23.01.23 Deriverte av trig. funksjoner.

$$(\sin(x))' = \cos(x) \qquad (\cos(x))' = -\sin(x)$$

x har enheter radianer.

Kjerneregel

$$f \circ u(x) = f(u(x))$$

$$(f \circ u)'(x) = f'(u(x)) \cdot u'(x)$$

$$(\sin(\underbrace{3x+1}_{u}))' = \cos(3x+1) \cdot \underbrace{u'(x)}_{(3x+1)' = 3}$$

$$= 3\cos(3x+1)$$

Hvis vi bruker grader i stedenfor radianer

$$\left(\sin\left(\underbrace{\frac{\pi}{180} \cdot x}_{u}\right)\right)' = \cos\left(\frac{\pi}{180} \cdot x\right)\left(\frac{\pi}{180} \cdot x\right)'$$

$$= \underline{\frac{\pi}{180} \cos\left(\frac{\pi}{180} \cdot x\right)}$$

Produktregelen $\quad (f(x)\cdot g(x))' = f'(x)\cdot g(x) + f(x)\cdot g'(x)$

$$(2\sin x\cdot\cos x)' = 2\big(\underbrace{(\sin x)'}_{\cos x}\cos x + \sin x\underbrace{(\cos x)'}_{-\sin x}\big)$$

$$= \underline{2(\cos^2 x - \sin^2 x)}.$$

er lik

$$= \underline{2\cos(2x)}$$

$$(\sin(2x))' = \cos(2x)\cdot(2x)' = 2\cos(2x).$$

$$(\sin^2 x + \cos^2 x)' = (1)' = 0$$

$\sin^2 x = (\sin x)^2$

$$(\sin^2 x)' + (\cos^2 x)'$$

alternativt

$$2\sin x\underbrace{(\sin x)'}_{\cos x} + 2\cos x\underbrace{(\cos(x))'}_{-\sin x} = 2\sin x\cos x - 2\sin x\cos x = 0$$

opg $f = 2\sin(3x) + 5\cos(4)$

$$f' = 2\underbrace{(\sin 3x)'}_{\cos(3x)\cdot(3x)'} + 5(\cos(4))'$$

$$+ 5 \cdot 0$$

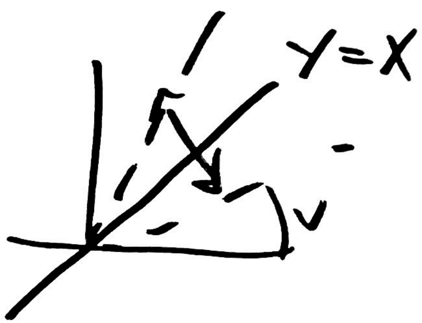
$$= 2\cos(3x)\cdot 3 = \underline{6\cos(3x)}.$$

Tangentlinjen til $\sin x$ i $(a, \sin a)$

er $y = \cos(a)(x-a) + \sin a$

Den deriverte til cos følger fra den deriverte til sin.

$$\sin x = \cos\left(\frac{\pi}{2} - x\right)$$



$$\cos(x) = \sin\left(\frac{\pi}{2} - x\right).$$

så $(\cos x)' = \left(\sin\left(\frac{\pi}{2} - x\right)\right)' = \cos\left(\frac{\pi}{2} - x\right)\cdot\underbrace{(-1)}_{\left(\frac{\pi}{2}-x\right)'} = \underline{-\sin x}$

$$\tan x = \frac{\sin x}{\cos x} \quad \left(\text{definert når } \cos x \neq 0\right)$$

$$= \sin x \cdot \frac{1}{\cos x} = \sin x \cdot (\cos x)^{-1}$$

$$(\tan x)' = \underbrace{(\sin x)'}_{\cos x} \cdot (\cos x)^{-1} + \sin x \cdot \underbrace{\left((\cos(x))^{-1}\right)'}_{-1(\cos x)^{-2}(-\sin x)}$$

$$= \frac{\cos x}{\cos x} + \frac{(\sin x)^2}{(\cos x)^2} = \frac{\overbrace{\cos^2 x + \sin^2 x}^{1}}{\cos^2 x}$$

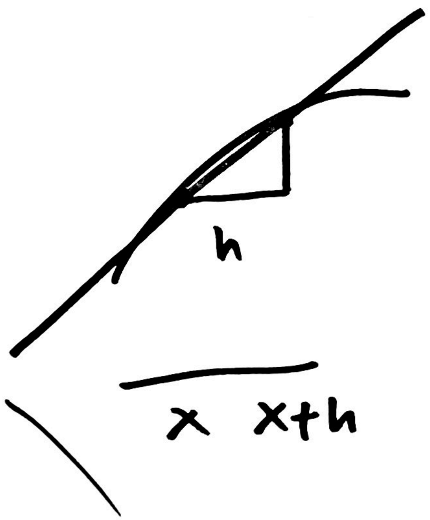
$$\boxed{\begin{aligned}(\tan x)' &= 1 + \tan^2 x \\ &= \frac{1}{\cos^2 x}\end{aligned}}$$

Deriver $e^{-x}\cos x$

$$\begin{aligned}
\left(e^{-x}\cos x\right)' &= (e^{-x})'\cos x + e^{-x}(\cos x)' \\
&= e^{-x}(-x)'\cos x + e^{-x}(-\sin x) \\
&= -1 \cdot e^{-x}\cos x - e^{-x}\sin x \\
&= \underline{-e^{-x}(\sin x + \cos x)}
\end{aligned}$$

Bevis for: $(\sin x)' = \cos x.$

$$(\sin x)' = \lim_{h\to 0} \frac{\sin(x+h) - \sin x}{h}$$



Addisjonsformelen

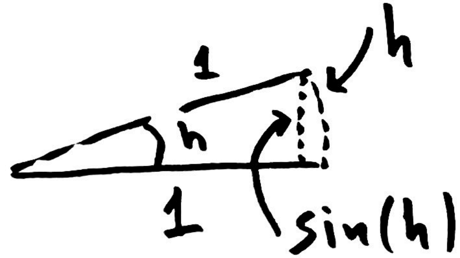
$$\sin(x+h) = \sin x \cdot \cos(h) + \cos x \cdot \sin(h)$$

$$(\sin x)' = \lim_{h\to 0} \frac{\sin x(\cos(h) - 1) + \cos x \sin(h)}{h}$$

$$= \sin x \underbrace{\lim_{h\to 0} \frac{\cos(h) - 1}{h}}_{0} + \cos x \underbrace{\lim_{h\to 0} \frac{\sin(h)}{h}}_{1}$$

$(\sin x)' = \underline{\cos x}$

Resultatet: $\lim_{h\to 0} \frac{\sin(h)}{h} = 1.$



Dette fører til at $\lim_{h\to 0} \frac{1-\cos h}{h^2} = \frac{1}{2}$

fordi $\lim_{h\to 0} \frac{1-\cos(h)}{h^2} \cdot \frac{1+\cos(h)}{1+\cos(h)}$

$$= \lim_{h\to 0} \frac{1-\cos^2 h}{h^2} \cdot \frac{1}{1+\cos(h)}$$

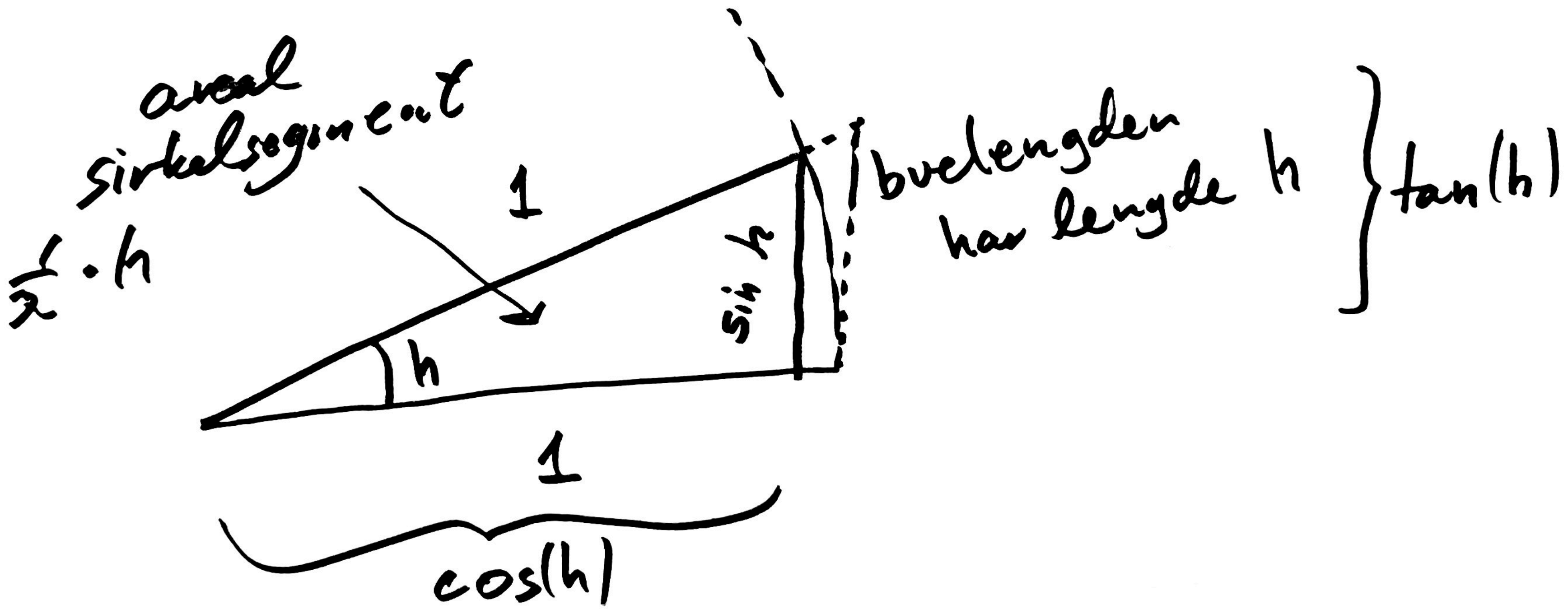
$$\lim_{h\to 0}\left(\left(\frac{\sin(h)}{h}\right)^2 \cdot \frac{1}{1+\cos(h)}\right) = \frac{1}{2}.$$

Derfor er $\lim_{h\to 0} \frac{\cos(h)-1}{h} = \lim_{h\to 0} \frac{\cos(h)-1}{h^2} \cdot h = 0$

$$\frac{\sin(-h)}{-h} = \frac{-\sin(h)}{-h} = \frac{\sin(h)}{h}$$

så $\lim\limits_{h\to 0^-} \frac{\sin(h)}{h} = \lim\limits_{h\to 0^+} \frac{\sin(h)}{h}$.

Det er tilstrekkelig å syne $\lim\limits_{h\to 0^+} \frac{\sin(h)}{h}$.



$$\underbrace{\frac{1}{2}\sin(h)\cos(h)}_{\substack{\text{areal}\\ \text{minste } \Delta}} \leq \underbrace{\frac{1}{2}h}_{\substack{\text{areal}\\ \text{sirkelsegment}}} \leq \underbrace{\frac{1}{2}\tan(h)}_{\substack{\text{areal}\\ \text{største } \Delta}}. \qquad 0 < h < \frac{\pi}{4}$$

deler med $h/2$

$$\underbrace{\frac{\sin h}{h}\cos h \leq 1}_{1/\cos h} \leq \underbrace{\frac{\sin h}{h}\cdot\frac{1}{\cos h}}_{\cdot \cos(h)}$$

$$\cos(h) \leq \frac{\sin(h)}{h} \leq \frac{1}{\cos(h)}$$

siden $\cos(h) \to 1$, $\frac{1}{\cos(h)} \to 1$ når $h \to 0$ så må $\underline{\lim_{h\to 0} \frac{\sin(h)}{h} = 1}$

Eksempel på Taylor polynomer.

$p(x) = x - \frac{x^3}{6}$ har samme deriverte som $\sin(x)$

$p(x)^{(n)}$ for $n = 0, 1, 2, 3$ i $x = 0$

$p(0) = 0$

$p' = 1 - \frac{3x^2}{6} = 1 - \frac{x^2}{2}$

$p'(0) = 1 = (\sin x)' = \cos x \,|_{x=0}$

$p''(x) = -\frac{2x}{2} = -x.$ $\qquad p''(0) = 0$

$(\sin x)'' = (\cos x)' = -\underline{\sin x}$ $\qquad (\sin''(x))|_{x=0} = 0$

$(\sin x)''' = -\cos x$ $\qquad$ i $x=0$ $(\sin'''(x))_{x=0} = -1$

$p'''(0) = (-x)'|_{x=0} = -1$ ✓

$q(x) = x - \frac{x^3}{6} + \frac{x^5}{120}$

$r(x) = x - \frac{x^3}{6} + \frac{x^5}{120} - \frac{x^7}{5040}$

samme deriverte i $x=0$ som $\sin x$ $\Big\}$ opp til 5-deriverte

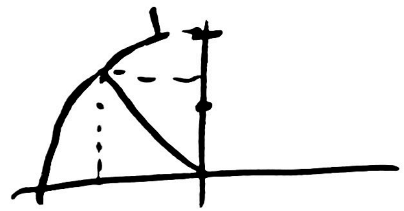
7 deriverte.

$$\underbrace{\sin^2 x}_{1-\cos^2 x} + \cos x - 1$$

$$= \cos x - \cos^2 x$$

$$= \cos x(1 - \cos x)$$

$\sin^2 x + \cos^2 x = 1$ Enhetsformelen = Pytagoras.



$\underline{\sin x = \frac{3}{5} = 0.6}$ $90 < x < 180$.

Finn $\tan x$ og $\cos x$

$$\cos^2 x = 1 - \underbrace{\frac{9}{25}}_{\sin^2 x} = \frac{25-9}{25} = \frac{16}{25}$$

$$\underbrace{\sin^2 x}_{\left(\frac{3}{5}\right)^2} + \cos^2 x = 1$$

$$\cos x = \pm\sqrt{\frac{16}{25}} = \pm\frac{4}{5} = \pm 0.8$$

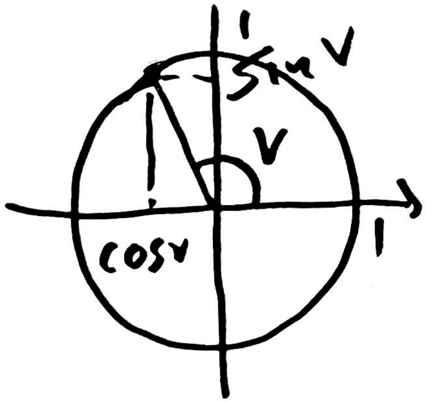
$90^\circ < x < 180^\circ$ så $\cos x < 0$ Derfor $\underline{\cos x = \frac{-4}{5}}$.

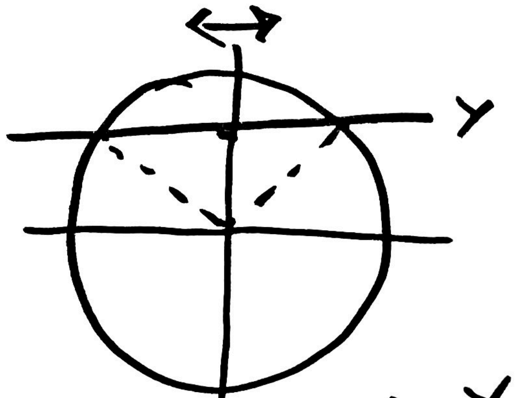
$$\tan x = \frac{\sin x}{\cos x} = \frac{3/5}{-4/5} = \underline{-\frac{3}{4}} = \underline{\underline{-0{,}75}}$$

oppg. Deriver $\frac{1}{\cos x}$ $(= \sec x)$

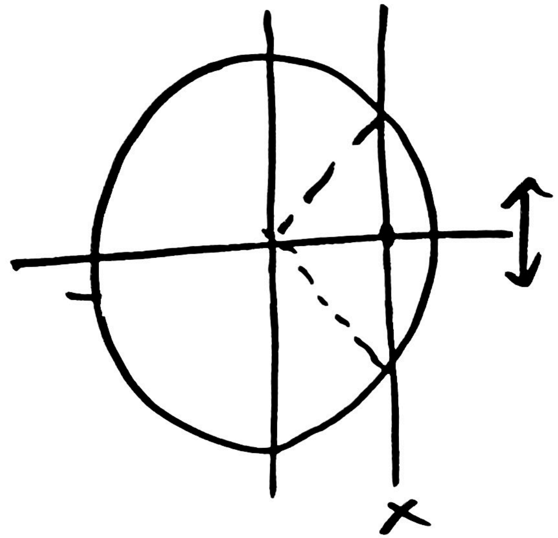
$$\left(\frac{1}{\cos x}\right)' = \left((\cos x)^{-1}\right)' = -1\,(\cos x)^{-2}\,\underbrace{(\cos x)'}_{-\sin x}$$

$$= \underline{\frac{\sin x}{\cos^2 x}} = \tan x \cdot \frac{1}{\cos x}$$





$\sin V = Y$



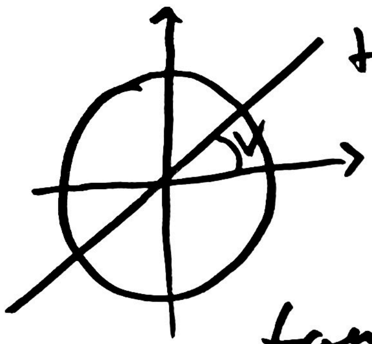
$\underbrace{\arcsin(Y)}_{V_1} + 2\pi \cdot n$

$\pi - V_1 + 2\pi \cdot n.$

$\cos V = X$

$V_1 = \arccos X + 2\pi \cdot n$

$-V_1 + 2\pi \cdot n$



tan v : stigningstallet

$\tan(v) = z$

Løsningen $v = \arctan(z) + \pi \cdot n$

opg

$$\sin^2 x + \cos x - 1 = 0$$

$$\cos x(1 - \cos x) = 0$$

$x = 0, \frac{-\pi}{2}, \frac{\pi}{2}$ og til hele omløp.