Eksamen 8 des 2015

1. $x-a \mid p(x) \iff p(a)=0$

a) betyr deler $\frac{p(x)}{x-a} = \text{polynom} + \frac{p(a)}{x-a}$

$p(x) = 2x^3 - 12x^2 - 2x + a$

$x-3$ er en faktor i $p \iff p(3)=0$

$\underbrace{2\cdot 3^3 - 4\cdot 3\cdot 3^2}_{-2\cdot 27} - 2\cdot 3 + a = 0$

$-2\cdot 27 \quad - 6 + a = 0$

$-54 - 6 + a = -60 + a = 0$

så $\underline{a = 60}$

b) Faktorisere $p(x) : (x-3) = 2x^2 - 6x - 20$

1)

$2x^2 - 6x - 20 = 2(x^2 - 3x - 10)$
$2\ (x-5)(x+2)$

$p(x) = 2(x-3)(x-5)(x+2)$

2) $p(x) < 0$ Benytter fortegnsskjema

	-2	0	3	5	
$2(x-3)$	- - - -	- - - -	- - 0	———	———
$(x-5)$	- - - -	- - - -	- - - -	- - 0	———
$(x+2)$	- - - - 0	———	———	———	———
$p(x)$	- - - - 0	———	——— 0	- - - 0	———

så $p(x) < 0$ for $\underline{x \in \langle -\infty, -2\rangle \cup \langle 3, 5\rangle}$

Vi viser polynom divisjonen:

$$
\begin{aligned}
&2x^3 - 12x^2 - 2x + 60 : x-3 = 2x^2 - 6x - 20 \\
&\underline{2x^3 - 6x^2} \\
&\quad -6x^2 - 2x + 60 \\
&\quad \underline{-6x^2 + 18x} \\
&\qquad -20x + 60 \\
&\qquad \underline{-20x + 60} \\
&\qquad\qquad 0 \quad \text{ingen rest}
\end{aligned}
$$

2 a) $\dfrac{2x^2-8}{2x+8} : \dfrac{3x+6}{x+4} = \dfrac{\cancel{2}(x^2-4)}{\cancel{2}(x+4)} \cdot \dfrac{x+4}{3(x+2)}$

delt på   gange

$= \dfrac{(x-2)(x+2)}{x+4} \cdot \dfrac{(x+4)}{3(x+2)}$ kanselere ut faktoren. $x+2$ og $x+4$

$= \underline{\dfrac{x-2}{3}}$

2 b) $\dfrac{2a^{-2} \cdot b \cdot 3\sqrt{a}}{6a^{-3} \cdot \sqrt[3]{b^2}} = \dfrac{\cancel{2 \cdot 3} \cdot a^{-2} \cdot b \cdot a^{1/2}}{\cancel{6} \cdot a^{-3} \cdot (b^2)^{1/3}}$

$= a^{-2} \cdot b\, a^{1/2} \left(a^{-3} \cdot b^{2/3}\right)^{-1} = a^{-2} \cdot a^{1/2} \cdot b \cdot \left(a^{-3}\right)^{-1} \left(b^{2/3}\right)^{-1}$

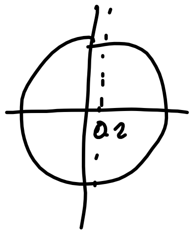
$= a^{-2} \cdot a^{1/2} \cdot a^{3} \cdot b \cdot b^{-2/3} = a^{-2+1/2+3} \cdot b^{1-2/3}$

$= \underline{a^{3/2}\, b^{1/3}} = \underline{a\sqrt{a} \cdot \sqrt[3]{b}}$

$$\left(\sqrt[n]{x} = x^{1/n} \qquad \frac{1}{x} = x^{-1} \qquad \frac{1}{a^{-n}} = \left(a^{-n}\right)^{-1} = a^{n} \right)$$

3a) $1 - 5\cos v = 0$ $\quad x \in [0, 360°]$

$\Leftrightarrow \quad \cos v = \frac{1}{5} = 0.2.$



$v = \arccos(0.2)$ og $-\arccos(0.2)$
opp til hele omløp. I intervallet $[0.360°)$
er løsningene derfor: $v = \underline{78.5°}$ og $\underline{281.5°}$

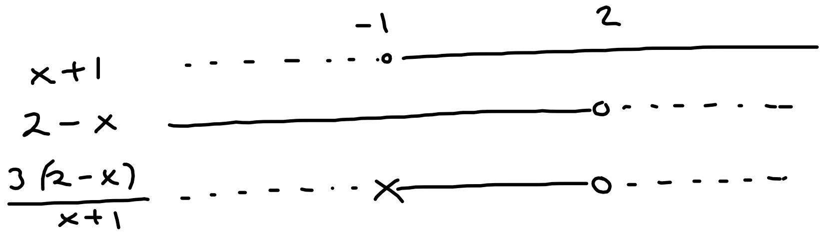
3 b) $\frac{8-x}{x+1} \geq 2$

Ikke gang opp med $x+1$: $\quad 8 - x \geq 2(x+1)$!!
bare riktig når $x+1 > 0$
$x+1 < 0$: fortegnet snus.

$$\frac{8-x}{x+1} - 2 \geq 0$$

$$\frac{8-x}{x+1} - \frac{2(x+1)}{x+1} \geq 0 \qquad \frac{8-x-2x-2}{x+1} \geq 0$$

$$\frac{-3x+6}{x+1} \geq 0 \qquad \frac{3(2-x)}{x+1} \geq 0$$



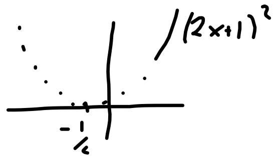
Løsningen er $\underline{\langle -1, 2]}$
(Her er 3 lagt til begge sider av =)

c) $2\sqrt{2x+2} = 2x+3$ kvadrer begge side

$\Rightarrow \quad 4(2x+2) = (2x+3)^2 = 4x^2 + 12x + 9$

Samler alle ledd på en side av likhetstegnet:

$$4x^2 + 12x - 8x + 9 - 8 = 0$$
$$4x^2 + 4x + 1 = 0$$
$$(2x+1)^2 = 0$$



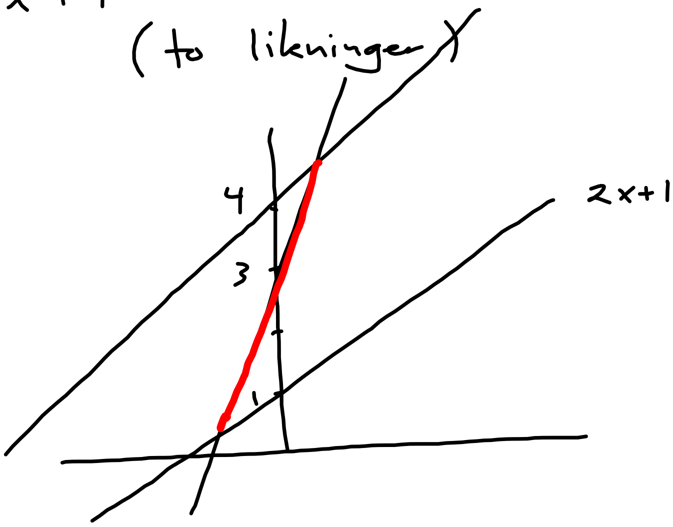
Her er det bare én løsning $x = -\frac{1}{2}$.
Vi sjekker nå om dette er en falsk løsning.

VS $\quad 2\sqrt{2(-\frac{1}{2})+2} - 3 = -1$ ✓ så løsningen

HS $\quad 2(-\frac{1}{2}) = -1$

til likningen er $\underline{x = -\frac{1}{2}}$

3 d) $2x+1 < 5x+3 \leq 3x+4$
(to likninger)



$2x+1 < 5x+3$ gir $1-3 < 5x-2x = 3x$
$-2 < 3x$

så $\underline{x > -2/3}$

$5x+3 \leq 3x+4$ $5x-3x \leq 4-3$
$2x \leq 1$

så $x \leq 1/2$

Løsningsmengden er $\underline{-\frac{2}{3} < x \leq \frac{1}{2}}$

4 setter inn $x = 2$ og $y = -2$

1 $ax + by = -2$

$bx - ay = 10$

I $2a - 2b = -2$

II $2b + 2a = 10$

I+II $2a \underbrace{-2b + 2b}_{0} + 2a = -2 + 10 = 8$

$4a = 8$

så $\underline{a = 2}$

$2b = 10 - 2a = 10 - 2 \cdot 2 = 6$

$b = 6/2 = \underline{3}$

Parametrene må være $a = 2$, $\underline{\underline{b = 3}}$

$$f(x) = -5x^2 - 3x + 1$$

$$f'(2) = \lim_{h\to 0} \frac{f(2+h) - f(2)}{h} \quad \text{definsjonen}$$

$$= \lim_{h\to 0} \frac{\left(-5(2+h)^2 - 3(2+h) + 1\right) - \left(-5\cdot 2^2 - 3\cdot 2 + 1\right)}{h}$$

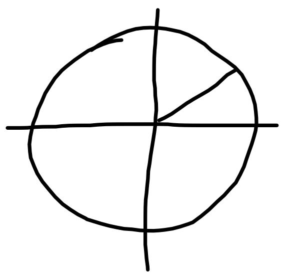
$$= \lim_{h\to 0} \frac{1}{h}\left[-5\left(2^2 + 2\cdot 2\cdot h + h^2 - 2^2\right) - 3h\right]$$

$$= \lim_{h\to 0} \frac{1}{h}\left[-5\left(4h + h^2\right) - 3h\right]$$

$$= \lim_{h\to 0} \frac{h}{h}\left(-23 - 5h\right) = \lim_{h\to 0} -23 - 5h$$

$$= \underline{\underline{-23}}$$

8. $$\frac{\sin(30^\circ) + \cos(30^\circ)}{1 - \tan(30^\circ)}$$



$$= \frac{\frac{1}{2} + \frac{\sqrt{3}}{2}}{1 - 1/\sqrt{3}}$$

$$= \frac{1 + \sqrt{3}}{2(1 - 1/\sqrt{3})} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{(1+\sqrt{3})\sqrt{3}}{2(\sqrt{3} - 1)} \cdot \frac{(\sqrt{3}+1)}{(\sqrt{3}+1)}$$

"den konjugert"

$$= \frac{\sqrt{3}(1+\sqrt{3})^2}{2 \cdot 2} = \frac{\sqrt{3}(1 + 3 + 2\sqrt{3})}{2 \cdot 2} = \frac{\sqrt{3}(2+\sqrt{3}) \cdot \cancel{2}}{2 \cdot 2}$$

$$= \frac{\sqrt{3}(2+\sqrt{3})}{2}$$

9 Vis at:

$$\frac{\sin 2x}{1 + \cos 2x} = \tan x$$

Setter inn:

$\sin(2x) = 2\sin x \cos x$

$\cos(2x) = \cos^2 x - \sin^2 x$

$$\frac{2\sin x \cos x}{1 + \cos^2 x - \sin^2 x}$$

benytter Pytagoras $1 - \sin^2 x = \cos^2 x$

$$= \frac{2\sin x \cdot \cos x}{\cos^2 x + \underbrace{\cos^2 x}_{(1-\sin^2 x)}} = \frac{2\sin x \cos x}{2\cos^2 x}$$

$$= \frac{2\sin x \cdot \cos x}{2\cos x \cdot \cos x}$$ (bare det. for $\cos x \neq 0$)

$$= \frac{\sin x}{\cos x} = \underline{\tan x}.$$ $\square$

10 f er kontinuerlig i $x=1$ hvis

$$\lim_{x\to 1^-} f(x) = f(1) = \lim_{x\to 1^+} f(x)$$

$f(1) = 5-2\cdot 1 = \underline{3}$ polynomer er kontinuerlige

$$\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-} 4-x^2 = 4-(+1)^2 = 3$$

$$\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} 5-2x = 5-2\cdot 1 = 3$$

Vi ser at kriteriet for kontinuitet i $x=1$ er oppfylt. Derfor er $f(x)$ kontinuerlig i $x=1$.

$\underline{5}$ $f(x) = \dfrac{x^3-x^2-2x}{x^2-9} = \dfrac{x^3-x^2-2x}{(x-3)(x+3)}$

Siden uttrykket i telleren, x^3-x^2-2x, er ulik 0 for $x=\pm 3$, så har vi vertikale asymptoter i $x=-3$ og i $x=3$.

$$\left(\begin{array}{ll} -3 \text{ gir} & (-3)^3-(-3)^2-2(-3) = -27-9+6 = -30 \\ 3 \text{ gir} & 3^3-3^2-2(3) = 27-9-6 = 12 \end{array}\right)$$

Polynomet i telleren har grad én mer enn polynomet i nevneren. Vi forventer derfor en skråasymptote:

Vi utfører polynomdivisjon for å finne asymptoten

$$x^3-x^2-2x : x^2-9 = x-1$$
$$x^3 - 9x$$
$$-x^2+7x$$
$$-x^2+9$$
$$7x-9$$

Så $f(x) = x-1+\underbrace{\dfrac{7x-9}{x^2-9}}$

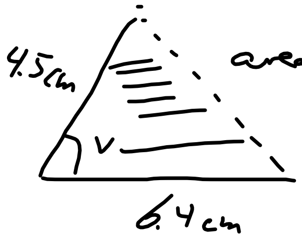
Dette leddet går mot 0 når $x\to\infty$ (og mot $-\infty$)

Derfor er $\lim_{x\to\infty}\left(f(x)-(x-1)\right)=0$

$\left(\lim_{x\to-\infty}\left(f(x)-(x-1)\right)=0\right)$

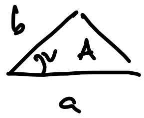
og $\underline{y=x-1}$ er en skrå asymptote.

6



arealet er $7.2\,cm^2$

$$A = \frac{1}{2} a \cdot b \cdot \sin v$$



$$7.2cm^2 = \frac{1}{2} \cdot 4.5cm \cdot 6.4cm \cdot \sin v$$

Så

$$= \frac{7.2}{4.5 \cdot 3.2} \cdot \frac{100}{100} = \frac{72 \cdot 10}{45 \cdot 32}$$

$$= \frac{8 \cdot 9 \cdot 10}{5 \cdot 9 \cdot 2 \cdot 2 \cdot 8} = \frac{8}{8} \cdot \frac{10}{5 \cdot 2} \cdot \frac{9}{9} \cdot \frac{1}{2}$$

$$= \frac{1}{2} = 0.5$$

Vinkelen må være

$$\arcsin(0.5) = \underline{\underline{30^\circ}} \quad (= \pi/6 \text{ rad})$$