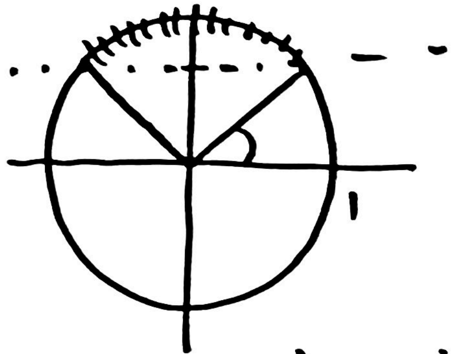
20.1.23

Trigonometriske ulikheter

$\sin v > \frac{1}{\sqrt{2}}$ $\qquad v \in [0, 2\pi]$

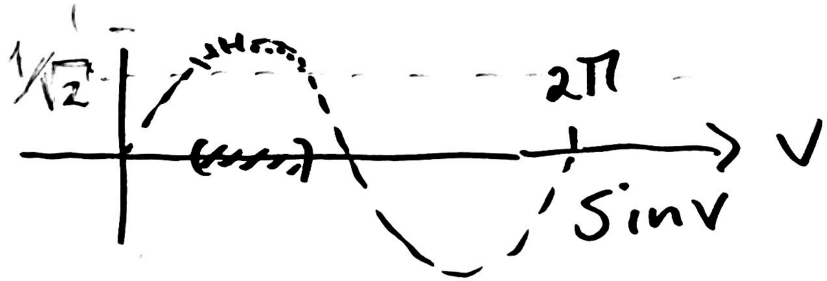


$\sin v = \frac{1}{\sqrt{2}}$

$v = \frac{\pi}{4}$ og $\frac{3\pi}{4}$

Løsningene er $\underline{v \in \langle \frac{\pi}{4}, \frac{3\pi}{4} \rangle}$

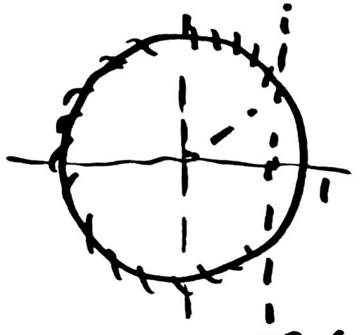
Alternativt:



$\cos v < \frac{\sqrt{3}}{2}$ $\qquad v \in [0, 2\pi]$

$\cos v = \frac{\sqrt{3}}{2}$

Løsningen til likningen $\frac{\pi}{6}$ og $\frac{11\pi}{6}$



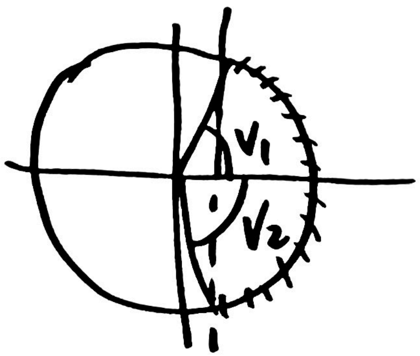
Løsningene er $\underline{v \in \langle \frac{\pi}{6}, \frac{11\pi}{6} \rangle}$

$\cos v \geq 0.2$ $\qquad v \in [0, 2\pi]$.

$v_1 = \arccos(0.2) \hat{=} 78.463 \sim \underline{1.369}$

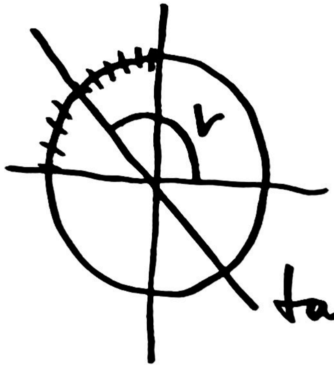
andre løsning til $\cos v = 0.2$

er $v_2 = 2\pi - v_1 = 4.9147\ldots$



Løsningene er $\underline{v \in [0, v_1] \cup [v_2, 2\pi]}$

opg. Løs $\tan v \leq 0$ $\quad v \in [0, 2\pi]$.



tan v er stigningstallet til linjen.

$$v \in \{0\} \cup \left\langle \frac{\pi}{2}, \pi \right] \cup \left\langle \frac{3\pi}{2}, 2\pi \right]$$

$x \in [0, 2\pi]$

$\sin(2x) = 2\sin x \cos x$

$$\sin x \cos x > \frac{1}{4} \quad | \cdot 2$$

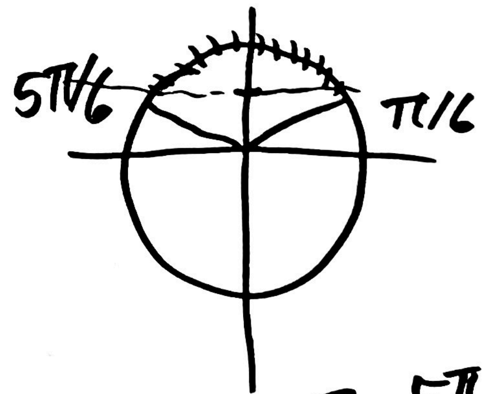
$$\frac{1}{2}\sin(2x) > \frac{1}{4}$$

$$\sin(2x) > \frac{1}{2}$$

$$x = v/2$$

$$v = 2x, \quad v \in [0, 4\pi].$$

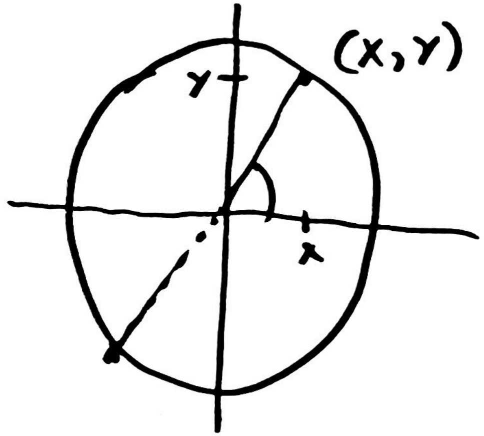
$$\sin(v) > \frac{1}{2}$$



$$v \in \left\langle \frac{\pi}{6}, \frac{5\pi}{6} \right\rangle \cup \left\langle \frac{13\pi}{6}, \frac{17\pi}{6} \right\rangle$$

$$x \in \left\langle \frac{\pi}{12}, \frac{5\pi}{12} \right\rangle \cup \left\langle \frac{13\pi}{12}, \frac{17\pi}{12} \right\rangle$$

$\sin x > \cos x$ $\quad x \in [0, 2\pi]$



Metode 1: $x \in \left\langle \frac{\pi}{4}, \frac{5\pi}{4} \right\rangle$ ved inspeksjon av enhetssirkelen.

Metode 2: $\cos x > 0$: Deler med $\cos x$ $\quad \tan x > 1$

$\left\langle \frac{\pi}{4}, \frac{\pi}{2} \right\rangle$

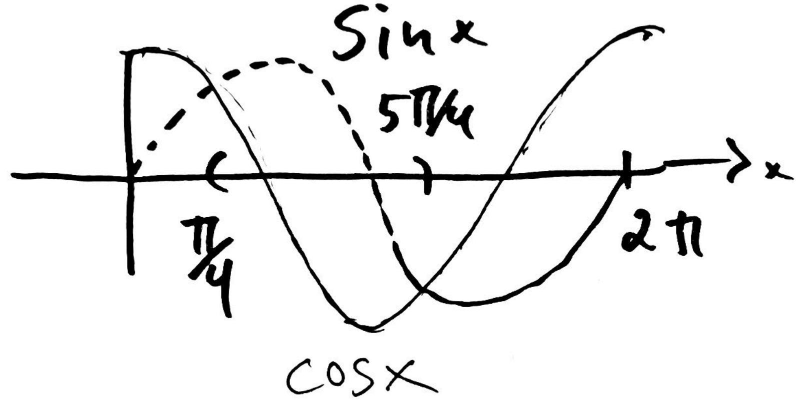
$\cos x < 0$ —‖— $\quad \tan x < 1$

$\left\langle \frac{\pi}{2}, \frac{5\pi}{4} \right\rangle$

$\cos x = 0$. løsning når $\sin x = 1$: $x = \pi/2$

Dette gir tilsammen løsningen $\underline{\left\langle \frac{\pi}{4}, \frac{5\pi}{4} \right\rangle}$

Metode 3



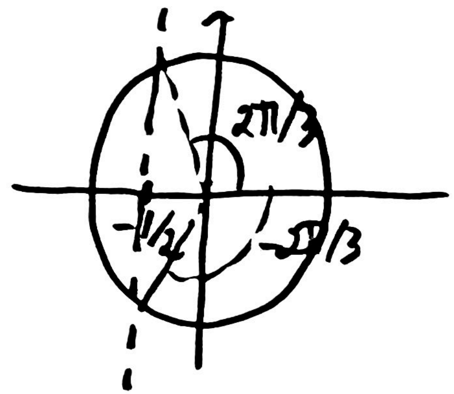
Opg.

$$\cos(\pi x) + \frac{1}{2} \geq 0 \qquad x \in [0, 2]$$

La $v = \pi x$. Da er $x = \frac{v}{\pi}$

$v \in [0, 2\pi]$

$$\cos(v) \geq -\frac{1}{2}.$$



$v_1 = 2\pi/3$

$v_2 = 4\pi/3 \left(= 2\pi - \frac{2\pi}{3}\right)$

$v \in [0, 2\pi/3] \cup [\frac{4\pi}{3}, 2\pi]$

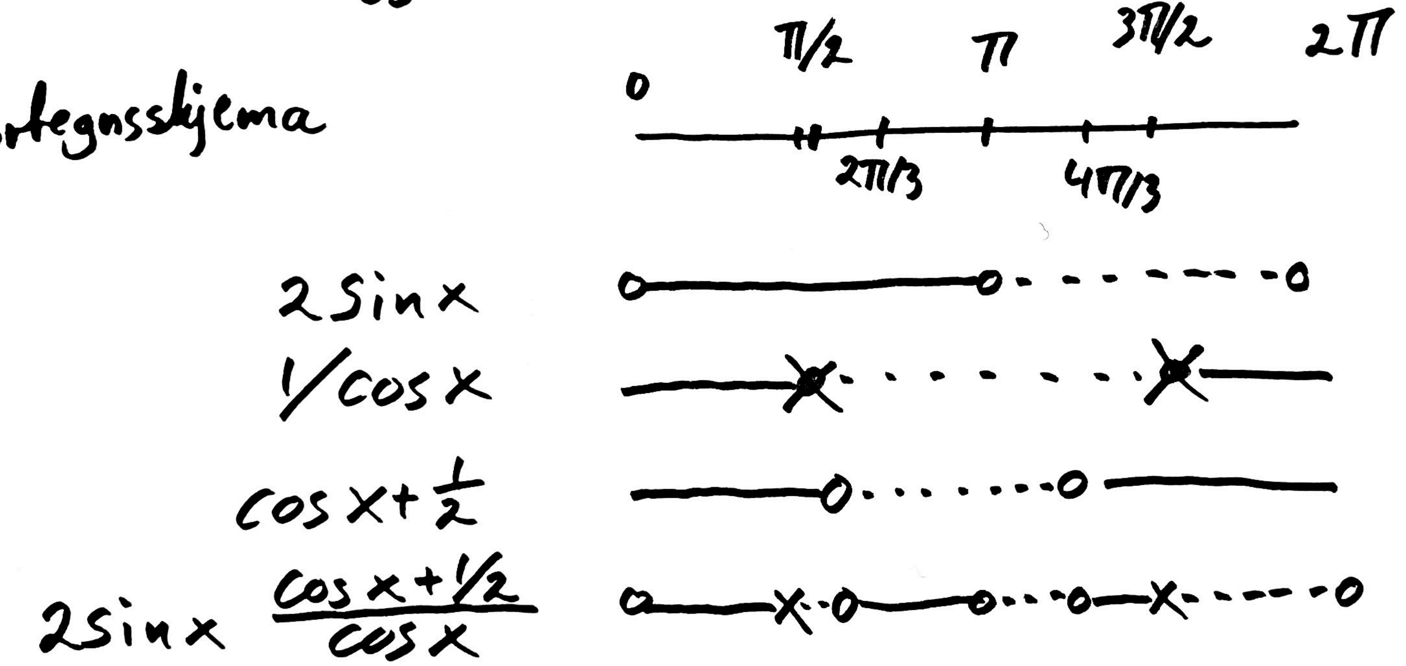
$\underline{x \in [0, \frac{2}{3}] \cup [\frac{4}{3}, 2]}$

$2\sin x + \tan x \geq 0$ $\qquad$ $\tan x = \frac{\sin x}{\cos x}$

$\sin x \left(2 + \frac{1}{\cos x}\right) \geq 0$

$2\sin x \, \frac{\cos x + \frac{1}{2}}{\cos x} \geq 0$

Fortegnsskjema



Løsningene er $\underline{\left[0, \frac{\pi}{2}\right\rangle \cup \left[\frac{2\pi}{3}, \pi\right] \cup \left[\frac{4\pi}{3}, \frac{3\pi}{2}\right\rangle \cup \{2\pi\}}$

Øving

Pytagoras
$\cos^2 x + \sin^2 x = 1$

10.75

c) $4\sin^2 x - 4\sin x\cos x + 4\cos^2 x = 1$

$4\underbrace{(\sin^2 x + \cos^2 x)}_{1} - 4\sin x\cos x = 1$

$(-4\sin x\cos x = 1 - 4 = -3)$

$\sin 2x = 2\sin x\cos x$

$4\sin x\cos x = 3$

$2\sin(2x) = 3$

$\sin(2x) = 3/2 = 1.5$

ingen reelle løsninger for x.

Alternativt del med $\cos^2 x$ ($\cos x \neq 0$ for løsningen)

$4\tan^2 x - 4\tan x + 4 = \frac{1}{\cos^2 x} = \tan^2 x + 1$

$3\tan^2 x - 4\tan x + 3 = 0$ La $z = \tan x$

$3z^2 - 4z + 3 = 0$ ingen reelle løsninger.

fordi diskriminanten er lik $(-4)^2 - 4\cdot 3\cdot 3 = -20 < 0$

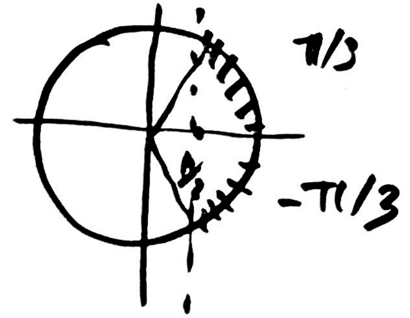
11.51

$2\cos x - 1 > 0$

$2\cos x > 1$

$\cos x > 1/2$

$x \in [-\pi, \pi]$

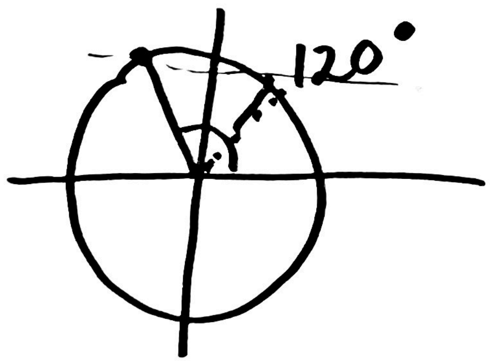


$\cos x = \frac{1}{2}$ $\quad x = \pm\frac{\pi}{3}$

Løsningene er $\underline{x \in \langle -\pi/3, \pi/3 \rangle}$.

10.35

Finn $V_1, V_2 \neq 120^\circ$, slik at $\sin(V_i) = \sin(120^\circ)$



Reflekterer om y-aksen

$180^\circ - 120^\circ = 60^\circ$

$\underline{V_1 = 60^\circ}$

Legger et helt omløp. $\quad 120^\circ + 360^\circ = \underline{480^\circ}$

$\underline{V_2 = 480^\circ}$