

20.sep
2021

Polynom division

$$\frac{p(x)}{q(x)} = S(x) + \frac{r(x)}{q(x)}$$

$\deg r < \deg q$.

(1) $\frac{p(x)}{x-x_0}$ tall

$$= S(x) + \frac{r}{x-x_0}$$

$x-x_0$

$$p(x) = S(x)(x-x_0) + r$$

Sette $x=x_0$

$$\text{Rullen } r = p(x_0)$$

$$\frac{x^2-1}{x-1} = \frac{x(x-1)+x}{x-1} = x + \frac{x}{x-1} = x + \frac{x-1+1}{x-1} = x + 1 + \frac{1}{x-1}$$

$$x-x_0 \text{ deler } p(x) \Leftrightarrow p(x_0) = 0$$

Eksempel Faktoriser: $p(x)=x^3-3x^2+4x-2$.

Vi ser at $p(1)=0$.

$x-1$ deler $p(x)$

$$x^3 - 3x^2 + 4x - 2 : x-1 = x^2 - 2x + 2$$

(2)

$$\begin{array}{r} x^3 - 3x^2 + 4x - 2 \\ \underline{- (x^3 - x^2)} \\ -2x^2 + 4x - 2 \\ \underline{- (-2x^2 + 2x)} \\ 2x - 2 \\ \underline{- (2x - 2)} \\ 0 \end{array}$$

$$\begin{aligned} & x^2 - 2x + 2 \\ &= (x-1)^2 + 1 \geq 1 \\ & \text{irreducible} \end{aligned}$$

$$p(x) = \underline{(x^2 - 2x + 2)(x-1)}$$

5.6 Doble ulikheter

$$(3) \quad x^2 + x - 2 < 5(x-1) \leq 4x-3$$

1. $x^2 + x - 2 < 5x - 5 \iff x^2 + x - 5x - 2 - (-5) < 0$

$$x^2 - 4x + 3 < 0$$

$$(x-1)(x-3) < 0$$

$$x-1$$

$$\dots\dots\dots\dots\dots\dots\dots$$

$$(x-1)(x-3) \dots\dots\dots\dots\dots\dots\dots$$

Første likning har løsning

$$x \in \underline{\langle 1, 3 \rangle}$$

2. $5x - 5 \leq 4x - 3 \iff 5x - 4x - 5 - (-3) \leq 0 \iff$

$$x - 2 \leq 0 \iff x \leq 2$$

Legger til x på begge sider av ulikhets tegnget.

Løsningene til den doble ulikheten er $\langle 1, 3 \rangle \cap \langle -\infty, 2 \rangle$

$$= \underline{\underline{\langle 1, 2 \rangle}}$$

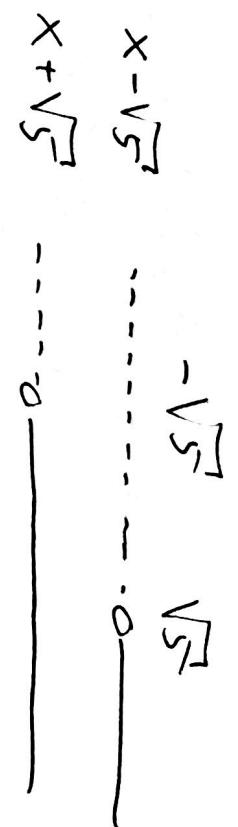
oppg.

$$0 < x^2 - 5 < 3x - 1 \quad \text{los ulikheter.}$$

$$1) 0 < x^2 - 5 = x^2 - (\sqrt{5})^2$$

$$= (x + \sqrt{5})(x - \sqrt{5})$$

Viillone til $x^2 - 5$

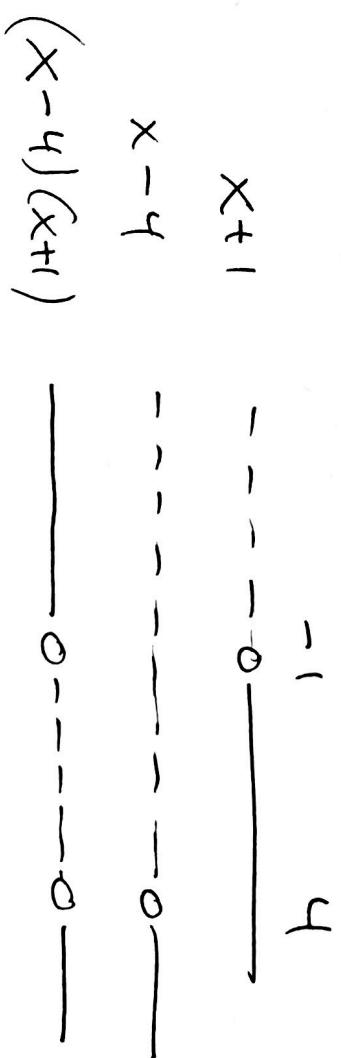


Løsningen $(-\infty, -\sqrt{5}) \cup (\sqrt{5}, \infty)$

$$2) x^2 - 5 < 3x - 1$$

$$x^2 - 3x - 4 < 0$$

$$(x-4)(x+1) < 0$$



Løsningene $< -1, 4 \rangle$.

Løsningene til den doble ulikheten er snittet av de

to løsningsmengdene $\underline{\underline{< \sqrt{5}, 4 \rangle}}$

5.7 Tredjegradsuløsninger.

$$P(x) = x^3 - 4x^2 - 2x + 3 \leq 0$$

$$P(-1) = (-1)^3 - 4(-1)^2 - 2(-1) + 3 = 0.$$

$$\quad \quad \quad -1 - 4 + 2 + 3$$

Se $x - (-1) = x + 1$ dele $P(x)$.

$$\begin{array}{r} x^3 - 4x^2 - 2x + 3 \\ x^3 + x^2 \\ \hline -5x^2 - 2x + 3 \end{array} : x + 1 = x^2 - 5x + 3$$

$$\begin{array}{r} 3x + 3 \\ 3x + 3 \\ \hline 0 \end{array}$$

$$P(x) = (x+1)(x^2 - 5x + 3)$$

$$x^2 - 5x + 3 = \left(x - \frac{5}{2}\right)^2 - \left(\frac{5}{2}\right)^2 + 3$$

Fullfaring av □

$$\begin{aligned}
 &= \left(x - \frac{5}{2} \right)^2 - \frac{25}{4} + \frac{3 \cdot 4}{4} \\
 &= \left(x - \frac{5}{2} \right)^2 - \frac{13}{4} = \left(x - \frac{5}{2} \right)^2 - \left(\frac{\sqrt{13}}{2} \right)^2 \\
 &\left(x - \frac{5}{2} + \frac{\sqrt{13}}{2} \right) \left(x - \frac{5}{2} - \frac{\sqrt{13}}{2} \right)
 \end{aligned}$$

6

$$P(x) = (x+1)\left(x - \left(\frac{5}{2} - \frac{\sqrt{13}}{2}\right)\right)\left(x - \left(\frac{5}{2} + \frac{\sqrt{13}}{2}\right)\right)$$

十一

$$X = \left(\frac{5}{2} - \frac{\sqrt{13}}{2} \right)$$

$$X = \left(\frac{5}{2} + \frac{\sqrt{13}}{2} \right)$$

- - - O - - - I - - O -

$$\frac{5 + \sqrt{13}}{2}$$

$$x \in (-\infty, -1] \cup \left[\frac{5-\sqrt{13}}{2}, 1 \right]$$

S^o $\rho(x) \leq 0$ for

$$\frac{x^2+1}{x+1} \geq \frac{2x-3}{x-2} \Leftrightarrow$$

$$\frac{x^2+1}{x+1} - \frac{2x-3}{x-2} \geq 0$$

$$\geq 0$$

$$\frac{x^2+1}{x+1} - \frac{2x-3}{x-2} \geq 0$$

$$\frac{x^2+1}{x+1} - \frac{2x-3}{x-2} \geq 0$$

$$\geq 0$$

(7)

$$\frac{\frac{x^2+1}{x+1} - \frac{2x-3}{x-2}}{(x+1)(x-2)} \geq 0$$

$$\frac{P(x)}{q(x)} = \frac{x^3 - 4x^2 + 2x + 1}{(x+1)(x-2)}$$

$$\geq 0$$

$$P(4) = 0 \quad \text{Se } x-1 \text{ deler } P(x)$$

$$x^3 - 4x^2 + 2x + 1 : x-1 = x^2 - 3x - 1$$

$$\frac{x^3 - x^2}{-3x^2 + 2x + 1}$$

$$\text{Se } P(x) = (x-1)(x^2 - 3x - 1)$$

$$\begin{array}{r} x^2 - 3x - 1 = \\ \hline -x+1 \\ -x+1 \\ \hline 0 \end{array}$$

$$\begin{aligned} x^2 - 3x - 1 &= \\ \left(x - \frac{3}{2}\right)^2 - \left(\frac{3}{2}\right)^2 - 1 &= \\ \left(x - \frac{3}{2}\right)^2 - \frac{9}{4} - \frac{4}{4} &= \\ \left(x - \frac{3}{2}\right)^2 - \frac{13}{4} & \end{aligned}$$

$$P(x) = (x-1) \left(x - \left(\underbrace{\frac{3}{2} - \frac{\sqrt{13}}{2}}_{a} \right) \right) \left(x - \left(\underbrace{\frac{3}{2} + \frac{\sqrt{13}}{2}}_{b} \right) \right)$$

(8)

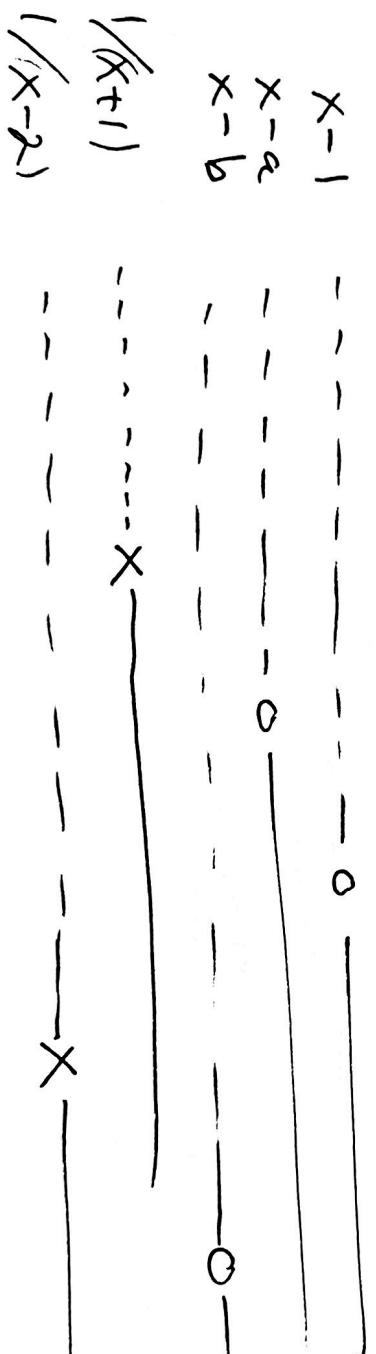
litt mindre
enn 0

litt større enn 3.

$$\frac{(x-1)(x-a)(x-b)}{(x+1)(x-2)} \geq 0$$

Fordelingskjema

-1 a 1 2 b



$$\frac{P(x)}{q(x)}$$

$$\text{Så } \frac{P(x)}{q(x)} \geq 0 \text{ for } x \in (-\infty, -1] \cup [1, 2] \cup [b, \infty)$$

Oppg. Los ulikheter

$$p(x) = x^3 + 3x^2 - 4x - 12 < 0$$

($p(x)$ har en likeartet helbeting løsning)

$$p(0) = -12, \quad p(1) = -12, \quad p(-1) = -6, \quad p(2) = 8 + 12 - 8 - 12 = 0 \quad \checkmark$$

$$x-2 \text{ deler } p(x) : \quad x^3 + 3x^2 - 4x - 12 : x-2 = x^2 + 5x + 6$$

Spørsmål:
 $(x+5)(x+1) = x^2 + 6x + 5$

$$\begin{array}{r} x^3 - 2x^2 \\ 5x^2 - 4x - 12 \\ \hline 5x^2 - 10x \\ \hline 6x - 12 \\ 6x \quad \overline{0} \\ \hline -3 \quad -2 \quad 2 \\ x-2 \cdots \cdots \cdots \cdots \cdots 0 \\ x+2 \cdots \cdots \cdots 0 \\ x+3 \cdots \cdots 0 \\ \hline p(x) = 0 \cdots 0 \cdots 0 \end{array}$$

Så $p(x) < 0$ for $x \in (-\infty, -3) \cup (-2, 2)$

Steg

$$(x-2)(x+3) \geq 0$$

Førstegnsljema
2

$$x-2$$

$$\dots -0$$

$$x+3$$

$$\dots -0$$

$$(x-2)(x+3)$$

$$\dots -0-\dots 0$$

$$(x \leq -3 \text{ og for } x \geq 2 \dots)$$

$$\text{Løsningene } x \in \underline{\langle -\infty, -3 \rangle} \cup \underline{[2, \infty)}$$

Løs den irrasjonale ulikheten

$$\sqrt{9-x} = x-3.$$

$$\Rightarrow (\sqrt{9-x})^2 = (x-3)^2$$

$$9-x = x^2-6x+9 \Leftrightarrow x^2-5x=0$$

$$\left\{ \begin{array}{l} a^2 = b^2 \Rightarrow \\ a^2 = b^2 \end{array} \right.$$

implikasjon

Oppg

$$x(x-5) = 0 \quad \text{Så } x=0$$

$$x=5$$

Tesker: sette $x=0$ og 5 inn i opprinnelig likning
False løsning

$$\begin{aligned} x=0 &: \sqrt{9} = 3 \neq 0 \cdot 3 = -3 \\ x=5 &: \sqrt{9-5} = \sqrt{4} = 2 = 5-3 \end{aligned}$$

Så løsningene er 0 og 5 .

Gyldne snitt (førholt)

$$\frac{x}{1-x}$$

Beskrem x skilakatt $\frac{x}{1} = \frac{1-x}{x}$

$$\Leftrightarrow x^2 = 1-x \quad \Leftrightarrow x^2 + x - 1 = 0$$

$$x = \frac{-1 \pm \sqrt{1^2 - 4 \cdot 1 \cdot (-1)}}{2} = \frac{-1 \pm \sqrt{5}}{2}$$

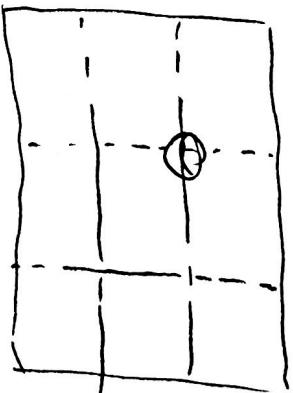
$$x = \frac{x}{1} = \frac{\sqrt{5}-1}{2} = 0.618\dots \text{ dimensionalt}$$

-1.618\dots

(den andre løsningen)

Dette
gyldne forhold.

"golden ration".



13

$$a < b \Leftrightarrow b - a > 0$$

$$a^2 < b^2 \Leftrightarrow b^2 - a^2 > 0$$

$$\begin{aligned} a^2 < b^2 &\Leftrightarrow \\ (b-a)(b+a) &> 0 \end{aligned}$$

$$\begin{aligned} \sqrt{4+9} &< \sqrt{4} + \sqrt{9} \\ \sqrt{13} &< 2 + 3 = 5 \end{aligned}$$

$$\left(\sqrt{x_1} + \sqrt{y_1} \right)^2 \leq x_1 + y_1$$

$$6 \quad 3) \quad f(x) = y = ax^2 + bx + c$$

$$f(2) = y(-2) = 5 \quad : \quad 4a - 2b + c = 5$$

$$f(-1) = y(-1) = -2 \quad : \quad a - b + c = -2$$

$$f(1) = y(1) = 2 \quad : \quad a + b + c = 2$$

3 lineare Gleichungen
3 Variablen.

Variant av oblig 1 #6 4)

$$\text{Parabel gitt med punktene } (0,0) \text{ og } (2,3).$$

$$Y = ax^2 + bx + c \quad \text{Parabel}$$

$$Y(0) = c = 0$$

$$Y(2) = 3$$

$$a \cdot 2^2 + b \cdot 2 = 3$$

$$4a + 2b = 3.$$

b kan uttrykkes v.h.a. a.

$$b = \frac{1}{2}(3 - 4a)$$

$$Y = a x^2 + \frac{1}{2}(3 - 4a) \cdot x \quad a \in \mathbb{R}$$

Kunne finne parameterne
ved å sette inn koefisientene
Som gitt gitt med punktene $(0,0)$
og $(2,3)$.