,1]\cdot[-3,-1]}{\sqrt{2}\sqrt{9+1}} = \frac{-3-1}{\sqrt{2}\sqrt{10}} = \frac{-4}{\sqrt{2}\sqrt{2}\sqrt{5}} = \underline{\frac{-2}{\sqrt{5}}}$$

$$u \approx \underline{153.435^\circ}$$

Vinkel mellom 2 linjer er i intervallet $[0, 90^\circ]$
(benytter den minste av de to vinklene)

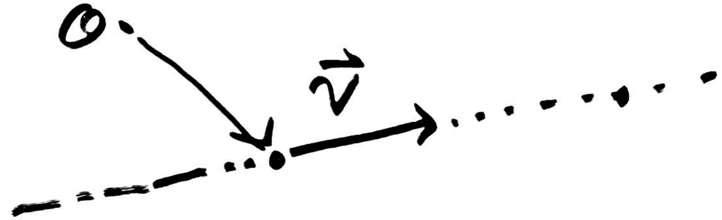
$$v = 180^\circ - u = \underline{26.565^\circ}$$

posisjon ved $t=0$ S_0, $(x_0, y_0) = (2m, -3m)$

Hastighetsvektor $\vec{v} = [3 m/s, 4 m/s]$

$(|\vec{v}| = 5 m/s)$

posisjonen ved tiden t :



$\overrightarrow{OS}(t) = \overrightarrow{OS_0} + t\vec{v}$

$= [2m, -3m] + [3 m/s \cdot t, 4 m/s \cdot t]$

$S(t) = \underline{(2 + 3 m/s \cdot t, -3m + 4 m/s \cdot t)}$

Gitt $\vec{a} = [-1, 1]$

$\vec{b} = [1, 2]$

Finn t s. a. $\vec{a} + \vec{b}t$ blir minst mulig.

Finn lengden til $\vec{a} + \vec{b}t$ da.

$$|\vec{a} + \vec{b}t| = |[-1+t, 1+2t]|$$

$$= \sqrt{\underbrace{(-1+t)^2 + (1+2t)^2}_{U(t)}}$$

Vi kan derivere og finne ut hva funksjonen $|\vec{a} + \vec{b}t|$ blir minst

$$|\vec{a} + \vec{b}t|' = \left((U(t))^{1/2}\right)' = \frac{1}{2\sqrt{U}}\left(2\overbrace{(-1+t)'}^{1}(-1+t) + 2\overbrace{(1+2t)'}^{2}(1+2t)\right)$$

$$= \frac{1}{2\sqrt{U}}\left(2(-1+t) + 2 \cdot 2(1+2t)\right)$$

$$= \frac{2}{2\sqrt{U}} \left(t-1 + 2\cdot 1 + 2\cdot 2\cdot t \right)$$

$$= \frac{1}{\sqrt{U}} \left(t + 4t - 1 + 2 \right)$$

$$= \frac{1}{\sqrt{U}} \left(5t + 1 \right)$$

Den deriverte er lik 0 når $5t+1=0$

$\underline{t = -1/5}$

$$\vec{a} + t\vec{b} = \vec{a} - \frac{1}{5}\vec{b} = [-1, 1] - \frac{1}{5}[1, 2]$$

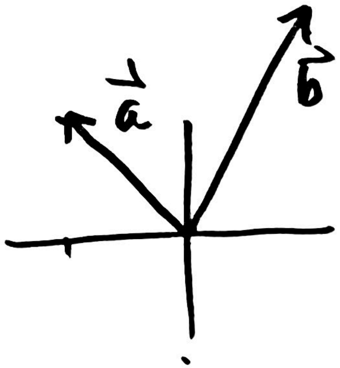
$$= \left[-\frac{6}{5}, \frac{3}{5}\right] = \frac{3}{5}[-2, 1]$$

Lengden er $\frac{3}{5}\left|[-2, 1]\right| = \frac{3}{5}\sqrt{4+1}$

$$= \frac{3}{5}\sqrt{5} = \frac{3}{\sqrt{5}} \approx \underline{1.3416\ldots}$$

$\vec{a} = [-1, 1]$

$\vec{b} = [1, 2]$



$\vec{a} + t\vec{b}$ minst når den er vinkelrett på $\vec{b}$

$\Leftrightarrow (\vec{a} + t\vec{b}) \cdot \vec{b} = 0$

$\vec{a} \cdot \vec{b} + t\, \vec{b} \cdot \vec{b} = 0$

$t = \dfrac{-\vec{a} \cdot \vec{b}}{|\vec{b}|^2}$

$$t = \frac{[-1, 1] \cdot [1, 2]}{|[1, 2]|^2}$$

$$= \frac{-1 + 2}{1 + 4} = \frac{-1}{5}$$

$3\vec{AB} = [6, 9]$ $\quad$ $B(2, -5)$

$C(5, -5)$

Opg 1) Finn A

2) Finn $\vec{AC}$

3) Vinkelen mellom $\vec{AB}$ og $\vec{AC}$.

---

1) $\vec{AB} = \frac{1}{3}(3\vec{AB}) = \frac{1}{3}[6, 9] = [2, 3]$

$\vec{OB} - \vec{OA} = \vec{AB}$ så $\vec{OA} = \vec{OB} \underbrace{- \vec{AB}}_{\vec{BA}}$

$\vec{OA} = [2, -5] - [2, 3] = \underline{[0, -8]}$

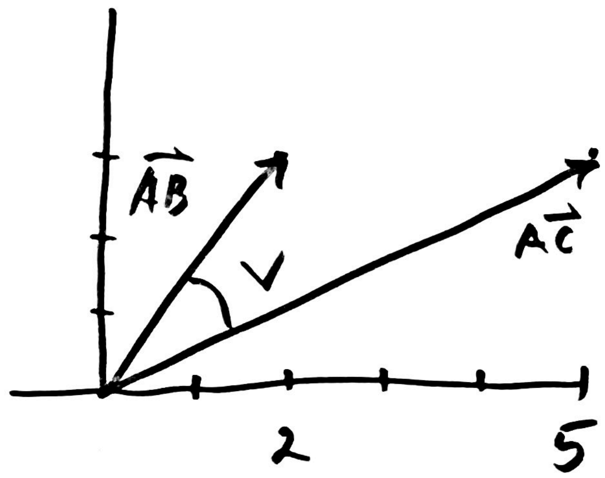
$\underline{A(0, -8)}$

2) $\vec{AC} = \vec{OC} - \vec{OA} = [5, -5] - [0, -8]$

$\vec{AC} = \underline{[5, 3]}$

3) Vinkel mellom $\vec{AB} = [2, 3]$

$\vec{AC} = [5, 3]$.



$$\cos v = \frac{\vec{AB} \cdot \vec{AC}}{|\vec{AB}| \cdot |\vec{AC}|} = \frac{[2,3] \cdot [5,3]}{\sqrt{4+9}\,\sqrt{25+9}} = \frac{2 \cdot 5 + 3 \cdot 3}{\sqrt{13}\,\sqrt{34}} = \frac{19}{\sqrt{13}\,\sqrt{34}}$$

$$v = \arccos\left(\frac{19}{\sqrt{13}\,\sqrt{34}}\right) = \underline{25{,}346°}$$

opg

Parametriser linjen gjennom $C(5, -5)$ som står vinkelrett på linjen gjennom A og B. $\vec{AB} = [2, 3]$

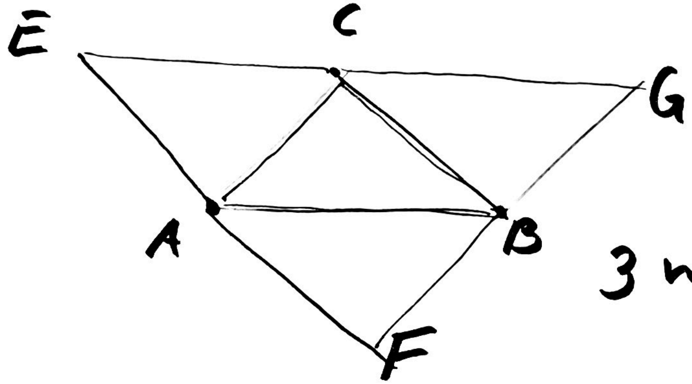
$\vec{n} = [-3, 2]$ er normal til $\vec{AB}$.

$$[x, y] = \vec{OC} + t\vec{n} = \underline{[5, -5] + t[-3, 2]}$$

$$\begin{cases} x = 5 - 3t \\ y = -5 + 2t \end{cases}.$$

parallellogram

Motstående sider er like lange og parallelle.



Mulige parallellogram med 3 gitte punkt som hjørner.

3 muligheter.

$\vec{AE} = \vec{BC}$ så $\vec{OE} = \vec{BC} + \vec{OA}$ etc

$\vec{OA} = \vec{OO} = \vec{0}$ $\vec{OB} = [2,0]$ $\vec{OC} = [1,1]$

$\vec{OE} = \vec{0} + \vec{OC} - \vec{OB}$
$= [1,1] - [2,0] = [-1,1]$

$E(-1,1)$

$\vec{AF} = \vec{CB}$

$$\vec{OF} = \vec{OA} + \vec{CB}$$

$$= \vec{0} + \vec{OB} - \vec{OC}$$

$$= [2,0] - [1,1]$$

$$= [1,-1]$$

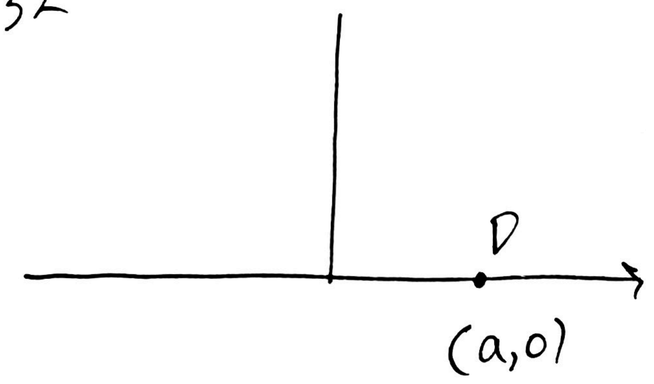
så $\underline{F(1,-1)}$

$$\vec{AG} = \vec{AB} + \vec{AC} = [2,0] + [1,1] = [3,1]$$

$\underline{G(3,1)}$

13.32

b)



$$\vec{DC} = \vec{OC} - \vec{OD}$$
$$= [5.5] - [a,0]$$
$$= [5-a, 5]$$

$$\vec{AC} = [7,4].$$

$$\vec{AC} \perp \vec{DC}$$
$$\vec{AC} \bullet \vec{DC} = [7,4] \bullet [5-a, 5] = 0$$

$$35 - 7a + 20 = 0$$
$$a = \frac{55}{7} \quad (= 8 - \tfrac{1}{7})$$

$D\left(\frac{55}{7}, 0\right)$