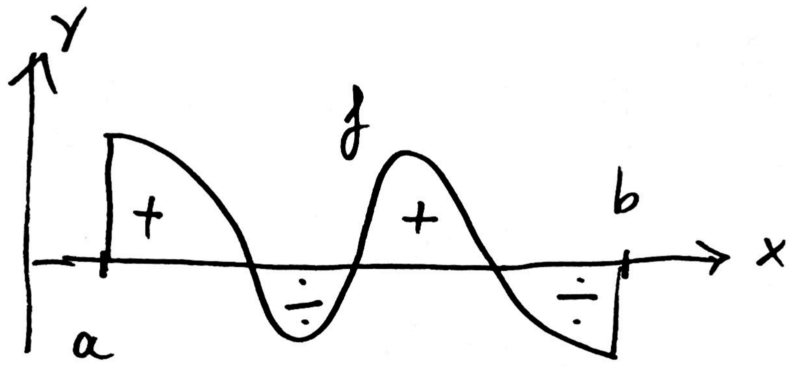
8 mars
2022

Fundamentalteoremet i kalkulus

Bestemt integral



$\int_a^b f(x)\,dx$ areal med fortegn av regionen mellom grafen til $f(x)$ og x-aksen.

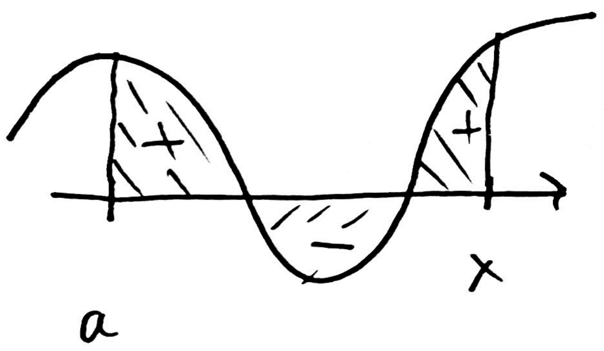
$\int f(x)\,dx$ = samlingen av alle antideriverte til $f(x)$

$= F(x) + C$

↑ én antiderivert

FUNDAMENTALTEOREMET

$F(x) = \int_a^x f(t)\,dt$ er en antiderivert til $f(x)$ for kontinuerlige funksjoner $f(x)$.



Andre formuleringer $\frac{d}{dx}\int_a^x f(t)\,dt = f(x)$

$$\int_a^b f(t)\,dt = F(b) - F(a) = \underline{F(x)\Big|_a^b}$$

F én antiderivert til f.

Hvis F er en antiderivert til $f(x)$.

Da er $F(x) = \int_a^x f(t)\,dt + C$

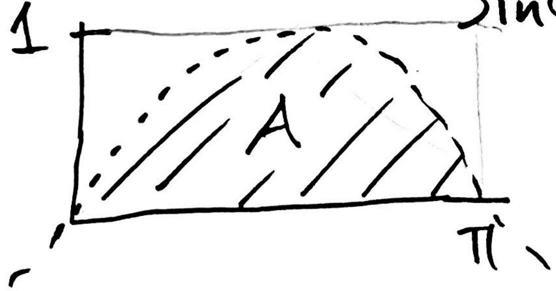
$$F(b) - F(a) = \left(\int_a^b f(t)\,dt + C\right) - \left(\underbrace{\int_a^a f(t)\,dt}_{0} + C\right)$$

kanselerer

$$= \int_a^b f(t)\,dt \underbrace{+ C - C}_{0}$$

$$\underline{F(b) - F(a) = \int_a^b f(t)\,dt}$$

Eks



Sin(x)

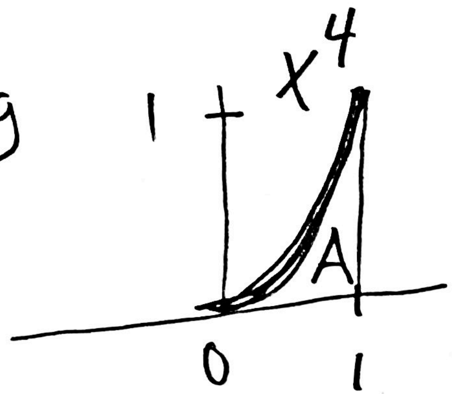
$$A = \int_0^{\pi} \sin x \, dx$$

$$= \underbrace{-\cos(x)}_{\text{antiderivert til } \sin x} \Big|_0^{\pi}$$

$$= -\cos(\pi) - (-\cos(0)) = \underline{2}$$

$\underline{A = 2}$

opg

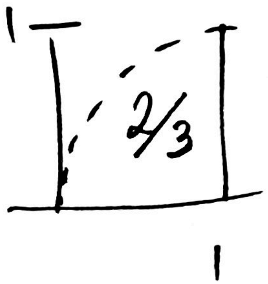


Finn arealet mellom x-aksen og grafen til x^4 fra $x=0$ til $x=1$.

$$A = \int_0^1 x^4\,dx = \frac{x^5}{5}\Big|_0^1 = \frac{1}{5} - 0 = \underline{\underline{\frac{1}{5}}}$$

Generelt x^n, $n \geq 0$

$$\int_0^1 x^n\,dx = \frac{x^{n+1}}{n+1}\Big|_0^1 = \underline{\underline{\frac{1}{n+1}}}$$

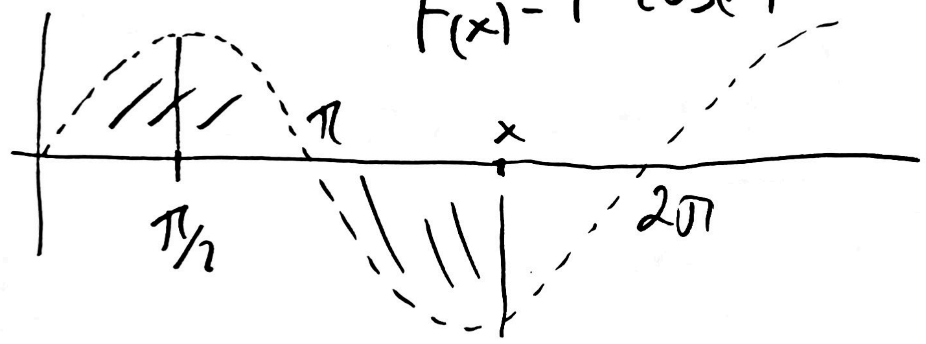


$$F(x) = \int_0^x \sin t \, dt = -\cos(t) \Big|_0^x = 1 - \cos(x)$$

$F(\pi/2) = 1$ ✓

$F(2\pi) = 0$ ✓

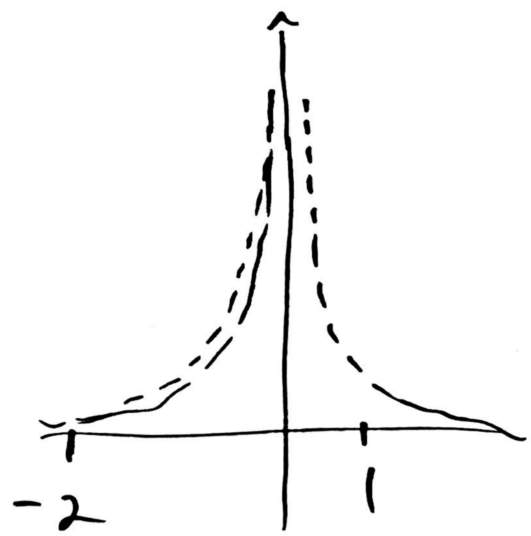
$F(x) = 1 - \cos(x)$



$$\int_{-2}^{1} \frac{1}{x^2} \, dx = \int_{-2}^{1} x^{-2} \, dx$$

$$= \frac{-1}{x} \Big|_{-2}^{1} = \frac{-1}{1} - \left(\frac{-1}{-2}\right)$$

$$= -1 + \frac{1}{2} = -\frac{1}{2} \; ?$$ GALT

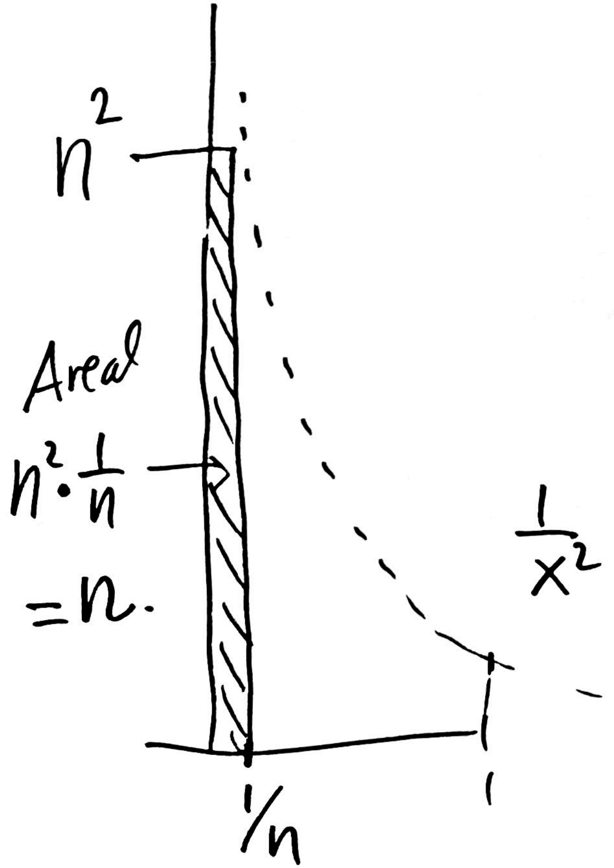


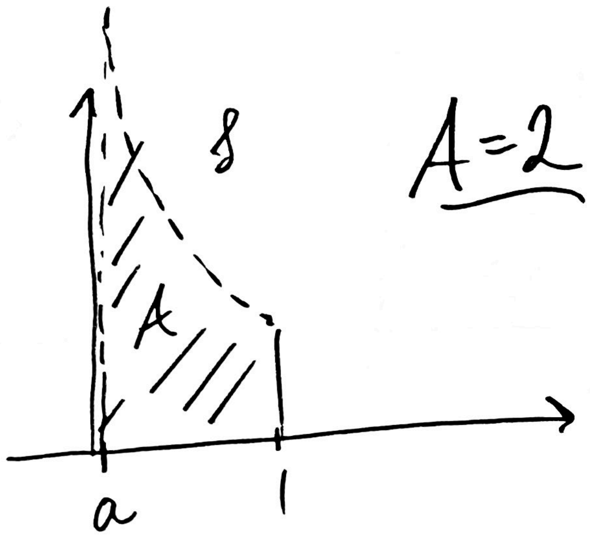
$\frac{1}{x^2}$ er ikkje en kont. funksjon på $[-2, 1]$

$\int_0^1 \frac{1}{x^2}\,dx$ eksisterer ikkje!

$\int_a^1 \frac{1}{x^2}\,dx$ $\quad a > 0$ $\quad$ på $[a, 1]$ er $1/x^2$ kontinuerlig

$= \frac{-1}{x}\Big|_a^1 = \underbrace{-1 + \frac{1}{a}}_{\text{går mot}}\ \infty$ når $a \to 0^+$.





$\underline{A=2}$

$$f(x) = \frac{1}{\sqrt{x}}$$

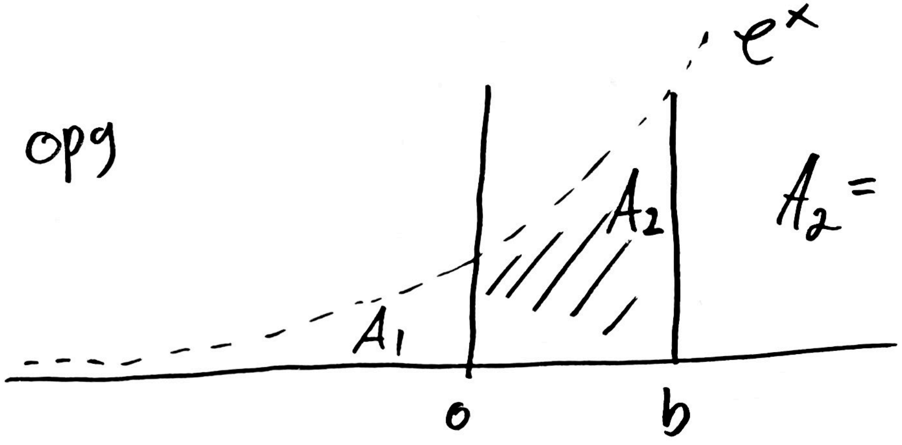
$$\int_0^1 f(x)\,dx = \lim_{a \to 0^+} \int_a^1 f(x)\,dx$$

$$\int_a^1 f(x)\,dx = \int_a^1 x^{-1/2}\,dx = \frac{x^{1/2}}{1/2}\Big|_a^1$$

$$= 2\sqrt{x}\,\Big|_a^1 = \underline{2 - 2\sqrt{a}}$$

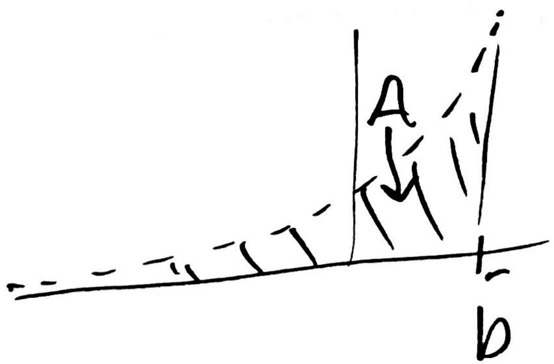
$$\int_0^1 \frac{1}{\sqrt{x}}\,dx = \lim_{a \to 0^+} 2 - 2\sqrt{a} = \underline{\underline{2}}$$

opg

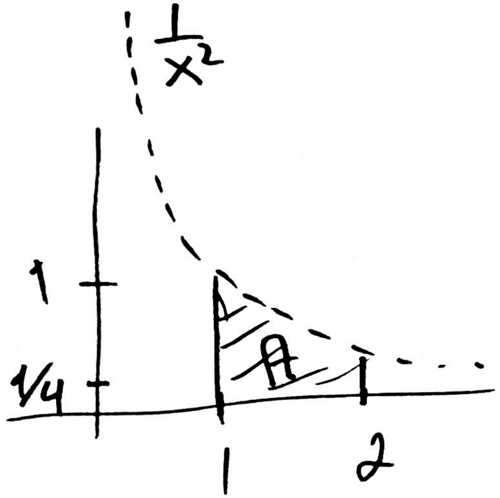


$$A_2 = \int_0^b e^x\,dx = e^x \Big|_0^b$$
$$= e^b - e^0$$
$$= \underline{e^b - 1}$$

$$A_1 = \int_{-\infty}^0 e^x\,dx = \lim_{a\to-\infty} \int_a^0 e^x\,dx$$
$$= \lim_{a\to-\infty} e^x \Big|_a^0 = \lim_{a\to-\infty} (e^0 - e^a) = e^0 = 1$$

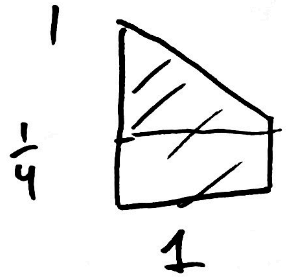


$$A = \int_{-\infty}^b e^x\,dx$$
$$= \underline{e^b}$$



$$A = \int_1^2 \frac{1}{x^2}\,dx$$

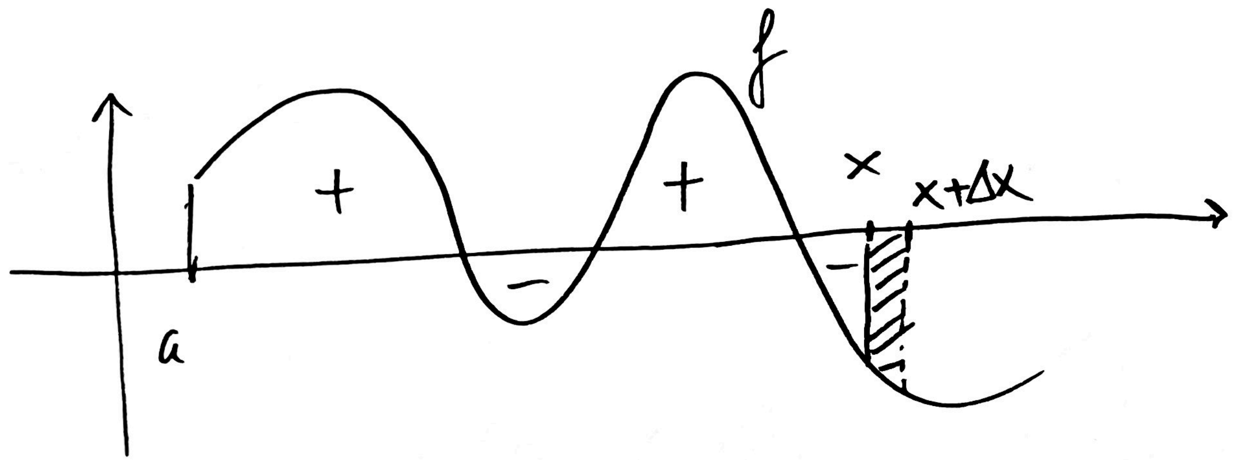
(Areal trapes :



$$1 \cdot \frac{1+1/4}{2} = \frac{5}{8} > A \;)$$

$$A = \int_1^2 \frac{1}{x^2}\,dx = \int_1^2 x^{-2}\,dx = \left.\frac{-1}{x}\right|_1^2$$

$$= \frac{-1}{2} - \left(\frac{-1}{1}\right) = 1 - \frac{1}{2} = \underline{\underline{\frac{1}{2}}}$$



f kontinuerlig

$$F(x) = \int_a^x f(t)\,dt$$

$$F'(x) = \lim_{\Delta x \to 0} \frac{F(x+\Delta x) - F(x)}{\Delta x} = \lim_{\Delta x \to 0} \frac{\int_x^{x+\Delta x} f(t)\,dt}{\Delta x}$$

$$\lim_{\Delta x \to 0} \frac{\Delta x \cdot f(y)}{\Delta x} \qquad x \leq y \leq x + \Delta x \text{ for en}$$

$$= f(x) \qquad \text{siden } f(x) \text{ er kontinuerlig.}$$

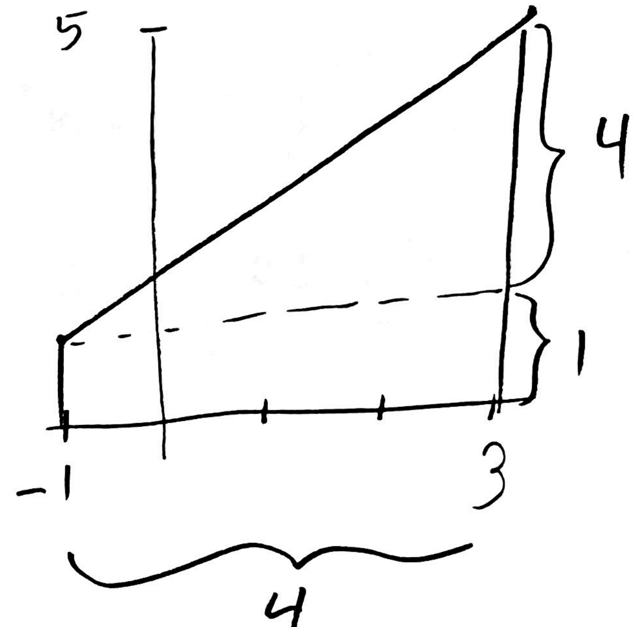
Derfor er $F(x) = \int_a^x f(t)\,dt$ en antiderivert til $f(x)$.

$$A = \int_{-1}^{3} 2 + x \, dx$$

$$= 2x + \frac{x^2}{2} \Big|_{-1}^{3}$$

$$= \underbrace{2(3-(-1))}_{8} + \frac{1}{2}\underbrace{(9-(-1)^2)}_{8}$$

$$= 8 + \frac{1}{2} \cdot 8 = \underline{12}$$



$$A = 4 + \frac{4 \cdot 4}{2} = \underline{12}$$

$$\int_0^{2\pi} \sin^2 x \, dx$$

og $\int_0^x \sin^2 t \, dt$.

$$\int_0^{2\pi} \cos^2 x\, dx = \int_0^{2\pi} \sin^2 x\, dx$$

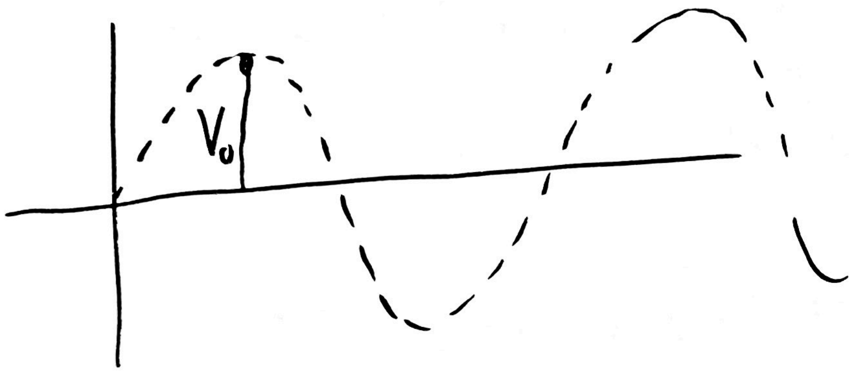
$$= \frac{1}{2}\int_0^{2\pi} \underbrace{\cos^2 x + \sin^2 x}_{1}\, dx = \underline{\underline{\pi}}$$

$$\sin^2 t = \frac{1}{2}\left(1 - \cos(2t)\right)$$

$$\int_0^x \sin^2 t\, dx = \int_0^x \frac{1}{2}\left(1 - \cos(2t)\right) dt$$

$$= \frac{1}{2}\left(t - \frac{1}{2}\sin(2t)\right)\Big|_0^x$$

$$= \underline{\frac{1}{2}\left(x - \frac{1}{2}\sin(2x)\right)}$$



$V(t) = V_0 \sin(\underline{50} \cdot 2\pi t)$

50 Hz svingninger per sek

$V_{rms} = 220V$

$I \cdot V$ effekt.

Ohm lov

$V = R \cdot I$

$I = \frac{V}{R}$

Effekt $= \frac{1}{R} V^2$

$V_{rms}^2 =$ gjennomsnitt av $V(t)^2$.

$V_{rms} = \sqrt{\text{gjennomsnitt av } V(t)^2}$

$V(t) = V_0 \sin(100\pi t)$

$V^2(t) = V_0^2 \sin^2(100\pi t)$

Gjennomsnittet av $\sin^2(kt)$ er $\frac{1}{2}$ $\left(\frac{\int_0^{2\pi} \sin^2 t \, dt}{2\pi} = \frac{\pi}{2\pi} = \frac{1}{2} \right)$

$V_{rms}^2 = V_0^2 \cdot \frac{1}{2}$

$V_{rms} = \frac{V_0}{\sqrt{2}}$

$V_0 = \sqrt{2} \cdot 220V = \underline{\underline{311 V}}$