10.des. Løsningsforslag oblig 4 2021

1 a)

$$y - y^2 + y^3 - y^4 + y^5 - y^6 \qquad - 1$$

$$1 - y + y^2 - y^3 + y^4 - y^5$$

① $= 1 - 1 - y^6 = \underline{-y^6}$

(kan og benytte $\sum_{n=0}^{5} (-y)^n = \frac{1-(-y)^6}{1-(-y)}$)

b)

$4^2 + b \cdot 4 + 3 = 0 \iff 4$ er rot

$$16 + 4b + 3 = 0$$

$$4b = -19$$

$$\underline{b = \frac{-19}{4}} = -5 + \frac{1}{4}$$

$$= \underline{-4.75}$$

c)

$$\frac{a\sqrt[3]{a}\,\sqrt[5]{a^2}}{\sqrt{a^3}} = \frac{a \cdot a^{1/3} \cdot (a^2)^{1/5}}{(a^3)^{1/2}}$$

$\left(\frac{1}{a} = a^{-1}\right)$

$$= a^{1 + \frac{1}{3} + \frac{2}{5} - \frac{3}{2}} = a^{\frac{1}{30}(30 + 10 + 12 - 45)}$$

$$= a^{\frac{1}{30}(7)} = \underline{a^{7/30}}$$

1 d) $\dfrac{-6(x-3)^2(\sqrt{5}+x)(\sqrt{5}-x)}{15(x+\sqrt{5})((x-2)(x-3))^2}$

2

$= \dfrac{-2\cdot(\sqrt{5}-x)}{5\,(x-2)^2}$

$= \dfrac{2(x-\sqrt{5})}{5(x-2)^2}$

e) $\sqrt{2x+6} = x+2$ kvadrerer begge sider av likhetstegnet.

$\Rightarrow \quad 2x+6 = (x+2)^2$

$= x^2+4x+4$

$\Leftrightarrow \quad x^2+2x-2 = 0$ Fullfører kvadratet

$\Leftrightarrow \quad (x+1)^2 - 3 = 0$

$(x+1)^2 = 3$

$(x+1) = \pm\sqrt{3}$

$x_1 = -1-\sqrt{3}$ ~ -2.73

$x_2 = \sqrt{3}-1$ ~ 0.73

sjekker løsningene.

x_1 falsk x_2 er løsning.

Den irrasjonale likningen har løsning

$\underline{x = \sqrt{3}-1}$

1f) L1 $x^2+3y = 7$

L2 $x+y = 1$

L2 gir $y = 1-x$

setter inn i L1: $x^2+3(1-x) = 7$

$$x^2-3x+3-7=0$$

3

$$x^2-3x-4=0$$

$$(x+1)(x-4)=0$$

$x=-1$ og $y = 1-(-1) = 2$

$x=4$ og $y = 1-4 = -3.$

Løsningene er $(x,y) = \underline{(-1,2)}$

og $(x,y) = \underline{(4,-3)}$

g) $\underbrace{\sin(2x)}$ $= \cos x$ $0 \le x \le 2\pi$

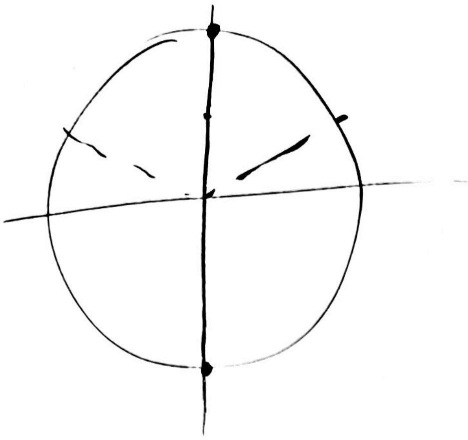
$$2\sin x \cdot \cos x = \cos x$$

$$\cos x(2\sin x - 1) = 0$$

$\Leftrightarrow$ $\cos x = 0$

eller $2\sin x - 1 = 0 \Leftrightarrow \sin x = \frac{1}{2}.$

1g



4

$\cos x = 0$: $x = \frac{\pi}{2}$ eller $\frac{3\pi}{2}$

$\sin x = \frac{1}{2}$ $x = \frac{\pi}{6}$ eller $\frac{5\pi}{6}$

Løsningene er $\underline{x \in \left\{ \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}, \frac{3\pi}{2} \right\}}$

h)

$$x^3 - 2x^2 + x - 2 : x^2 - 2 = \underline{x - 2 + \frac{3(x-2)}{x^2-2}}$$

$$\begin{array}{l} x^3 - 2x^2 + x - 2 \\ \underline{x^3 \qquad\quad - 2x} \\ -2x^2 + 3x - 2 \\ \underline{-2x^2 \qquad + 4} \\ \qquad 3x - 6 \end{array}$$

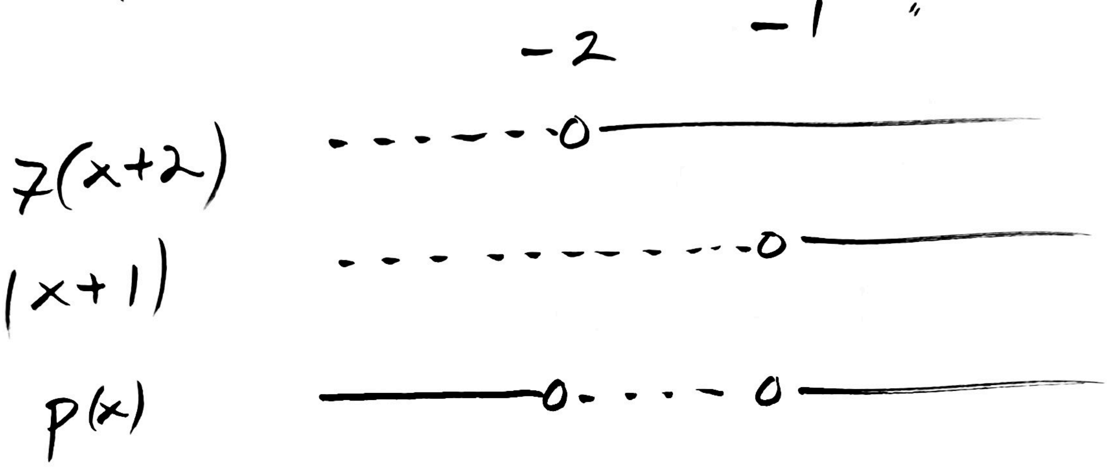
Når x blir stor vil resten $\frac{3(x-2)}{x^2-2} \to 0$

$|q(x) - (x-2)| \to 0$ når $x \to \infty$, $x \to -\infty$

så grafen til $q(x)$ nærmer seg grafen til linjen $y = x - 2$ for $|x|$ stor.

2a) $p(x) = 7(x^2 + 3x + 2)$
$= 7(x+2)(x+1)$

5 $7(x+2)(x+1) \leq 0$



Løsningen er $\underline{x \in [-2, -1]}$

2b) $$\frac{x+1}{x^2-4} \geq 1$$

$$\frac{x+1}{x^2-4} - 1 \geq 0$$

$$\frac{x+1}{x^2-4} - \frac{x^2-4}{x^2-4} \geq 0$$

$$\frac{-(x^2-x-5)}{(x+2)(x-2)} \geq 0$$

$$x^2 - x - 5 = \left(x - \frac{1}{2}\right)^2 - \frac{1}{4} - \frac{5 \cdot 4}{4}$$
$$= \left(x - \frac{1}{2}\right)^2 - \frac{21}{4}$$
$$= \left(x - \frac{1}{2}\right)^2 - \left(\frac{\sqrt{21}}{2}\right)^2$$

Ulikheten:

6

$$(*) \quad \frac{+\left(x - \frac{1}{2} + \frac{\sqrt{21}}{2}\right)\left(x - \frac{1}{2} - \frac{\sqrt{21}}{2}\right)}{(x+2)(x-2)} \leq 0$$

$\left(\sqrt{21} \sim 4{,}58\right)$

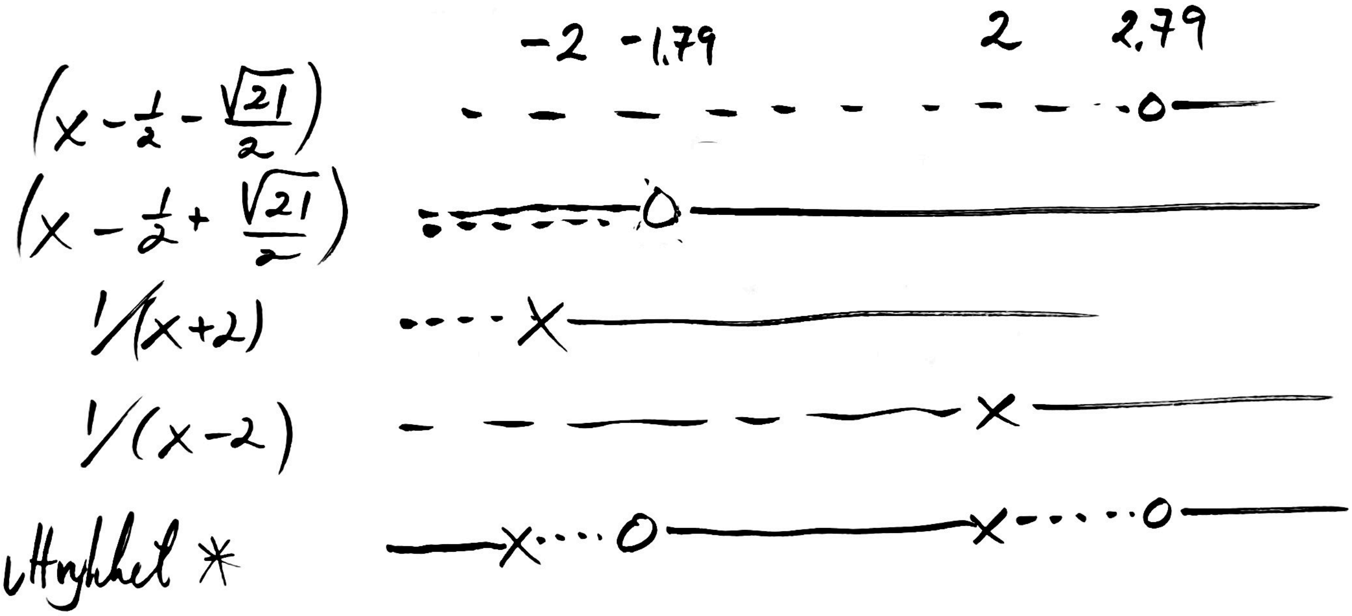
Telleren har røtter: $\frac{1}{2} + \frac{\sqrt{21}}{2}$

~ 2.79

og $\frac{1}{2} - \frac{\sqrt{21}}{2}$

$= 1 - \left(\frac{1}{2} + \frac{\sqrt{21}}{2}\right)$

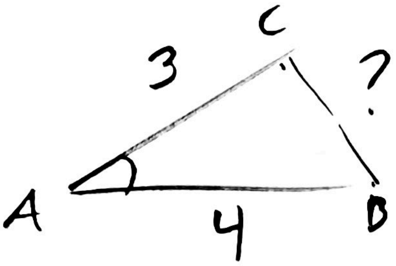
~ -1.79



Løsningen er $x \in \left(-2, \frac{1}{2} - \frac{\sqrt{21}}{2}\right] \cup \left(2, \frac{1}{2} + \frac{\sqrt{21}}{2}\right]$

3 a)

7



cos setning

$$|BC|^2 = 3^2 + 4^2 - 2 \cdot 3 \cdot 4 \overbrace{\cos(\angle A)}^{\sqrt{3}/2}$$

$$= 25 - 12 \cdot \sqrt{3}$$

$$|BC| = \underline{\sqrt{25 - 12\sqrt{3}}} \sim \underline{2.053}$$

Sin setning $\quad \dfrac{\sin(\angle B)}{3} = \dfrac{\sin(\angle A)}{|BC|}$

$$\sin(\angle B) = \frac{3}{|BC|} \underbrace{\sin(30°)}_{1/2}$$

$$= \frac{3/2}{|BC|} \qquad (\angle B < 90°)$$

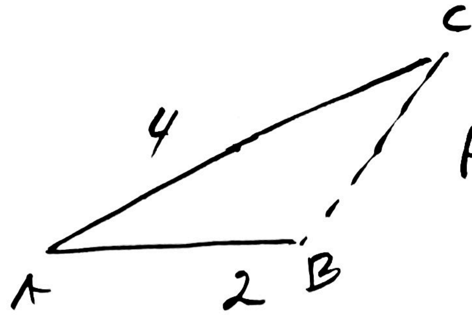
$$\sim 0.7306$$

$$\angle B = 46.9°$$

$$\angle C = 180° - \angle A - \angle B$$

$$= 180° - 30° - 46.9°$$

$$= \underline{103.1°}$$

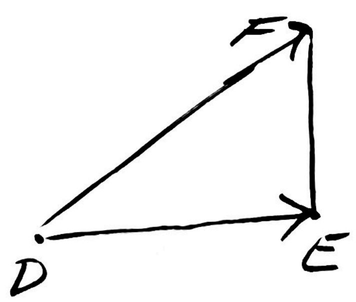


Areal $\sqrt{12}$
$= 2\sqrt{3}$.

$$\text{Areal} = \sqrt{12} = \frac{1}{2} \cdot 2 \cdot 4 \cdot \sin(\angle A)$$
$$2\sqrt{3} = 4 \sin(\angle A)$$
$$\sin(\angle A) = \frac{\sqrt{3}}{2}$$
$$\angle A = \underline{60^\circ} \quad \text{eller} \quad \underline{120^\circ}$$
$(180^\circ - 60^\circ)$

Mulige vinkler $\angle A$ er
$\underline{60^\circ \text{ og } 120^\circ}$.

3c) Trekant DEF
D (2,4,1) og F(-1,-3,2)
E (-3,3,1)



$$\text{Areal } A = \frac{1}{2} |\vec{DE} \times \vec{DF}|$$

$$\vec{DE} = \vec{OE} - \vec{OD}$$
$$= [-3,3,1] - [2,4,1]$$
$$= [-5,-1,0]$$

$$\vec{DF} = \vec{OF} - \vec{OD}$$
$$= [-1, -3, 2] - [2, 4, 1]$$
$$= [-3, -7, 1]$$

(9)

$$\vec{DE} \times \vec{DF} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -5 & -1 & 0 \\ -3 & -7 & 1 \end{vmatrix}$$

$$= (-1)^2 \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 5 & 1 & 0 \\ 3 & 7 & -1 \end{vmatrix}$$

$$= [-1, -(-5), 32]$$

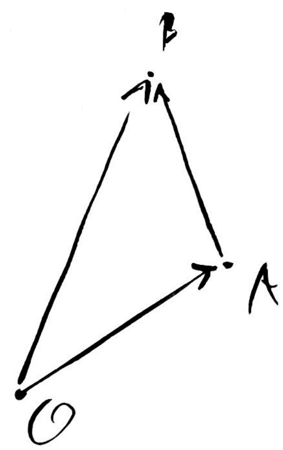
Arealet til ΔDEF er lik

$$\frac{1}{2} |[-1, 5, 32]|$$
$$= \frac{1}{2}\sqrt{1 + 25 + 1024} \; \underline{\underline{= \frac{1}{2}\sqrt{1050}}}$$

$$\left(\begin{aligned} (32)^2 = (2^5)^2 &= 2^{10} = 1024 \\ (30+2)^2 &= 900 + 2 \cdot 2 \cdot 30 + 2^2 \\ &= 1024 \end{aligned} \right)$$

4. $\vec{OB} = [3, -2, 0]$

$B(3, -2, 0)$



$\vec{AB} = [1, -1, 2]$

(10)

$= \vec{OB} - \vec{OA}$

Så $\vec{OA} = \vec{OB} - \vec{AB}$

$= [3, -2, 0] - [1, -1, 2]$

$= [2, -1, -2]$

$A(2, -1, -2)$

$3\vec{BC} = [0, -1, 3]$

$3\vec{OC} = 3\vec{OB} + 3\vec{BC}$

$= 3(\vec{OB} + \vec{BC})$

$= 3[3, -2, 0] + [0, -1, 3]$

$= [9, -7, 3]$

$\vec{OC} = \frac{1}{3}(3\vec{OC}) = \frac{1}{3}[9, -7, 3]$

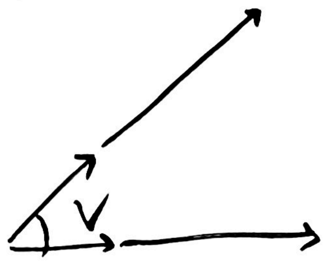
$= [3, \frac{-7}{3}, 1]$

$C(3, \frac{-7}{3}, 1)$

4b) Vinkel mellom $[10, 15, 5] = 5[2,3,1]$

og $[-4, 4, 2] = 2[-2,2,1]$

(11)



samme som vinkelen mellom

$[2,3,1]$ og $[-2,2,1]$

$$\cos V = \frac{[2,3,1] \cdot [-2,2,1]}{|[2,3,1]| \cdot |[-2,2,1]|}$$

$$= \frac{-4+6+1}{\sqrt{4+9+1}\sqrt{4+4+1}}$$

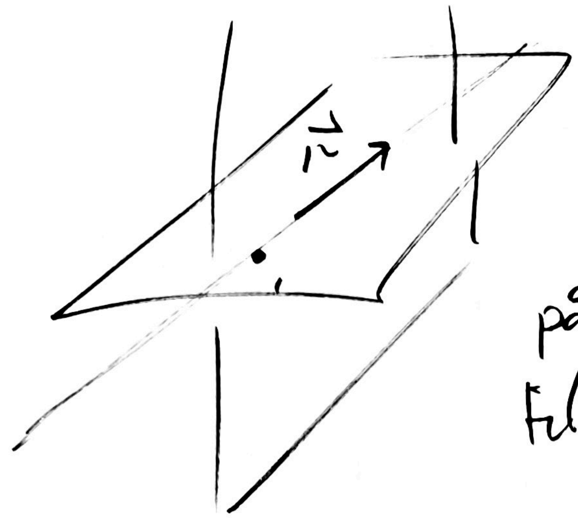
$$= \frac{3}{\sqrt{14} \cdot \sqrt{9}} = \underline{\frac{1}{\sqrt{14}}}$$

$$V = \underline{\arccos\left(\frac{1}{\sqrt{14}}\right)}$$

$$= \underline{74{,}499^\circ}$$

4c)

12



$\vec{r} \perp$ på normalvektorene til begge plana.

$\vec{n}_1 = [1, -3, 1]$ $\qquad x - 3y + z = 2$

$\vec{n}_2 = [2, 3, -4]$ $\qquad 2x + 3y - 4z = 1$

$$\vec{n}_1 \times \vec{n}_2 = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & -3 & 1 \\ 2 & 3 & -4 \end{vmatrix}$$

$$= [9, +6, 9] = 3[3, +2, 3].$$

Velger $\vec{r} = \underline{[3, +2, 3]}$

Finner felles punkt summen av likningene

$3x - 3z = 3$

$x - z = 1$

$x = 1 + z$

$\left(\frac{3}{2}, 0, \frac{1}{2}\right)$ eller $\left(1, -\frac{1}{3}, 0\right)$

$1+z-3y+z=2$

$2z-3y=2-1=1$

$y=-1, \quad z=-1$

$x=1+z=0$

13

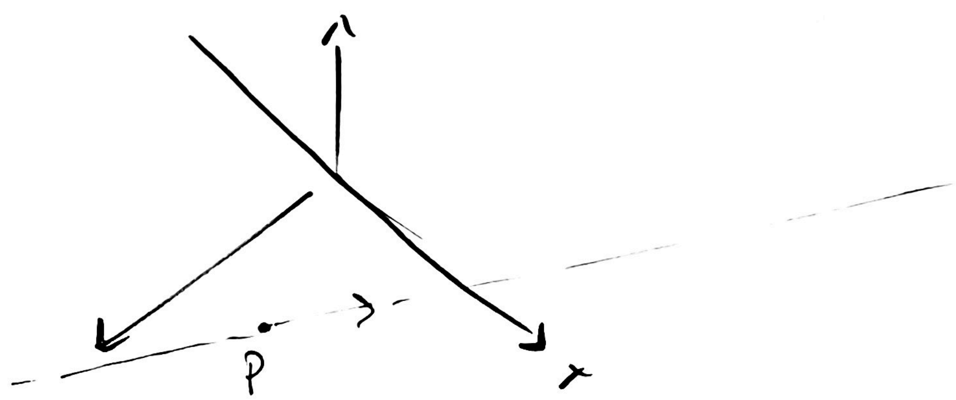
$(0,-1,-1)$ er og i begge plana.

Linja er parametrisert av

$[x,y,z]=[0,-1,-1]+t[3,2,3]$

$$x = 3t$$
$$y = -1+2t$$
$$z = -1+3t$$

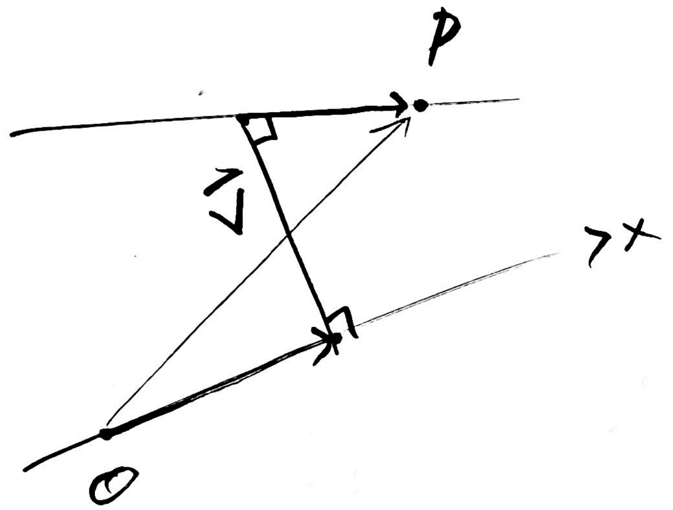
d) Kortest avstand fra x-aksen til linjen gjennom $P(1,0,3)$ med retningsvektor $\vec{r}=[2,1,-4]$.



Linjestykke som realiserer korteste avstand står vinkelrett på begge linjene.

Linjen x-aksen har retningsvektor [1,0,0] sevat $\vec{v} = [0,4,1]$ er en slik vektor (alternativt ta $[1,0,0] \times [2,1,4]$)

(14)



| Komponenten til $\vec{OP}$ langs $\vec{v}$ |
= korteste avstand mellom linjene.

$\vec{OP} = [1,0,3]$ $\quad \vec{v} = [0,4,1]$

korteste avstand: $\left| \dfrac{\vec{OP} \cdot \vec{v}}{|\vec{v}|} \right|$

$$= \left| \frac{[1,0,3] \cdot [0,4,1]}{|[0,4,1]|} \right| = \underline{\underline{\frac{3}{\sqrt{17}}}}$$

(litt kortere enn 3/4)

5 $\quad 6 = 2\cdot 3 \qquad 15 = 3\cdot 5$

n er delelig med 6 og 15

$\Leftrightarrow$ n delelig med $2\cdot 3\cdot 5 = \underline{30}$

15 Sum av tall på forma $30\cdot n$, $n \in \mathbb{Z}$

mellom 1000 og 4000.

Hvilke verdi starter vi med?

$30\cdot 30 = 900$

$30\cdot 33 = 990 \qquad 30\cdot 34 = \underline{1020}$

$30\cdot 133 = 3000 + 990 \qquad 30\cdot 134 = 4020$

$= \underline{3990}$

Summa e $\sum_{n=34}^{133} 30\cdot n$

$$= 30 \underbrace{\sum_{n=34}^{133} n}_{\sum_{m=1}^{100} (33+m)}$$

$$= 30\left[33\cdot 100 + \underbrace{\sum_{m=1}^{100} m}_{\frac{100\cdot 101}{2}}\right]$$

$$= 30\left[3300 + 5050\right]$$

$$= 30(8350) = \begin{pmatrix} 24\,000 \\ +\ 1\,050 \end{pmatrix}\cdot 10$$

$$= \underline{250\,500}$$

Legger sammen sum av etterfølgende par

$$\sum_{m=1}^{10^9/2} \left(\underbrace{-(2m)^2 + (2m-1)^2}_{-4m^2 + 4m^2 - 4m + 1} \right)$$

(16)

$$\sum_{m=1}^{10^9/2} -4m + 1$$

$$= -4 \sum_{m=1}^{10^9/2} m + \sum_{m=1}^{10^9/2} 1$$

$$= -4 \frac{10^9/2\,(10^9/2 + 1)}{2} + 10^9/2$$

$$\frac{10^9}{2}\left(-2\left(\frac{10^9}{2} + 1 \right) + 1 \right)$$

$$= \frac{10^9}{2}\left(-2\cdot\frac{10^9}{2} - 1 \right)$$

$$= -\frac{10^9}{2}(10^9 + 1)$$

6 $\quad S = 10{,}000$

P_n pengemengde etter n år.

$P_1 = S(1{,}05)$

(17)

$P_2 = S(1{,}05)^2 + S(1{,}05)$

$\vdots$

$P_n = S\left((1{,}05) + (1{,}05)^2 + \dots (1{,}05)^n\right)$

$= S(1{,}05)\left(1 + (1{,}05) + \dots (1{,}05)^{n-1}\right)$

$= S(1{,}05)\dfrac{(1{,}05)^n - 1}{1{,}05 - 1}$ $\quad // \dfrac{1}{1.05-1} = \dfrac{1}{0.05} = 20$

$= S\cdot(1{,}05)\cdot 20\,(1{,}05^n - 1)$

minste n slik at

$P_n \geq 200\,000$

$(1{,}05)\cdot 20\,((1{,}05)^n - 1) \geq \dfrac{200000}{10\,000} = 20$

$(1{,}05)^n - 1 \geq \dfrac{1}{1{,}05}$

$(1{,}05)^n \geq 1 + \dfrac{1}{1{,}05}$

minste heltallig n er $\underline{n = 14}$

Vi passer 200 000 kr i $\underline{2035}$