(1) Eksamen 2015 august

Treningsoppgaver (som ligner på eksamensoppgavene)

1 a) $A = \begin{bmatrix} 1 & 2 & 3 \\ -1 & 0 & 4 \end{bmatrix}$

Finn $-5A$ og $A \cdot A^T$

$$-5A = \begin{bmatrix} -5 & -10 & -15 \\ 5 & 0 & -20 \end{bmatrix}$$

$$A \cdot A^T = \begin{bmatrix} 1 & 2 & 3 \\ -1 & 0 & 4 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 2 & 0 \\ 3 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 1+2^2+3^2 \, , & -1 \cdot 1 + 0 + 3 \cdot 4 \\ -1 \cdot 1 + 0 + 4 \cdot 3 \, , & (-1)^2 + 0 + 4 \cdot 4 \end{bmatrix}$$

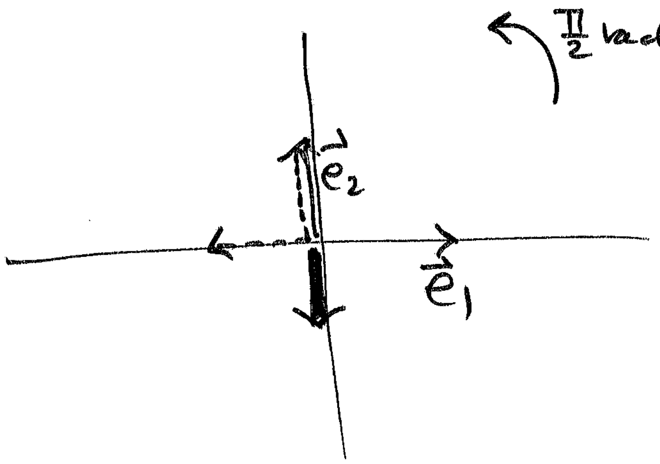
$$= \begin{bmatrix} 14 & 11 \\ 11 & 17 \end{bmatrix}$$

(2)

$$M\begin{bmatrix} x \\ y \end{bmatrix} = T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right)$$

$$M = \left[\vec{V}_1, \vec{V}_2\right] \qquad M\begin{bmatrix} x \\ y \end{bmatrix} = x\vec{V}_1 + y\vec{V}_2$$

$$\vec{V}_1 = T\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right) = T(\vec{e}_1), \quad V_2 = T\left(\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right) = T(\vec{e}_2).$$




$$T(\vec{e}_1) = -\vec{e}_2$$
$$T(\vec{e}_2) = -\vec{e}_1$$

Standardmatrisen til $T$ er

$$\left[T(\vec{e}_1), T(\vec{e}_2)\right] = \underline{\begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}}$$

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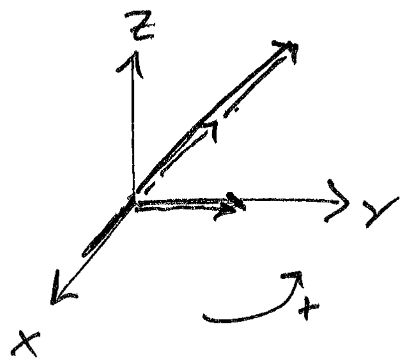
$$T : \mathbb{R}^3 \longrightarrow \mathbb{R}^3$$

1) Roter $\frac{\pi}{2}$ rad i pos. retning om $z$-aksen
2) skaler med en faktor 2 langs $x$-aksen.




$$T(\vec{e}_3) = \vec{e}_3$$
$$T(\vec{e}_1) = \vec{e}_2$$
$$T(\vec{e}_2) = -2\vec{e}_1$$

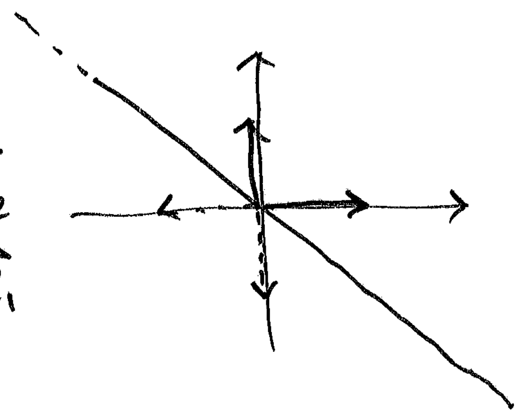
standardmatrisen er

$$\begin{bmatrix} 0 & -2 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

2015 H

1 b) Refleksjon (speiling) om aksen $y = -x$ er en lineær transformasjon.

③ Finn standardmatrisen.



Transformasjonen sender $\vec{e}_1$ til $-\vec{e}_2$

$\vec{e}_2$ til $-\vec{e}_1$

Standardmatrisen er $\underline{\underline{\begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}}}$

2 $$f(x) = \begin{cases} a \ln x & x \geq 1 \\ x^2 + b & x < 1 \end{cases}$$

Bestem parametrene a og b slik at $f(x)$ er deriverbar for alle $x$.

Kontinuitet i $x = 1$: $\lim_{x \to 1^-} f(x) = f(1) = \lim_{x \to 1^+} f(x)$

$$a \cdot \ln(1) = \lim_{x \to 1^-} x^2 + b = 1 + b$$

$$0 = 1 + b. \qquad \text{så } b = -1.$$

Deriverbar i $x = 1$: $\lim_{h \to 0^+} \frac{f(1+h) - f(1)}{h} = \lim_{h \to 0^-} \frac{f(1+h) - f(1)}{h}$

derivert fra høyre side

$$\lim_{h\to 0^+} \frac{a\cdot \ln(1+h) - (a\ln(1))}{h} = (a\ln x)'\Big|_{x=1}$$

$$= a\frac{1}{x}\Big|_{x=1} = \underline{a}$$

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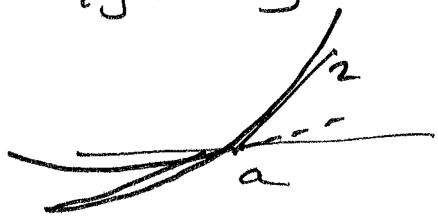
derivert fra venstre side

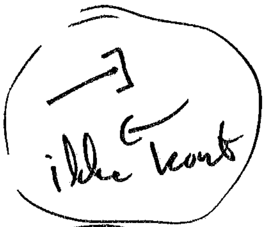
$$\lim_{h\to 0^-} \frac{(1+h)^2 + b - (a\ln 1)}{h}$$

$$= \lim_{h\to 0^-} \frac{(1+h)^2 - 1}{h} = (x^2+b)'\Big|_{x=1}$$

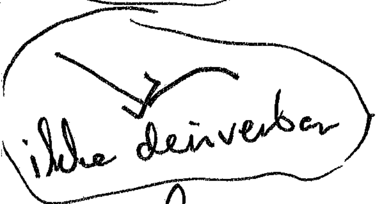
$$= 2x\Big|_{x=1} = 2$$

De deriverte fra høyre og venstre skal være lik




$$\underline{a = 2}$$




$$f(x) = \begin{cases} 2\ln x & x \geq 1 \\ x^2 - 1 & x < 1 \end{cases}$$

er deriverbar for alle $x$.

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$$f'(x) = \begin{cases} a/x & x > 1 \\ 2x & x < 1 \end{cases}$$

sjekker at $\lim_{x\to 1^+} f'(x) = \lim_{x\to 1^-} f'(x)$

$$\underline{a = 2}$$

3 a) Løs likningen

5

$$3z+1 = 2z+2i$$

og skriv løsningen på kartesisk og polar form

b) Faktoriser $3z^3 - 12z$

og $3z^3 + 12z$

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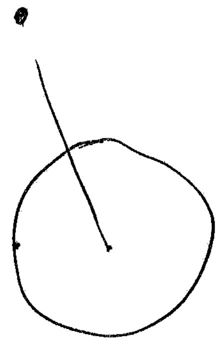


a) $$3z+1 = 2z+2i$$

$$3z-2z = -1+2i$$

$$\underline{z = -1+2i}$$

$$z = re^{i\theta}$$

$$r = \sqrt{|z|^2} = \sqrt{z\cdot\bar{z}} = \sqrt{(-1)^2+2^2} = \underline{\sqrt{5}}$$

$$\theta = \pi + \arctan\left(\frac{2}{-1}\right) = \pi + \arctan(-2)$$

$$\underline{z = \sqrt{5}\, e^{i(\pi+\arctan(-2))}}$$

b) * $$3z(z^2-4) = \underline{3z(z-2)(z+2)}$$

* $$3z(z^2+4) = 3z(z^2-(2i)^2)$$

$$= \underline{3z(z+2i)(z-2i)}$$

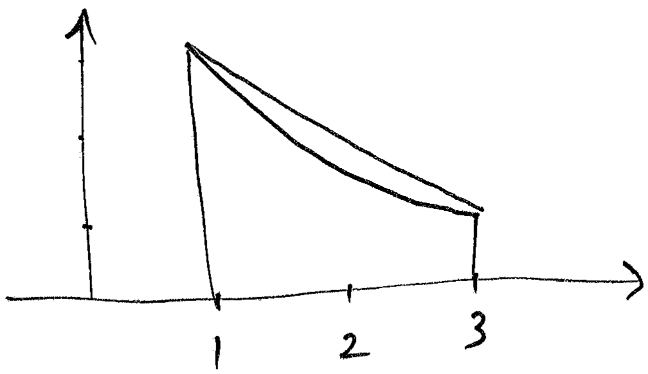
6 Estimer integralet $\int_1^3 \frac{1}{x}\,dx$

1) ved bruk av Simpsons metode (med ett dobbelt intervall)

$\left( \frac{1}{6}(1+4+1) \right)$

2) ved bruk av Trapesmetoden med ett intervall

(Eksakt verdi er $\int_1^3 \frac{1}{x}\,dx = \ln x \Big|_1^3 = \ln 3 \approx 1.0986\ldots$)




1) Estimat ved bruk av Simpsons metode

bredde

$$\underset{\text{bredde}}{2} \cdot \frac{1}{6}\left(1 \cdot \frac{1}{1} + 4 \cdot \frac{1}{2} + 1 \cdot \frac{1}{3}\right)$$

$$= \frac{1}{3}\left(1 + 2 + \frac{1}{3}\right) = 1 + \frac{1}{9} = \underline{\underline{1.111\ldots}}$$

2) Estimat ved bruk av trapesmetoden

$$2 \cdot \frac{1}{2}\left(\frac{1}{1} + \frac{1}{3}\right) = 1 + \frac{1}{3} = \underline{\underline{1.33\ldots}}$$

7

$$\int_0^1 x\,\underbrace{(1+x^2)^6}_{u^6}\,dx \qquad \frac{u'}{2}\cdot u^6$$

$u = 1+x^2$

$u' = 2x$

7

$$\int_0^1 \frac{1}{2}u^6 \underbrace{u'dx}_{du} = \int_{u(0)}^{u(1)} \frac{1}{2}u^6\,du$$

$$= \frac{1}{2}\frac{u^7}{7}\Big|_1^2 = \frac{1}{14}(2^7 - 1^7) = \underline{\underline{\frac{127}{14}}}$$

b) $\int_0^2 \frac{x^2}{x^2+4}\,dx$ ser på det ubestemte integralet

$$= \int \frac{x^2+4-4}{x^2+4}\,dx = \int 1 - \frac{4}{x^2+4}\,dx$$

( husk at $\int \frac{1}{1+x^2}dx = \arctan(x) + c$ )

$$\int 1 - \frac{4/4}{\frac{x^2}{4}+\frac{4}{4}}\,dx = \int 1 - \frac{1}{(\frac{x}{2})^2+1}\,dx$$

$$= x - \int \frac{1}{(\frac{x}{2})^2+1}\,dx$$

La $u = \frac{x}{2}$

$du = \frac{1}{2}dx$

$2du = dx$

$$= x - \int \frac{1}{u^2+1}\,2du$$

$$= x - 2\arctan(u) + c$$

$$= \underline{x - 2\arctan\left(\frac{x}{2}\right) + c}$$

$$\int_0^2 \frac{x^2}{x^2+4}\,dx = \left[ x - 2\arctan\left(\frac{x}{2}\right) + c \right]_0^2$$

$$= 2 - 2\arctan\left(\frac{2}{2}\right) - \left(0 - 2\arctan(0)\right)$$

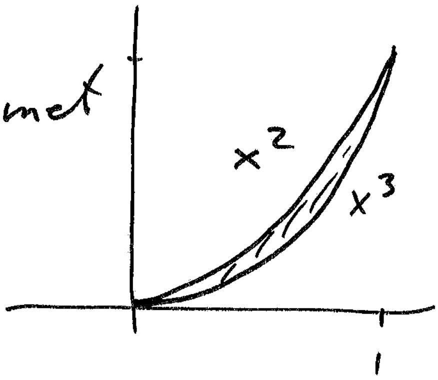
$$= 2 - 2 \cdot \frac{\pi}{4} - 0$$

$$= 2 - \frac{\pi}{2} \sim \underline{0.43}$$

8

8

(9) R er regionen mellom grafen til $y = x^2$ og $y = x^3$

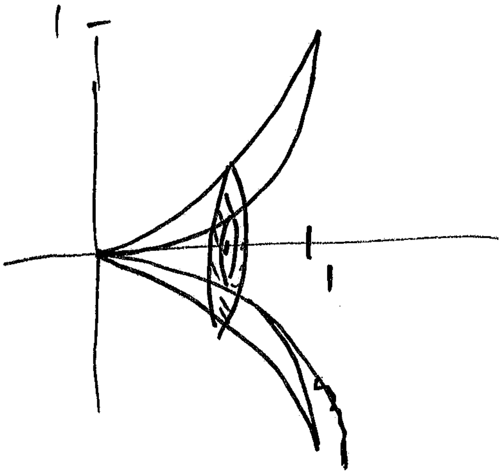



Finn volumet til rotasjonslegemet som fremkommer ved

a) Rotasjon om x-aksen

b) Rotasjon om y-aksen.

a)



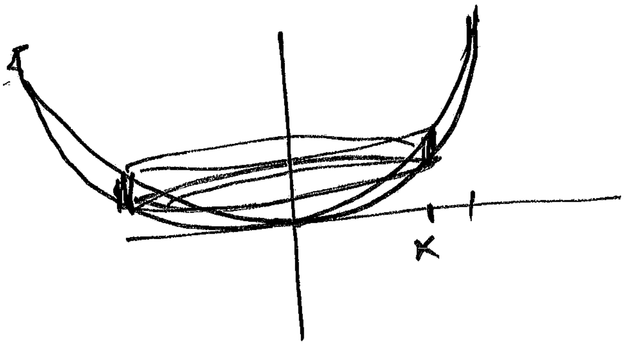

$$V = \int_0^1 \pi\left((x^2)^2 - (x^3)^2\right) dx$$

$$= \pi \int_0^1 x^4 - x^6 \, dx$$

$$= \pi \left[ \frac{x^5}{5} - \frac{x^7}{7} \right]_0^1$$

$$= \pi\left(\frac{1}{5} - \frac{1}{7}\right) = \underline{\underline{\frac{2\pi}{35}}}$$

b)




$$V = \int_0^1 2\pi x \left(x^2 - x^3\right) dx$$

$$= 2\pi \int_0^1 x^3 - x^4 \, dx$$

$$= 2\pi \left(\frac{x^4}{4} - \frac{x^5}{5}\right) \Big|_0^1$$

$$= 2\pi\left(\frac{5-4}{20}\right) = \underline{\underline{\frac{\pi}{10}}}$$

# Eksamen august 2013

2 c) i) $\lim_{x \to 1} \frac{x^2-1}{x^2+1} = \frac{0}{2} = 0$

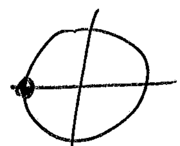
$$\left( \lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f(a) + f'(a)(x-a) + \cdots}{g(a) + g'(a)(x-a) + \cdots} \right)$$

ii) $\lim_{x \to 1} \frac{x-1}{x^2-1}$ (type $\frac{0}{0}$)

L'Hopital

$$= \lim_{x \to 1} \frac{(x-1)'}{(x^2-1)'} = \lim_{x \to 1} \frac{1}{2x} = \underline{\underline{\frac{1}{2}}}$$

Alternativt: $\lim_{x \to 1} \frac{x-1}{(x-1)(x+1)} = \lim_{x \to 1} \frac{1}{x+1} = \underline{\underline{\frac{1}{2}}}$



iii) $\lim_{x \to 1} \frac{\ln x}{\sin(\pi x)}$ (type $\frac{0}{0}$)

L'Hopital

$$\lim_{x \to 1} \frac{(\ln x)'}{(\sin(\pi x))'} = \lim_{x \to 1} \frac{1/x}{\pi \cos(\pi x)} = \frac{1}{\pi(-1)}$$

Så $\lim_{x \to 1} \frac{\ln x}{\sin(\pi x)} = \underline{\underline{\frac{-1}{\pi}}}$