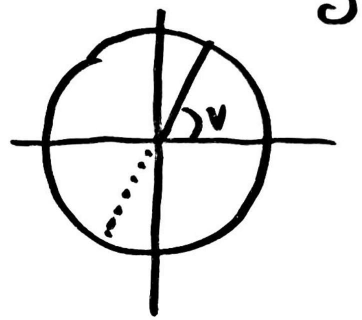
13.01.23

Trigonometriske likninger

$\tan v = 8$



$\tan v = \frac{\sin v}{\cos v}$

arctan(y) er definert for alle y.

$-\frac{\pi}{2} < \arctan y < \frac{\pi}{2}$

$\tan(\arctan(y)) = y$

$v_1 = \arctan 8$

$= 82.87\ldots^\circ$

$= 1.446\ldots$

$v_2 = v_1 + \pi$

Løsningene $v_1 + 2\pi \cdot n$ $\quad n \in \mathbb{Z}$

$(v_1 + \pi) + 2\pi \cdot n$

Dette er det samme som

$v_1 + \pi \cdot n$ $\quad n \in \mathbb{Z}$

$\underline{1.446 + \pi \cdot n}$

oppg (*) $\sin v + \sqrt{3}\cos v = 0$ (hint gjør om til tan-likning)

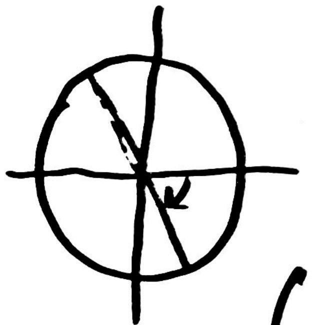
$v \in [0, 2\pi]$.

hvis $\cos v = 0$ da er $\sin v$ lik -1 eller 1, så ingen løsning.

Vi kan derfor anta at $\cos v \neq 0$.

Deler med $\cos v$: $\tan v + \sqrt{3} = 0$

$\tan v = -\sqrt{3}$ er ekvivalent til (*)



$$v = \underbrace{\arctan(-\sqrt{3})}_{-\pi/3} + \pi \cdot n$$

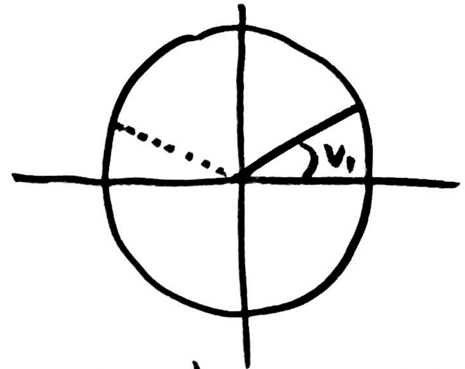
$$\left(\tan(-\pi/3) = \frac{\sin(-\pi/3)}{\cos(-\pi/3)} = \frac{-\sqrt{3}/2}{1/2} = -\sqrt{3} \right)$$

I intervallet $[0, 2\pi]$ er løsningene : $\underline{\frac{2\pi}{3}, \frac{5\pi}{3}}$

$\sin v = y$ Løsninger

$v_1 = \arcsin y + 2\pi \cdot n$

$v_2 = \pi - \arcsin y + 2\pi \cdot n$

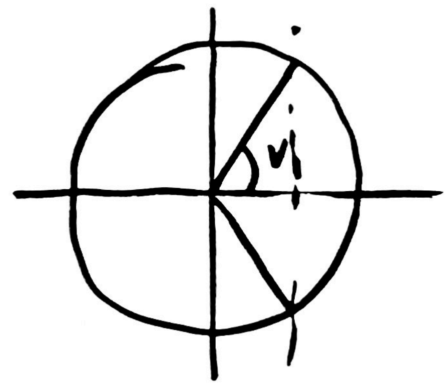


speglev om y-aksen

$\cos v = x$ Løsninger

$v_1 = \arccos(x) + 2\pi \cdot n$

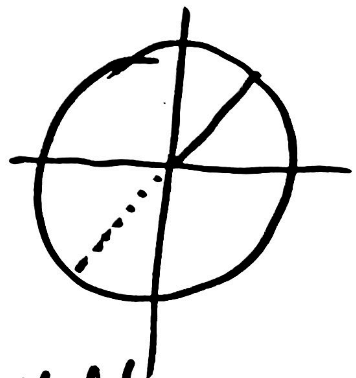
$v_2 = -\arccos(x) + 2\pi \cdot n$



Reflekterer om x-aksen

$\tan v = a$ Løsninger

$v = \arctan(a) + \pi \cdot n$



Reflekterer om origo

(*) $\sin^2 v + 1.5 \sin v - 1 = 0$

$\sin v = z$,

$z^2 + 1.5z - 1 = 0$

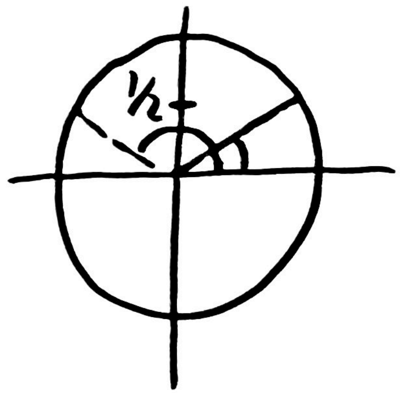
$= (z+2)(z - \frac{1}{2})$

Røttene er $z = -2$

$z = 1/2$

(*) $\Leftrightarrow$ $\sin v = -2$ ingen løsning.

$\sin v = 1/2$



$v_1 = \arcsin(\frac{1}{2}) = \frac{\pi}{6}$ $(30°)$

$v_2 = \pi - v_1 = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$

Løsningene er: $\underline{v_1 = \frac{\pi}{6} + 2\pi \cdot n}$

$\underline{v_2 = \frac{5\pi}{6} + 2\pi \cdot n}$

$$\sin^2 v + \cos v \sin v - 12\cos^2 v = 0$$

$\cos v = 0$ da er $|\sin v| = 1$: ingen løsning

Vi kan anta $\cos v \neq 0$. Deler med $\cos^2 v$:

$$\tan^2 v + \tan v - 12 = 0$$

$$(\tan v + 4)(\tan v - 3) = 0$$

Løsningene er de kombinerte løsningene til $\tan v = -4$, $\tan v = 3$

$$v = \arctan(-4) + \pi \cdot n = -1.325\ldots + \pi \cdot n \quad (-75.96^\circ)$$

og $$v = \arctan(3) + \pi \cdot n = 1.249\ldots + \pi \cdot n \quad (71.56^\circ)$$

$$3\sin^2 v + 2\underbrace{\cos^2 v}_{1-\sin^2 v} = 2.5$$

$$3\sin^2 v + 2 - 2\sin^2 v = 2.5$$

$$\sin^2 v = 2.5 - 2 = 0.5 = 1/2$$

$$\sin^2 v = \frac{1}{2} \qquad \left(\begin{aligned} \sin v &= z \\ z^2 &= \frac{1}{2} \\ z &= \pm\sqrt{\frac{1}{2}} = \frac{\pm 1}{\sqrt{2}} \end{aligned} \right)$$

$$\Updownarrow$$

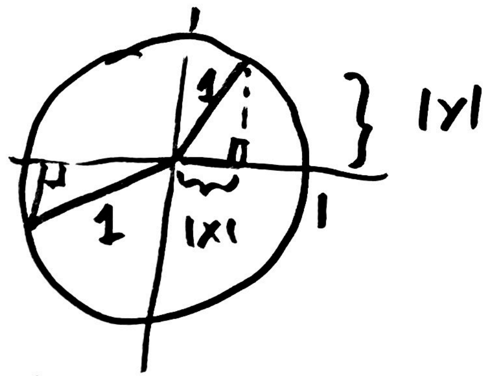
$$\sin v = \frac{1}{\sqrt{2}}$$

eller $\sin v = \frac{-1}{\sqrt{2}}$

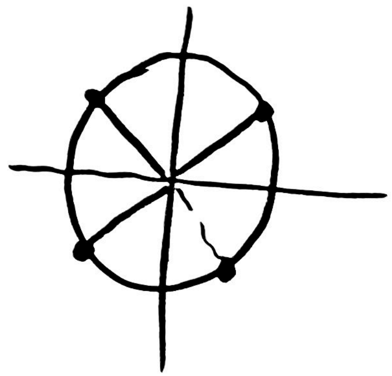
Løsningene er

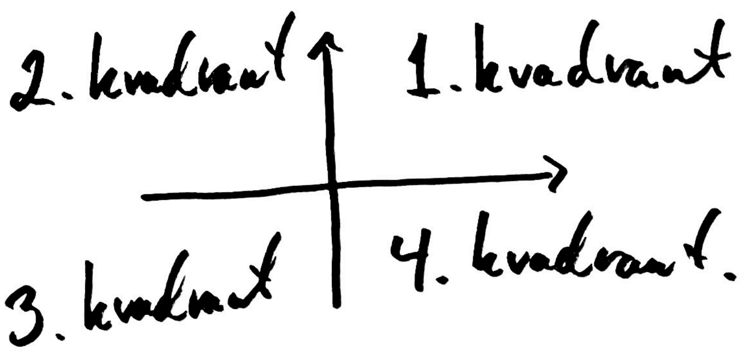
$$v = 45^\circ + 90^\circ \cdot n$$

$$\left(= \frac{\pi}{4} + \frac{\pi}{2} \cdot n \right)$$



Pythagoras
$\cos^2 v + \sin^2 v = 1$
for alle v.



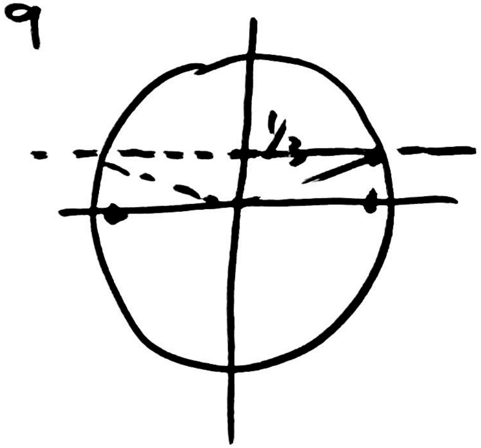


+ retning

$\sin v = \frac{1}{3}$ Hva kan $\cos v$ være?

$\cos^2 v = 1 - \sin^2 v = 1 - \left(\frac{1}{3}\right)^2 = \frac{8}{9}$

$\cos v = \pm\sqrt{\frac{8}{9}} = \underline{\pm \frac{2\sqrt{2}}{3}}$



Finn $\cos v$ når v ligger i 2. kvadrant og $\sin v = \frac{1}{3}$.

$\cos v$ er negativ i 2. kvadrant så $\cos v = \underline{-\frac{2\sqrt{2}}{3}}$

Addisjonsformlene for sin og cos.

$$\sin(u+v) = \sin(u)\cos(v) + \sin(v)\cos(u)$$

$$\cos(u+v) = \cos(u)\cos(v) - \sin(u)\sin(v).$$

$u = v$

$$\sin(2u) = 2\sin(u)\cos(u)$$

$$\cos(2u) = \cos^2(u) - \underbrace{\sin^2(u)}_{1-\cos^2 u}$$

$$= 2\cos^2(u) - 1$$

$$\cos(2u) = (1-\sin^2 u) - \sin^2 u$$

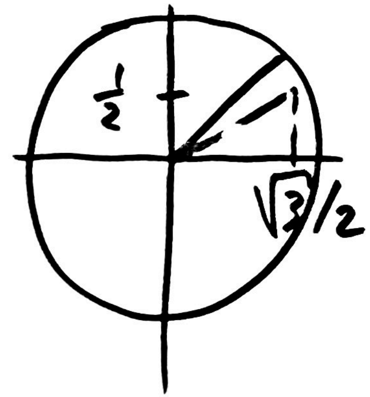
$$= 1 - 2\sin^2 u$$

$$\sin(75^\circ) = \sin(30^\circ + 45^\circ)$$

$$= \underbrace{\sin(30^\circ)}_{\frac{1}{2}} \underbrace{\cos(45^\circ)}_{\frac{1}{\sqrt{2}}} + \underbrace{\sin(45^\circ)}_{\frac{1}{\sqrt{2}}} \underbrace{\cos(30^\circ)}_{\frac{\sqrt{3}}{2}}$$

$$= \frac{1}{2\sqrt{2}} + \frac{\sqrt{3}}{2\sqrt{2}}$$

$$= \frac{1+\sqrt{3}}{2\sqrt{2}} \sim \underline{0.966}$$

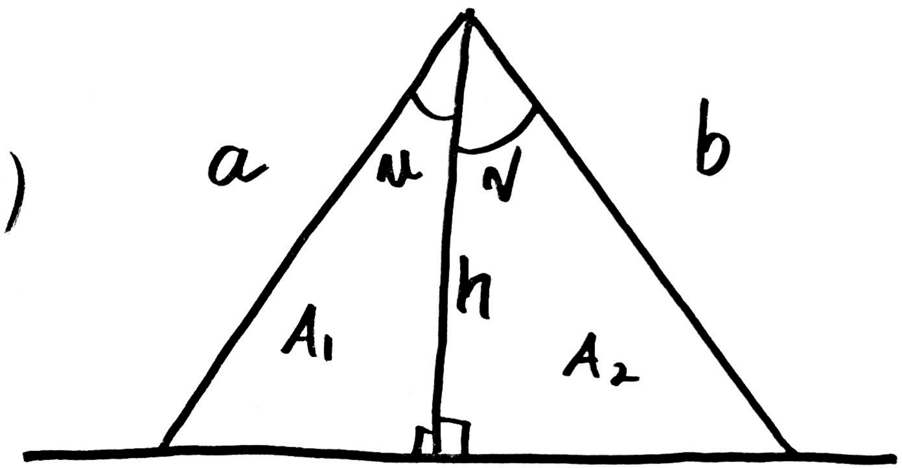


Bevis for addisjonsformelen for sin

$0 \le u, v \le 90°$

Areal til den store Δ er lik $\underline{\frac{1}{2} a \cdot b \cdot \sin(u+v)}$

Det er lik summen av arealet til de to mindre Δ-ene.



$h = a \cos u$
$= b \cos v$

$A_1 = \frac{1}{2} a \cdot h \cdot \sin u$

$A_2 = \frac{1}{2} b \cdot h \cdot \sin v$

$$\frac{1}{2} ab \sin(u+v) = A_1 + A_2 = \frac{1}{2} a \overbrace{(b \cos v)}^{h} \sin u + \frac{1}{2} b \underbrace{(a \cos u)}_{h} \cdot \sin v \quad \Big| \cdot 2$$

$$ab\sin(u+v) = ab\cos v \cdot \sin u + ab\cos u \cdot \sin v \quad \Big| \ \frac{1}{a \cdot b}$$

$$\underline{\sin(u+v) = \sin u \cos v + \sin v \cos u}$$

Addisjonsformel for tan :

$$\tan(u+v) = \frac{\sin(u+v)}{\cos(u+v)} = \frac{\sin(u)\cos(v) + \sin(v)\cos(u)}{\cos(u)\cos(v) - \sin(u)\sin(v)}$$

$$= \frac{\tan(u) + \tan(v)}{1 - \tan(u)\tan(v)}$$

Øvingstime : 10.53 , 10.62 , 10.73

Gjennomgang 11:30.

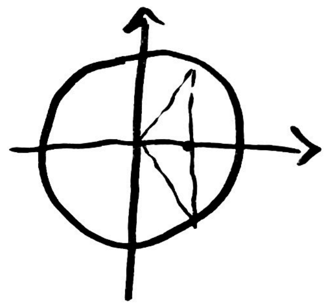
10.53 a) $4\cos(2x) = 2$ $\quad x \in [0, \pi]$.

$$\cos(\underbrace{2x}_{v}) = \frac{2}{4} = \frac{1}{2}$$

$$\cos(v) = \frac{1}{2} \qquad v \in [0, 2\pi].$$

1.

$\arccos\left(\frac{1}{2}\right) = \frac{\pi}{3}$



$v = \frac{\pi}{3} + 2\pi \cdot n$

$v = -\frac{\pi}{3} + 2\pi \cdot n$

i $[0, 2\pi]$ er løsningene: $v = \frac{\pi}{3}, \frac{5\pi}{3}$

2. $v = 2x$ så $x = \frac{v}{2}$

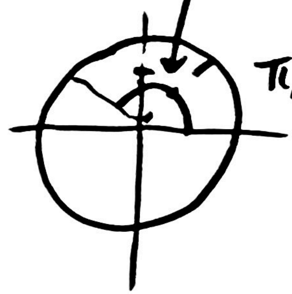
Løsningene er $\underline{x = \frac{\pi}{6}}$ og $\underline{x = \frac{5\pi}{6}}$

10.53 c) $4\sin(\pi x) = 2\sqrt{2}$ $\quad x \in [0, 2]$

$\Leftrightarrow \sin(\pi x) = \frac{2\sqrt{2}}{4} = \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}}$

$\pi x = v$ $\quad v \in [0, 2\pi]$

$x = v/\pi$



1. $\sin(v) = \frac{1}{\sqrt{2}}$

$\arcsin\left(\frac{1}{\sqrt{2}}\right) = \frac{\pi}{4}$

$v = \frac{\pi}{4}, \frac{3\pi}{4}$

2. $x = v/\pi$

Løsningene er $\underline{x = \frac{1}{4}}$ og $\underline{x = \frac{3}{4}}$.

10.85 a) Vis $\sin(3v) = 3\sin v - 4\sin^3 v$

$3v = v + 2v$

$$\sin(3v) = \sin(v+2v) = \sin(v)\underbrace{\cos(2v)}_{\cos^2 v - \sin^2 v} + \underbrace{\sin(2v)}_{2\sin v\cos v}\cos v$$

$$= \sin v(\cos^2 v - \sin^2 v) + 2\sin v\cos^2 v$$

$$= 3\sin v\underbrace{\cos^2 v}_{1-\sin^2 v} - \sin^3 v$$

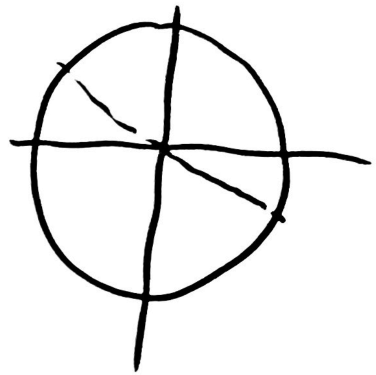
(Pytagoras $\sin^2 v + \cos^2 v = 1$)

$$= \underline{3\sin v - 4\sin^3 v}.$$

b) $\cos(3v) = \cdots$

10.62 d) $3\sin x + 2\cos x = 0$ $x \in [-\pi, \pi]$

$\cos x \neq 0$ for løsning



$$3\tan x + 2 = 0$$

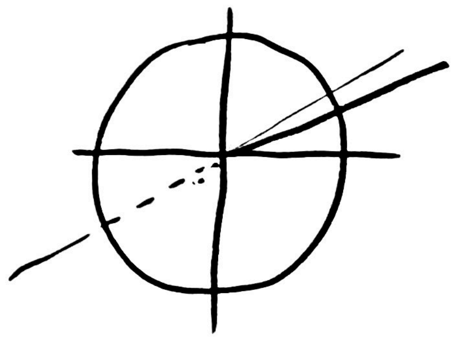
$$\tan x = \frac{-2}{3}$$

$$\arctan\left(\frac{-2}{3}\right) \sim -0.588$$

$(-33.7°)$

Løsningene er $\underline{v_1 = -0.588}$ og $\underline{v_2 = v_1 + \pi}$

a) $2\tan x = 1$ $x \in [-\pi, \pi]$



$$\tan x = \frac{1}{2}$$

$v_1 = \arctan\left(\frac{1}{2}\right) = 0.4636\ldots$ $(26.565°)$

$v_1 + \pi \cdot n$ ligger i $[-\pi, \pi]$

for $n = -1$ og 0

Løsningene er

$\underline{x \in \{v_1 - \pi, v_1\}}$

10.73 a) (*) $2\sin^2 x + \sin x - 1 = 0$ $x \in [0, 2\pi]$

$z = \sin x$

$2z^2 + z - 1 = 0$

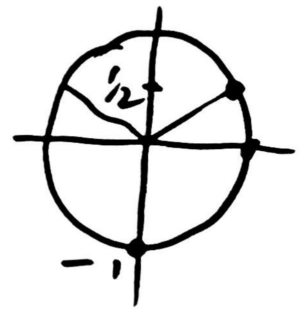
$(z+1)(2z-1) = 0$

$z = -1$ eller $z = 1/2$.

(*) $\Leftrightarrow$ $\sin x = -1$ eller $\sin x = 1/2$.

$x = \frac{3\pi}{2}$ $x = \pi/6$

$x = \pi - \pi/6 = \frac{5\pi}{6}$



Løsningene er $\underline{x = \frac{\pi}{6}, \frac{5\pi}{6} \text{ og } \frac{3\pi}{2}}$