8.12
2023

oblig 4

1 a) $x^3 - 2x^2 = x$

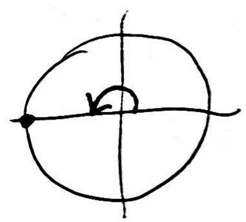
$x(x^2 - 2x - 1) = 0$

$x((x-1)^2 - 1 - 1) = x((x-1)^2 - 2) = 0$

$x = 0$ eller $(x-1)^2 = 2$

$x - 1 = \pm\sqrt{2} \iff x = 1 \pm \sqrt{2}$

Løsningene er $\underline{\{1-\sqrt{2}, 0, 1+\sqrt{2}\}}$



b) $\cos(v) + 1 = 0$ alle v

$\cos(v) = -1$ $v = \pi$ rad.

Løsningene er $v = \underline{\pi + 2\pi \cdot n}$, $n \in \mathbb{Z}$.

c) $\sqrt{3x+1} = \sqrt{x}$

$a = b \Rightarrow a^2 = b^2$

kvadrere $3x+1 = x$

$3x - x + 1 = 2x + 1 = 0$ så $x = \frac{-1}{2}$,

$\sqrt{\frac{-1}{2}}$ ikke def. (blant reelle tall)

Løsningsmengden er $\underline{\text{tom}}$.

d) $\sqrt{4x-1} = x$ (def når $x \geq 1/4$) $\Rightarrow$

$4x - 1 = x^2$

$x^2 - 4x + 1 = 0$

$(x-2)^2 - 4 + 1 = 0$

$(x-2)^2 = 3$

$x = 2 \pm \sqrt{3}$.

abc-formelen

$a=1,\ b=-4,\ c=1$

$x = \frac{4 \pm \sqrt{16-4}}{2 \cdot 1}$

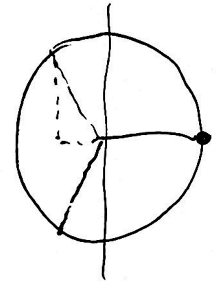
$= \frac{4 \pm \sqrt{4 \cdot 3}}{2}$

$= 2 \pm \sqrt{3}$

Begge er ekte løsninger.

Løsningene er $\begin{cases} x = 2 - \sqrt{3} \sim \underline{0.268} \\ x = 2 + \sqrt{3} \sim \underline{3.732} \end{cases}$

1e) $\cos(2z) = \cos(z)$ $\qquad z \in [0, \pi]$



$$\cos(2z) = \cos^2(z) - \underbrace{\sin^2(z)}_{1-\cos^2(z)}$$

$$= 2\cos^2 z - 1.$$

$$2\cos^2 z - 1 = \cos z$$

$$2\cos^2 z - \cos z - 1 = 0$$

$$2(\cos z)^2 - (\cos z) - 1 = 0 \quad \text{2.grads likning i } \cos z.$$

$$(\cos z - 1)(2\cos z + 1) = 0$$

$\cos z = 1$ og $\cos z = -1/2.$

$z = 0$ $\qquad z = 2\pi/3$

Løsningene er $\underline{z \in \{0, \frac{2\pi}{3}\}}$

2 Pris P

$(1.15)(1.3)(1.2)P = 897$

gir $\underline{P = 500\text{ kr}}$

3. $p(x) = x^2 + ax + b$

delelig med $(x-3) \Leftrightarrow P(3) = 0 \Leftrightarrow 9 + 3a + b = 0$

$p(1) = 1 + a + b = 3$

2 lineære likninger med to ukjente a og b.

$a + b = 2$

$3a + b = -9$

L2 − L1: $2a = -9 - 2 = -11$ så $\underline{\underline{a = \frac{-11}{2}}}$

$b = 2 - a = 2 + \frac{11}{2} = \frac{4+11}{2} = \underline{\underline{\frac{15}{2}}}$

$\underline{\underline{a = \frac{-11}{2}}}$ $\underline{\underline{b = \frac{15}{2}}}$

4 a) $\dfrac{x^3+3x-4}{x^2-3x+2}$.

nevner x^2-3x+2

$= (x-1)(x-2)$

sjekker om teller er delelig med disse faktorene

$1^3+3\cdot 1-4 = 0$ så delelig med $x-1$

$2^3+3\cdot 2-4 = 10 \neq 0$ ikke — $x-2$.

$$x^3+\underline{0}+3x-4 : x-1 = \underline{x^2+x+4}$$

$$\begin{array}{l} \underline{x^3-x^2} \\ \quad x^2+3x-4 \\ \quad \underline{x^2-x} \\ \qquad 4x-4 \\ \qquad \underline{4x-4} \\ \qquad\quad 0 \end{array}$$

$$\frac{x^3+3x-4}{x^2-3x+2} = \frac{(x^2+x+4)(x-1)}{(x-2)(x-1)} = \underline{\underline{\frac{x^2+x+4}{x-2}}}$$

4b) $\sqrt{9a\sqrt{4b}} \cdot \sqrt[3]{\frac{-8b}{a}}$

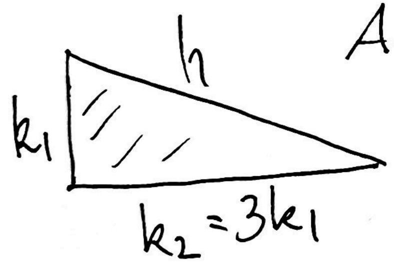
$= \sqrt{9}\sqrt{a}\sqrt{\sqrt{4}\sqrt{b}} \cdot \sqrt[3]{-8}\ \sqrt[3]{b}\ \frac{1}{\sqrt[3]{a}}$

$= 3a^{1/2}\sqrt{2}\left(b^{1/2}\right)^{1/2} \cdot -2b^{1/3}\left(a^{1/3}\right)^{-1}$

$= -6\sqrt{2}\ a^{1/2}a^{-1/3}\ b^{1/4} \cdot b^{1/3}$

$= -6\sqrt{2}\ a^{1/2-1/3}\ b^{1/4+1/3} = \underline{-6\sqrt{2}\ a^{1/6}\ b^{7/12}}$

5



Areal 12

Arealet $A = \frac{1}{2} k_1 \cdot k_2$

$12 = \frac{1}{2} k_1 (3k_1)$

$\frac{12 \cdot 2}{3} = k_1^2$

$k_1 > 0$

$k_1^2 = 8$

$k_1 = \sqrt{8} = \sqrt{4}\sqrt{2} = \underline{\underline{2\sqrt{2}}}$

$k_2 = 3k_1 = \underline{\underline{6\sqrt{2}}}$

$h^2 = k_1^2 + k_2^2 = k_1^2 + (3k_1)^2$

$= 10k_1^2 = 10 \cdot 8 = 80$

$= 5 \cdot 16$

hypotenus $h = \sqrt{16 \cdot 5} = \underline{\underline{4\sqrt{5}}}$

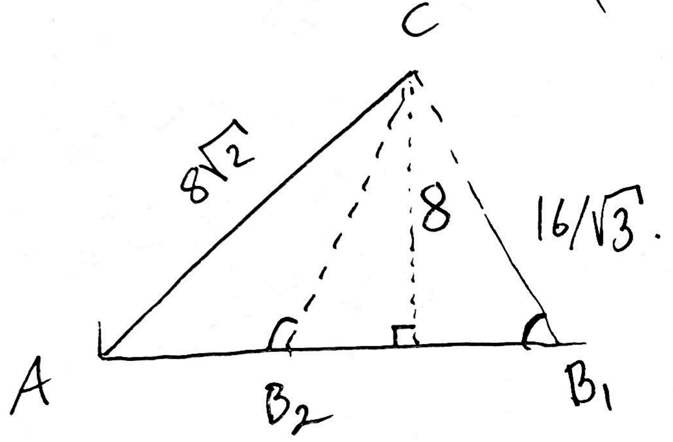
6 $\triangle ABC$

$|AC| = 8\sqrt{2}$

$|BC| = 16/\sqrt{3}$

$\angle A = 45^\circ$

Bestem vinklene.



(høyden $8\sqrt{2} \cdot \sin(45^\circ)$
$= 8$)

$\sin(\angle B) \cdot \frac{16}{\sqrt{3}} = 8$

$\sin \angle B = \frac{8}{16} \cdot \sqrt{3} = \frac{\sqrt{3}}{2}$.

så $\angle B_1 = \underline{60^\circ}$

$\angle B_2 = 180^\circ - 60^\circ = \underline{120^\circ}$

To $\triangle ABC$:

$\angle A = 45^\circ$	$\angle B_1 = 60^\circ$	$\angle C_1 = 75^\circ$
$\angle A = 45^\circ$	$\angle B_2 = 120^\circ$	$\angle C_2 = \underline{15^\circ}$

7 a) $x^2 < 3x \leq 4$

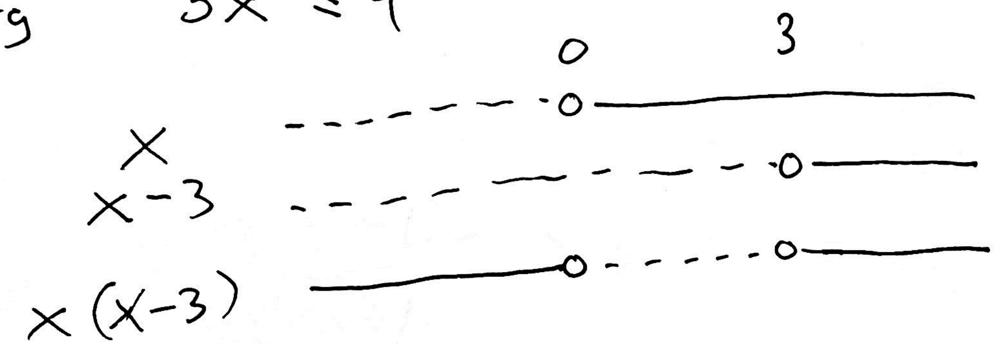
$\Leftrightarrow \quad x^2 < 3x \quad$ og $\quad 3x \leq 4$

I

$x^2 < 3x$

$x^2 - 3x < 0$

$x(x-3) < 0$



II

$3x \leq 4 \quad \Leftrightarrow \quad x \leq \frac{4}{3}$

Løsningsmengden er $\langle 0, 3\rangle \cap \langle -\infty, 4/3]$

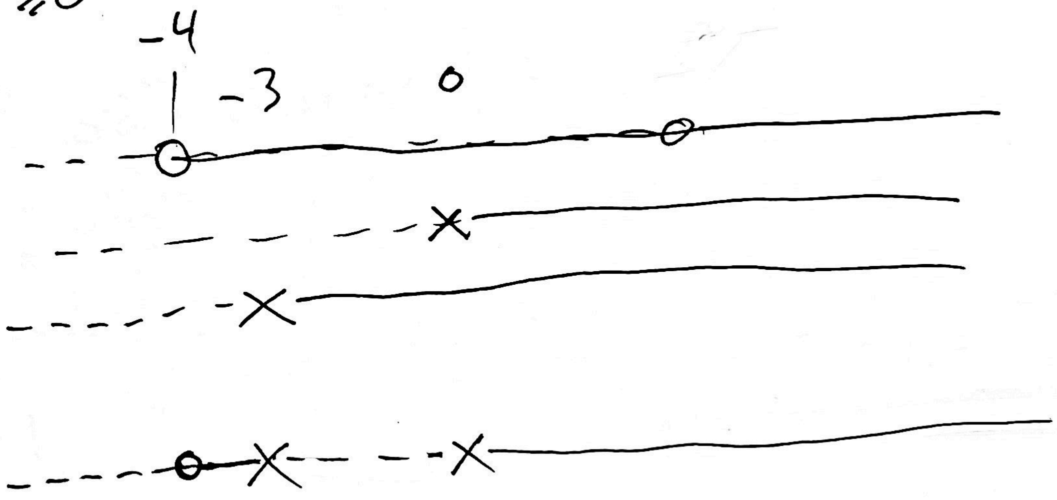
$\underline{\langle 0, 4/3]}$

7b) $\frac{4}{x} \geq \frac{1}{x+3}$ $\qquad x \neq 0, -3$

$\frac{4}{x} - \frac{1}{x+3} \geq 0$

$\frac{4(x+3) - x}{x(x+3)} \geq 0 \iff \frac{3x+12}{x(x+3)} \geq 0$

$\frac{3(x+4)}{x(x+3)} \geq 0$



$3(x+4)$

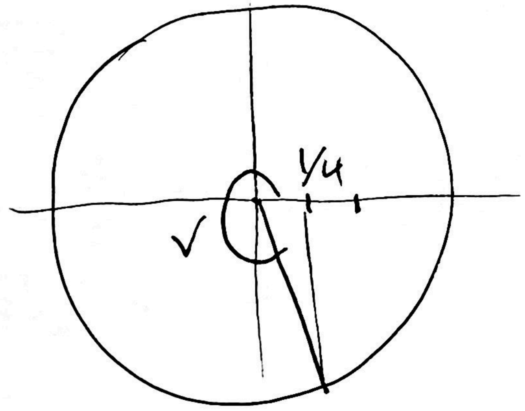
$1/x$

$1/(x+3)$

$\frac{3(x-4)}{x(x+3)}$

Løsningsmengden er $\underline{\underline{x \in [-4,-3\rangle \cup [0, \infty\rangle}}$

8 $\cos v = \frac{1}{4}$ I fjerde kvadrant



Pytagoras: $\sin^2 v + \cos^2 v = 1$

$\sin^2 v + \left(\frac{1}{4}\right)^2 = 1$

$\sin^2 v = 1 - \frac{1}{16} = \frac{15}{16}$

$\sin v = \pm\sqrt{\frac{15}{16}} = \pm\frac{\sqrt{15}}{\sqrt{16}} = \pm\frac{\sqrt{15}}{4}$

I fjerde kvadrant, så $\sin v < 0$

$\underline{\sin v = \frac{-\sqrt{15}}{4}}$

$\cos(2v) = 2\cos^2 v - 1 = 2\cdot\left(\frac{1}{4}\right)^2 - 1 = \frac{1}{8} - 1 = \underline{\frac{-7}{8}}$

8 $\tan v = \frac{\sin v}{\cos v} = \frac{-\sqrt{15}/4}{1/4} = \underline{-\sqrt{15}}$

$\sin\left(v+\frac{\pi}{3}\right) = \sin v \cdot \cos\frac{\pi}{3} + \sin\frac{\pi}{3} \cdot \cos v$

$\frac{-\sqrt{15}}{4} \cdot 1/2 + \sqrt{3}/2 \cdot \frac{1}{4}$

$-\frac{\sqrt{15}}{8} + \frac{\sqrt{3}}{8}$

$= \underline{\frac{1}{8}\left(\sqrt{3}-\sqrt{15}\right)}$

9 a) $A(1,1,-1)$ $\quad B(3,0,2)$

$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = [3,0,2]-[1,1,-1]$

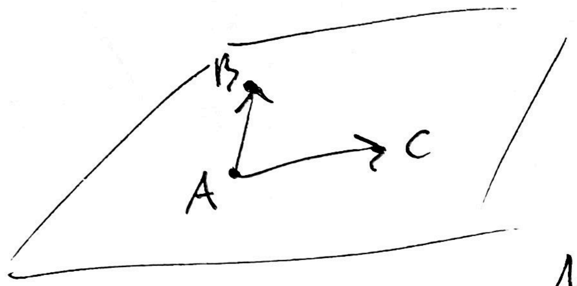
$\overrightarrow{AB} = \underline{[2,-1,3]}$

$|\overrightarrow{AB}| = \sqrt{2^2+(-1)^2+3^2} = \sqrt{4+1+9} = \underline{\sqrt{14}}$

9b) $\vec{AB} = [2,-1,3]$ og $\vec{AC} = \vec{OC} - \vec{OA}$

$= [3,2,1] - [1,1,-1]$

$= [2,1,2]$



Normal vektor til planet:

$$\vec{n} = \vec{AB} \times \vec{AC} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & -1 & 3 \\ 2 & 1 & 2 \end{vmatrix} = \left[\begin{vmatrix} -1 & 3 \\ 1 & 2 \end{vmatrix}, -\begin{vmatrix} 2 & 3 \\ 2 & 2 \end{vmatrix}, \begin{vmatrix} 2 & -1 \\ 2 & 1 \end{vmatrix} \right]$$

$$\vec{n} = [-5, 2, 4]$$

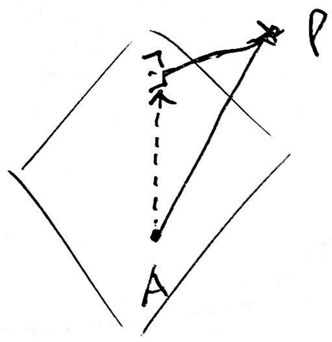
Planet: $-5x + 2y + 4z = d$

$(3,0,2)$ ligger i planet, så $-15 + 0 + 8 = d$

$-7 = d$

Planet gjennom A, B og C : $\underline{\underline{-5x + 2y + 4z = -7}}$

9c)



$P(1,0,2)$

$\vec{AP} = \vec{OP} - \vec{OA}$

$= [1,0,2] - [1,1,-1]$

$= \underline{[0,-1,3]}$

$\vec{n} = [-5,2,4]$

Komponenten til $\vec{AP}$ langs $\vec{n}$:

$$\frac{\vec{AP} \cdot \vec{n}}{|\vec{n}|^2}\vec{n}$$

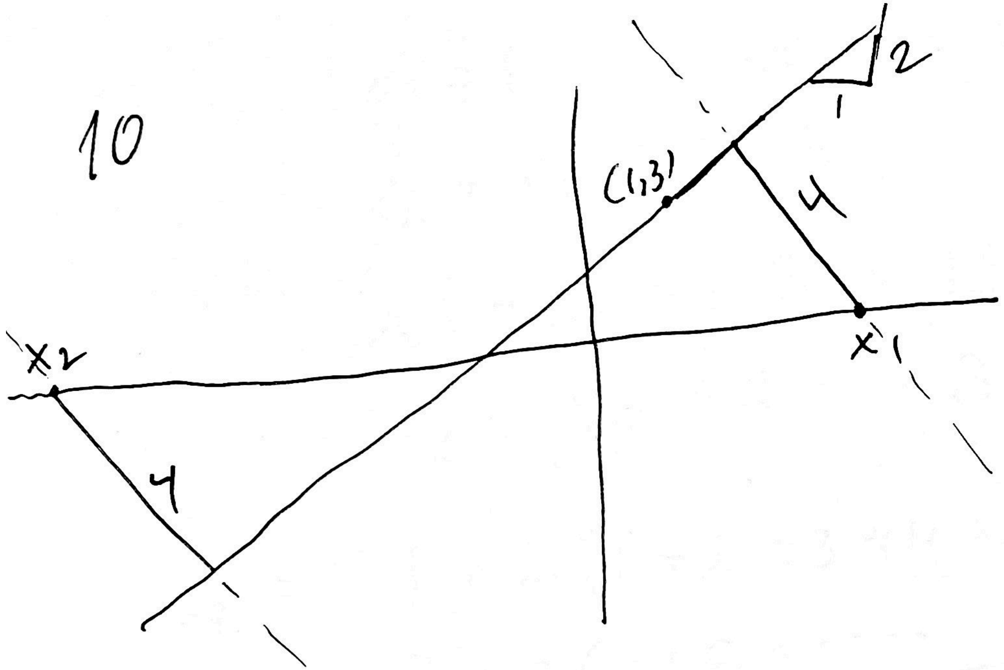
Korteste avstand fra P til planet

$$\left|\frac{\vec{AP} \cdot \vec{n}}{|\vec{n}|^2}\vec{n}\right| = \left|\frac{\vec{AP} \cdot \vec{n}}{|\vec{n}|}\right|$$

$$= \left|\frac{[0,-1,3] \cdot [-5,2,4]}{\sqrt{25+4+16}}\right| = \frac{|-2+12|}{\sqrt{45}}$$

$$= \frac{10}{\sqrt{9 \cdot 5}} = \frac{10}{3\sqrt{5}} = \frac{2 \cdot 5}{3\sqrt{5}} = \underline{\frac{2\sqrt{5}}{3}}$$

10



Likning for linjen

normalvektor $[-2,1]$

$-2x+y = \text{konst}.$

gjennom $(1,3)$

$-2\cdot 1+3 = 1$

$$\underline{-2x+y=1}$$

fra $(x,0)$ gå ut en lengde 4 langs normalvektoren $[-2,1]$.

$$[x,0] \pm 4\frac{[-2,1]}{\sqrt{5}} = \left[x \pm \frac{-2\cdot 4}{\sqrt{5}}, \pm\frac{4}{\sqrt{5}}\right]$$

Disse ligger på linjen når

$$-2\left(x \pm \frac{-8}{\sqrt{5}}\right) + \pm\frac{4}{\sqrt{5}} = 1$$

$$-2x \underbrace{\pm \frac{16}{\sqrt{5}} \pm \frac{4}{\sqrt{5}}} = 1$$

$$-2x \pm \frac{20}{\sqrt{5}} = 1$$

så $2x = -1 \pm \frac{20}{\sqrt{5}}$

$x = \frac{1}{2}(-1 \pm 4\sqrt{5})$

$$\underline{\underline{x = \frac{-1}{2} \pm 2\sqrt{5}}}$$

11

$x_1 = 1 \qquad x_{n+1} = x_n + 2n$

$x_2 = x_1 + 2 \cdot 1 = 3 \qquad x_3 = x_2 + 2 \cdot 2 = 7$

$x_4 = x_3 + 2 \cdot 3 = 13 \qquad x_5 = x_4 + 2 \cdot 4 = 21$

$x_6 = x_5 + 2 \cdot 5 = 31 \qquad x_7 = x_6 + 2 \cdot 6 = 43.$

$x_7 = 1 + 2(1 + 2 + 3 + 4 + 5 + 6)$

$x_n = 1 + 2(\underbrace{1 + 2 + \cdots + (n-1)})$

$1 + 2 \cdot \frac{n(n-1)}{2} = n(n-1) + 1$

$x_n = \underline{\underline{n^2 - n + 1}}$

12

P_0

1 januar 2003 . . . . 1.01 2018

$P_0 + P_0(1+r) + P_0(1+r)^2 + \dots \quad P_0(1+r)^{15}$

pengemengden i 1.1.2018

Ved utgangen av 2023 :

$$(1+r)^6 P_0\Big(\underbrace{1+(1+r)+\dots(1+r)^{15}}_{\frac{(1+r)^{16}-1}{1+r-1}}\Big)$$

$$P = \frac{P_0(1+r)^6}{r}\cdot\left((1+r)^{16}-1\right)$$