Løsningsforslag eksamen matematikk forkurs 04.08.2025

Oppgave 1

a) $(5x-2)\left(\frac{1}{2}x+2\right)-2(x^2+4x)$

$=(\frac{5}{2}x^2+10x-x-4)-(2x^2+8x)$

$=\frac{5}{2}x^2+10x-x-4-2x^2-8x$

$=\frac{1}{2}x^2+x-4$

b) $\frac{(x+3)^{\frac{5}{2}}}{2(x+3)^{\frac{1}{2}}}=\frac{1}{2}(x+3)^{(\frac{5}{2}-\frac{1}{2})}=\frac{1}{2}(x+3)^{\frac{4}{2}}=\frac{1}{2}(x+3)^2$

$=\frac{1}{2}(x^2+6x+9)=\frac{1}{2}x^2+3x+\frac{9}{2}$

$a=\frac{1}{2} \qquad b=3 \qquad c=\frac{9}{2}$

Oppgave 2

a) $\frac{x}{x+1}-\frac{1}{x-1}=2 \qquad | \ (x+1)(x-1)$

$\frac{x(x+1)(x-1)}{x+1}-\frac{1(x+1)(x-1)}{x-1}=2(x+1)(x-1)$

$x^2-x-(x+1)=2(x^2-1)$

$x^2-x-x-1=2x^2-2$

$x^2+2x-1=0$

$x=\frac{-2\pm\sqrt{2^2-4\cdot1\cdot(-1)}}{2\cdot1}=\frac{-2\pm\sqrt{8}}{2}=\frac{-2\pm2\sqrt{2}}{2}=\frac{2(-1\pm\sqrt{2})}{2}=-1\pm\sqrt{2}$

$x = -1 + \sqrt{2}$ $\qquad$ $x = -1 - \sqrt{2}$

$L = \{-1 - \sqrt{2}, -1 + \sqrt{2}\}$

b) $\sin x \cdot cosx - cos^2 x = 0$ $\qquad$ $x \in [\pi, 2\pi\rangle$

$\cos x(\sin x - \cos x) = 0$

$\cos x = 0$ $\qquad$ $\sin x - \cos x = 0$

$cos^{-1}(0) = \frac{\pi}{2}$ $\qquad$ $\frac{\sin x}{\cos x} - \frac{\cos x}{\cos x} = 0$

$x = \frac{\pi}{2} + k2\pi$ $\qquad$ $\tan x = 1$

$x = -\frac{\pi}{2} + k2\pi$

$tan^{-1}(1) = \frac{\pi}{4}$

$x = -\frac{\pi}{2} + 1 \cdot 2\pi = \frac{3\pi}{2}$

$x = \frac{\pi}{4} + k\pi$

$x = \frac{\pi}{4} + 1 \cdot \pi = \frac{5\pi}{4}$

$L = \left\{\frac{5\pi}{4}, \frac{3\pi}{2}\right\}$

Oppgave 3

a) $f'(x) = 3x^2 - 2$

$f''(x) = 6\text{x}$

1. $f(-2) = (-2)^3 - 2(-2) = -4$
2. $f'(-2) = 3(-2)^2 - 2 = 10$
3. $f''(-2) = 6(-2) = -12$

b) Brøkregelen/kvotientregelen:

$u = cos2x \qquad u' = -2\sin2x$

$v = e^x \qquad v' = e^x$

$g'(x) = -\sin(2x) \cdot e^x + \cos(2x) \cdot e^x = e^x(\cos 2x - 2\sin 2x)$

c) $h(x) = \frac{2}{x} - \ln\left(x^2\sqrt{x}\right) = 2x^{-1} - \ln\left(x^{\frac{5}{2}}\right) = 2x^{-1} - \frac{5}{2}\ln x$

$$h'(x) = -2x^{-2} - \frac{5}{2} \cdot \frac{1}{x} = \frac{-2}{x^2} - \frac{5}{2x}$$

Oppgave 4

a) $\int \left(2x - \frac{1}{x}\right) dx = 2 \cdot \frac{1}{2}x^2 - \ln|x| + C = x^2 - \ln|x| + C$

b) $\int_0^1 \frac{x+5}{x^2-2x-3}\, dx$

$$x^2 - 2x - 3 = 0$$

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4 \cdot 1 \cdot (-3)}}{2 \cdot 1} = \frac{2 \pm 4}{2}$$

$$x_1 = \frac{2+4}{2} = 3 \qquad x_2 = \frac{2-4}{2} = -1$$

$$x^2 - 2x - 3 = (x+1)(x-3)$$

$$\frac{x+5}{x^2-2x-3} = \frac{x+5}{(x+1)(x-3)} = \frac{A}{x+1} + \frac{B}{x-3}$$

$$x + 5 = \frac{A(x+1)(x-3)}{x+1} + \frac{B(x+1)(x-3)}{x-3}$$

$$x + 5 = A(x-3) + B(x+1)$$

$$x = -1$$

$$-1+5=A(-1-3)+B(-1+1)$$

$$4=-4A$$

$$A=-1$$

$$x=3$$

$$3+5=A(3-3)+B(3+1)$$

$$8=4B$$

$$B=2$$

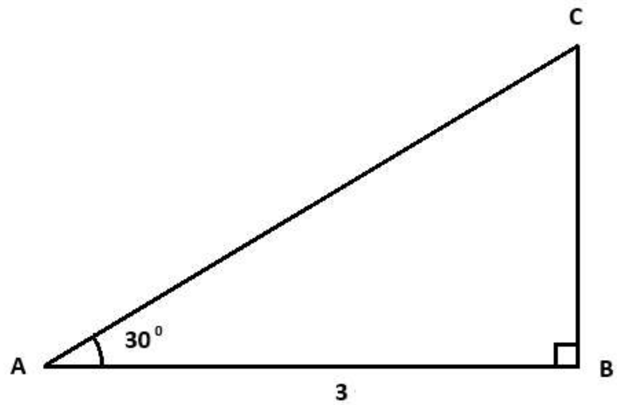
$$\int_0^1 \frac{x+5}{x^2-2x-3}\,dx=\int_0^1\left(\frac{-1}{x+1}+\frac{2}{x-3}\right)dx=\left[-\ln|x+1|+2\ln|x-3|\right]\begin{matrix}1\\0\end{matrix}$$

$$-\ln|1+1|+2\ln|1-3|-(-\ln|0+1|+2\ln|0-3|)=-\ln 2+2\ln 2-2\ln 3$$

$$=\ln 2-2\ln 3=\ln 2-\ln 9=ln\frac{2}{9}$$

Oppgave 5

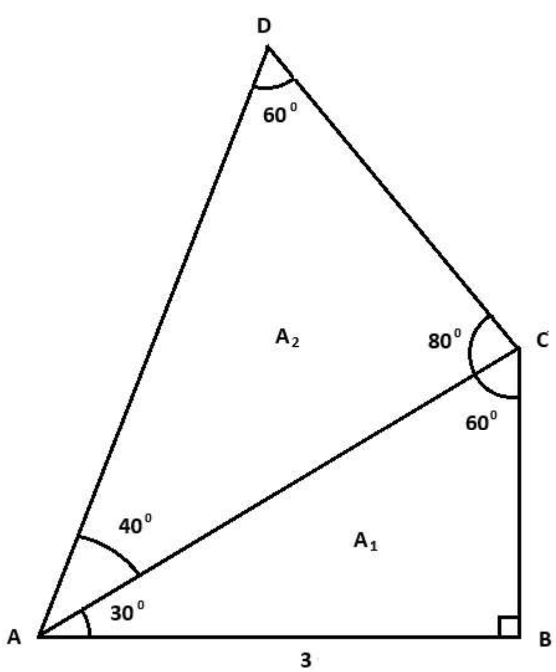
a)



$$\cos A = \frac{AB}{AC}$$

$$AC = \frac{AB}{\cos A} = \frac{3}{\cos 30°} = \frac{3}{\frac{\sqrt{3}}{2}} = \frac{2 \cdot 3}{\sqrt{3}} = \frac{2 \cdot 3}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{2 \cdot 3 \cdot \sqrt{3}}{3} = 2\sqrt{3} \approx 3{,}46$$

b)



$$\angle CAD = 70° - 30° = 40°$$

$$\angle BCD = 90° - 30° = 60°$$

$$\angle ACD = 140° - 60° = 80°$$

$$\angle CDA = 180° - 40° - 80° = 60°$$

$$\frac{AD}{\sin 80°} = \frac{2\sqrt{3}}{\sin 60°}$$

$$AD = \frac{2\sqrt{3}\sin 80°}{\sin 60°} = 3{,}94$$

c)

$$A = A_1 + A_2 = \frac{1}{2} \cdot 3 \cdot 2\sqrt{3} \cdot \sin 30° + \frac{1}{2} \cdot 2\sqrt{3} \cdot 3{,}94 \cdot \sin 40° = 6{,}98$$

Oppgave 6

a) $k = \dfrac{\frac{(x+1)^2}{4}}{\frac{(x+1)}{2}} = \dfrac{\frac{(x+1)^2}{4} \cdot 4}{\frac{(x+1)}{2} \cdot 4} = \dfrac{(x+1)^2}{2(x+1)} = \dfrac{x+1}{2}$

Rekka konvergerer når: $-1 < k < 1$

$$-1 < \frac{x+1}{2} < 1$$

$$-1 < \frac{x+1}{2} \quad \wedge \quad \frac{x+1}{2} < 1$$

$$-2 < x+1 \quad \wedge \quad x+1 < 2$$

$$-3 < x \quad \wedge \quad x < 1$$

Konvergensintevall: $\underline{\underline{x \in \langle -3, 1 \rangle}}$

evnt.: $\underline{\underline{-3 < x < 1}}$

b) $x = -2$:

Finner k når $x = -2$: $k = \frac{x+1}{2} = \frac{-2+1}{2} = -\frac{1}{2}$

Finner a_1 når $x = -2$: $a_1 = \frac{x+1}{2} = \frac{-2+1}{2} = -\frac{1}{2}$

$$s = \frac{a_1}{1-k} = \frac{-\frac{1}{2}}{1-(-\frac{1}{2})} = \frac{-\frac{1}{2}}{\frac{3}{2}} = -\frac{1}{3}$$

Oppgave 7

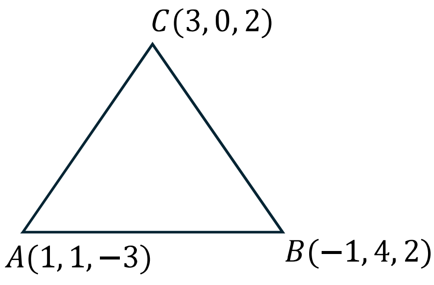
a) 3, 9 og 21

b) Kan løses på ulike måter. Her kommer tre alternativ:

Alternativ 1:	Alternativ 2:	Alternativ 3:

`tall = 0`
`while tall < 18:`
`    tall = tall + 2`
`    print(tall)`
	`tall = 0`
	`i = 1`
	`while tall < 18:`
	`    tall = i*2`
	`    i = i+1`
	`    print(tall)`
		`for i in range(1,10):`
		`    print(i*2)`

Oppgave 8



a) $\overrightarrow{BA} = [1-(-1), 1-4, -3-2] = [2, -3, -5]$

$\left|\overrightarrow{AB}\right| = \left|\overrightarrow{BA}\right| = \sqrt{2^2 + (-3)^2 + (-5)^2} = \sqrt{38} \approx 6{,}2$

b) $\overrightarrow{BC} = [3-(-1), 0-4, 2-2] = [4, -4, 0]$

$\overrightarrow{BA} \cdot \overrightarrow{BC} = \left|\overrightarrow{BA}\right| \cdot \left|\overrightarrow{BC}\right| \cdot \cos B$

$\cos B = \frac{[2,-3,-5]\cdot[4,-4,0]}{\sqrt{38}\cdot\sqrt{32}}$ $\qquad \left|\overrightarrow{BC}\right| = \sqrt{4^2 + (-4)^2 + 0^2} = \sqrt{32}$

$= \frac{2\cdot 4 + (-3)\cdot(-4) + (-5)\cdot 0}{\sqrt{38}\cdot\sqrt{32}} = \frac{20}{\sqrt{38}\cdot\sqrt{32}}$

$\angle B = \cos^{-1}\frac{20}{\sqrt{38}\cdot\sqrt{32}} \approx 55{,}0°$

c) $\overrightarrow{BA} \times \overrightarrow{BC} = \begin{vmatrix} \overrightarrow{e_x} & \overrightarrow{e_y} & \overrightarrow{e_z} \\ 2 & -3 & -5 \\ 4 & -4 & 0 \end{vmatrix} = \begin{vmatrix} -3 & -5 \\ -4 & 0 \end{vmatrix}\overrightarrow{e_x} - \begin{vmatrix} 2 & -5 \\ 4 & 0 \end{vmatrix}\overrightarrow{e_y} + \begin{vmatrix} 2 & -3 \\ 4 & -4 \end{vmatrix}\overrightarrow{e_z}$

$= (0-20)\overrightarrow{e_x} - \big(0-(-20)\big)\overrightarrow{e_y} + \big(-8-(-12)\big)\overrightarrow{e_z} = [-20, -20, 4]$

Finner normalvektoren som oppgaven spør etter:

$$[5, 5, -1] = k[-20, -20, 4]$$

Bruker x-koordinaten til å finne k: $5 = k \cdot (-20)$

$$k = \frac{5}{-20} = -\frac{1}{4}$$

$$\vec{n} = -\frac{1}{4}[-20, -20, 4] = [5, 5, -1]$$

Velger $C(3, 0, 2)$:

$$5(x-3) + 5(y-0) - 1(z-2) = 0$$
$$5x - 15 + 5y - z + 2 = 0$$
$$\underline{\underline{\alpha:\ 5x + 5y - z - 13 = 0}}$$

d) På x-aksen er $y = 0$ og $z = 0$.

Kaller et punkt på x-aksen $P(x, 0, 0)$:

$\overrightarrow{BP} = [x-(-1), y-4, z-2] = [x+1, 0-4, 0-2] = [x+1, -4, -2]$

Finner P ved å sette at lengden til $\overrightarrow{BP}$ er lik 6:

$$\left|\overrightarrow{BP}\right| = 6$$

$$\sqrt{(x+1)^2 + (-4)^2 + (-2)^2} = 6$$

$$\sqrt{(x+1)^2 + (-4)^2 + (-2)^2}^2 = 6^2$$

$$(x+1)^2 + (-4)^2 + (-2)^2 = 36$$

$$x^2 + 2x + 1 + 16 + 4 = 36$$

$$x^2 + 2x - 15 = 0$$

$$x = \frac{-2 \pm \sqrt{2^2 - 4 \cdot 1 \cdot (-15)}}{2 \cdot 1} = \frac{-2 \pm \sqrt{64}}{2} = \frac{-2 \pm 8}{2} = \begin{cases} 3 \\ -5 \end{cases}$$

$$x_1 = 3 \qquad x_2 = -5$$

De to punktene på x-aksen med avstand 6 fra punktet B er dermed

$$P_1 = (-5, 0, 0)$$

$$P_2 = (3, 0, 0)$$

Lar B og P_1 ligge på linja l, og B og P_2 på linja m.

Finner retningsvektor til l:

$\overrightarrow{BP_1} = [-5 - (-1), 0 - 4, 0 - 2 = [-4, -4, -2]$

Velger retningsvektor $\vec{r_l} = -\frac{1}{2}[-4, -4, -2] = [2, 2, 1]$

Velger punktet $P_1 = (-5, 0, 0)$ til parameterframstillinga av l:

$$l: \begin{cases} x = -5 + 2t \\ y = 2t \\ z = t \end{cases}$$

Finner retningsvektor til m:

$\overrightarrow{BP_2} = [3 - (-1), 0 - 4, 0 - 2 = [4, -4, -2]$

Velger retningsvektor $\overrightarrow{r_m} = \frac{1}{2}[4, -4, -2] = [2, -2, -1]$

Velger punktet $P_2 = (3, 0, 0)$ til parameterframstillinga av m:

$$m: \begin{cases} x = 3 + 2t \\ y = -2t \\ z = -t \end{cases}$$

Oppgave 9

a) Nullpunkt: $g(x) = 0 \iff 2x^2 - 4 = 0 \iff x = \pm\sqrt{2}$

b) Eventuelle vertikale asymptoter:

$x^2 - 4 = 0 \iff x = \pm 2$

Når $x = \pm 2$ blir teller $\neq 0$

Vertikale asymptoter: $x = 2$ og $x = -2$

<u>Alternativ 1:</u>

Polynomdivisjon gir:

$(2x^2 - 4) : (x^2 - 4) = 2 + \frac{4}{x^2-4}$

$\underline{-(2x^2 - 8)}$

4

$y = 2$ er en horisontal asymptote

<u>Alternativ 2:</u>

$$\lim_{x\to\pm\infty} \frac{2x^2 - 4}{x^2 - 4} = \lim_{x\to\pm\infty} \frac{(2x^2 - 4)\cdot\frac{1}{x^2}}{(x^2 - 4)\cdot\frac{1}{x^2}} = \lim_{x\to\pm\infty} \frac{2 - \frac{4}{x^2}}{1 - \frac{4}{x^2}} = 2$$

Dermed blir $y = 2$ en horisontal asymptote

c) Brøkregel/kvotientregel:

$u = 2x^2 - 4 \qquad u' = x^2 - 4$

$v = 4x \qquad v' = 2x$

$$g'(x) = \frac{4x\cdot(x^2 - 4) - (2x^2 - 4)\cdot 2x}{(x^2 - 4)^2}$$

$$= \frac{-8x}{(x^2-4)^2}$$

Topp- eller bunnpunkt: $g'(x) = 0$

$$\frac{-8x}{(x^2-4)^2} = 0$$

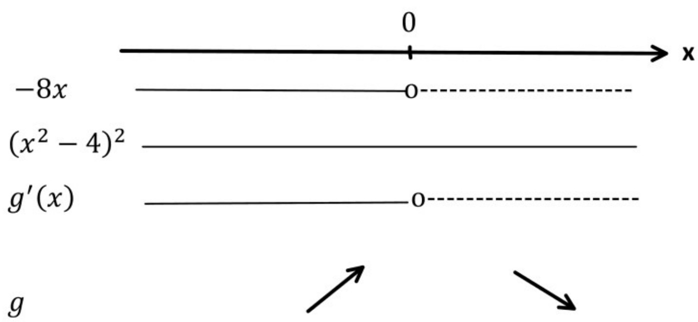
$-8x = 0$

$x = 0$

Må videre finne ut om $x = 0$ er et topp- eller bunnpunkt.

<u>Alternativ 1:</u>

Lager fortegnskjema:



$x = 0$ er et toppunkt

<u>Alternativ 2:</u>

Bruker $g''(x)$ som er oppgitt i d)

$$g''(0) = \frac{24 \cdot 0^2 + 32}{(0^2-4)^3} = \frac{32}{-64} < 0$$

$x = 0$ er et topppunkt

Finner tilhørende y-verdi:

$$g(0) = \frac{2 \cdot 0^2 - 4}{0^2 - 4} = \frac{-4}{-4} = 1$$

Toppunkt: $(0, 1)$

d) Brøkregel/kvotientregel:

$u = -8x \qquad u' = -8$

$v = (x^2 - 4)^2 \qquad v' = 2(x^2 - 4) \cdot 2x = 4x(x^2 - 4)$ (kjerneregel)

$$g''(x) = \frac{-8 \cdot (x^2 - 4)^2 - (-8x) \cdot 4x(x^2 - 4)}{(x^2 - 4)^4}$$

$$= \frac{(x^2-4)(-8(x^2-4)+32x^2)}{(x^2-4)^4} = \frac{-8x^2+32+32x^2}{(x^2-4)^3} = \frac{24x^2+32}{(x^2-4)^3}$$

Vendepunkt: $g''(x) = 0 \Rightarrow 24x^2 + 32 = 0 \Rightarrow$ Ingen løsning, så ingen vendepunkt.

e) Tegning av grafen: