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Mer delbrøkoppspalting

① $\int \frac{2x}{(2x+1)^2}\,dx$

$$\frac{2x}{(2x+1)^2} = \frac{A}{2x+1} + \frac{B}{(2x+1)^2}$$

Felles nevner gir: $2x = A(2x+1) + B$

$A = 1, \quad B = -1.$

$$\int \frac{2x}{(2x+1)^2}\,dx = \int \frac{1}{2x+1} - \frac{1}{(2x+1)^2}\,dx$$

Lineær substitusjon $\quad u = 2x+1 \quad u' = 2$

$du = 2dx$

$$= \int \left(\frac{1}{u} - \frac{1}{u^2}\right)\frac{1}{2}\,du$$

$$= \frac{1}{2}\int u^{-1} - u^{-2}\,du$$

$$= \frac{1}{2}\left(\ln|u| - \frac{u^{-1}}{-1}\right) + c$$

$$= \underline{\frac{1}{2}\left(\ln|2x+1| + \frac{1}{2x+1}\right) + c}$$

* $\int \frac{1}{x^4-1}\,dx$

Faktoriserer nevneren

$(x^4-1) = (x^2-1)(x^2+1)$ konjugat set.

$= (x-1)(x+1)(x^2+1)$ ← generelt polynom av grad 1

$$\frac{1}{x^4-1} = \frac{A}{x-1} + \frac{B}{x+1} + \frac{Cx+D}{x^2+1}$$

Felles nevner gir

$$1 = A(x+1)(x^2+1) + B(x-1)(x^2+1) + (x-1)(x+1)(Cx+D)$$

setter $x = 1 \quad 1 = A\cdot 4 \quad$ så $A = 1/4$

$x = -1 \quad 1 = -4B \quad$ så $B = -1/4$

$$1 = \frac{1}{4}(x^2+1)\Big(\underbrace{(x+1) - (x-1)}_{2}\Big) + (x-1)(x+1)(Cx+D)$$

2 så $C = 0$ (sammenlikner x^3-ledd)

Setter $x = 0$ $\quad 1 = \frac{1}{2} + -D \quad$ så $D = \frac{-1}{2}$

$$\int \frac{1}{x^4-1}\,dx = \int \frac{1/4}{x-1} - \frac{1/4}{x+1} - \frac{1/2}{x^2+1}\,dx$$

$$= \frac{1}{4}\ln|x-1| - \frac{1}{4}\ln|x+1| - \frac{1}{2}\arctan(x) + C$$

$$= \underline{\frac{1}{4}\ln\left|\frac{x-1}{x+1}\right| - \frac{1}{2}\arctan(x) + C}$$

eks. $\displaystyle\int \frac{x^2+x}{x^2+1}\,dx$

Polynomdivisjon

$$\frac{x^2+1+x-1}{x^2+1} = 1 + \frac{x-1}{x^2+1}$$

$$\int \frac{x-1}{x^2+1}\,dx = \int \frac{x}{x^2+1} - \frac{1}{x^2+1}\,dx$$

$\displaystyle\int \frac{x}{x^2+1}\,dx$ substitusjon $\quad u = x^2+1$

$du = 2x\,dx$

$\frac{1}{2}du = x\,dx$

$$= \int \frac{1}{u}\cdot\frac{1}{2}\,du$$

$$= \frac{1}{2}\ln|u| + C = \frac{1}{2}\ln(x^2+1) + C$$

$$\int \frac{x^2+x}{x^2+1}\,dx = \int 1 + \frac{x}{x^2+1} - \frac{1}{x^2+1}\,dx$$

$$= \underline{x + \frac{1}{2}\ln(x^2+1) - \arctan x + C}$$

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③

Test

1) $\int_{-2}^{3} 4\,dx$

2) $\int 2x^3 - \frac{3}{x^2}\,dx$

3) $\int \frac{4}{3x-5}\,dx$

4) $\int 2x \sin(7x)\,dx$

5) $\int 2x \sin(7x^2)\,dx$

6) $\int_{8}^{27} \frac{1}{\sqrt[3]{x}}\,dx$

4 1) $\int_{-2}^{3} 4\,dx = 20$ (ingen konstant c)

$$4x + c \Big|_{-2}^{3} = (4\cdot 3 + c) - (4\cdot(-2) + c)$$

$$= 20 + c - c = \underline{20}$$

2) $\int x^r\,dx = \begin{cases} \frac{x^{r+1}}{r+1} & r \neq -1 \\ \ln|x| & r = -1 \end{cases}$

$$\int 2x^3 - \frac{3}{x^2}\,dx = 2\int x^3\,dx - 3\int x^{-2}\,dx$$

$$= 2\,\frac{x^4}{4} - 3\,\frac{x^{-1}}{-1} + c$$

$$= \underline{\frac{x^4}{2} + \frac{3}{x} + c}$$

Feil 1) $\int \frac{1}{x^2}\,dx \cancel{=} \ln(x^2) + c$ $\left(\text{merk: } \frac{d}{dx}\ln(x^2) = \frac{2}{x}\right)$

$$\int \frac{u'}{u}\,dx = \int \frac{du}{u} = \ln|u| + c$$

2) $\cancel{\frac{1}{\ln x}}$?

3) $\int \frac{1}{x^2}\,dx \cancel{=} \frac{1}{x}$

$$\left(\int \frac{3}{x^2}\,dx = 3\int \frac{1}{x^2}\,dx\right)$$

3) $\int \frac{4}{3x-5}\, dx$

5

$= 4 \cdot \int \frac{1}{3x-5}\, dx$

$u = 3x-5$
$du = 3dx$
$dx = \frac{1}{3}du$

$= 4 \int \frac{1}{u} \cdot \frac{1}{3} du$

$= \frac{4}{3} \int \frac{1}{u} du = \frac{4}{3} \int u^{-1} du = \frac{4}{3} \ln|u| + c$

$= \underline{\frac{4}{3} \ln|3x-5| + c}$.

Veldig mange fikk $4 \cdot \ln|3x-5|$.
$\frac{1}{3}$ ble glemt!

4) $\int 2x \sin(7x)\, dx$ Delvis integrasjon

$u = 2x \qquad v' = \sin(7x)$

$u' = 2 \qquad v = \frac{-\cos(7x)}{7}$

Mer detaljert: $c + v = \int v' dx = \int \sin(7x)\, dx$

Linear substitusjon: $w = 7x \qquad \frac{dw}{dx} = 7$

$dw = 7dx$

$\frac{1}{7}dw = dx$

$\int \sin(w) \frac{1}{7} dw$

$= \frac{1}{7} \int \sin(w) dw = \frac{1}{7}(-\cos(w)) + c$

$v + c = \frac{-1}{7}\cos(7x) + c$

Velger $v = \frac{-1}{7}\cos(7x)$

$$\int u \cdot v' \, dx = u \cdot v - \int u' v \, dx$$

$$\int 2x \sin(7x) \, dx = 2x\left(\frac{-1}{7}\cos(7x)\right) - \int 2 \cdot \left(\frac{-1}{7}\right)\cos(7x) \, dx$$

6)

$$= -\frac{2x}{7}\cos(7x) + \frac{2}{7}\left(\frac{\sin(7x)}{7}\right) + C$$

$$= \frac{2}{7^2}\left(\sin(7x) - 7x\cos(7x)\right) + C$$

5)

$$\int 2x \sin(7x^2) \, dx$$

substitusjon

$U = 7x^2$

$U' = 14x$

$dU = 14x \, dx$

$\frac{1}{7} dU = 2x \, dx$

$$= \int \sin(U) \frac{1}{7} dU$$

$$= \frac{-1}{7}\cos(U) + C$$

$$= \frac{-1}{7}\cos(7x^2) + C$$

6)

$$\int_8^{27} \frac{1}{\sqrt[3]{x}} \, dx = \int_8^{27} x^{-1/3} \, dx$$

$$= \frac{x^{-1/3+1}}{2/3}\Bigg|_8^{27} = \frac{3x^{2/3}}{2}\Bigg|_8^{27}$$

$$= \frac{3}{2}\underbrace{\left(\sqrt[3]{x}\right)^2}_{\sqrt[3]{x^2}}\Bigg|_8^{27} = \frac{3}{2}\left(\left(\underbrace{\sqrt[3]{27}}_{3}\right)^2 - \left(\underbrace{\sqrt[3]{8}}_{2}\right)^2\right)$$

$$= \frac{3}{2}(9-4) = \frac{15}{2}$$