3.sep 2015

Inversmatriser

① $A \cdot A^{-1} = 1_n \qquad A^{-1}A = 1_n$

A^{-1} inversmatrisen til A.

Eksempel $\qquad 2x + ay = 2 \qquad$ a parameter
$\qquad 3x - y = a$

$$\overbrace{\begin{bmatrix} 2 & a \\ 3 & -1 \end{bmatrix}}^{A}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ a \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = A^{-1} \cdot A \begin{bmatrix} x \\ y \end{bmatrix} = A^{-1}\begin{bmatrix} 2 \\ a \end{bmatrix}$$

$$A^{-1} = \frac{1}{2(-1) - 3 \cdot a}\begin{bmatrix} -1 & -a \\ -3 & 2 \end{bmatrix} \qquad \left(\begin{bmatrix} a, b \\ c, d \end{bmatrix}^{-1} = \frac{1}{ad - bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix} \right)$$

$$A^{-1} = \frac{1}{-2 - 3a}\begin{bmatrix} -1 & -a \\ -3 & 2 \end{bmatrix} = \frac{1}{3a + 2}\begin{bmatrix} 1 & a \\ 3 & -2 \end{bmatrix}$$

Løsningen er: $\begin{bmatrix} x \\ y \end{bmatrix} = A^{-1}\begin{bmatrix} 2 \\ a \end{bmatrix} = \frac{1}{3a+2}\begin{bmatrix} 1 & a \\ 3 & -2 \end{bmatrix}\begin{bmatrix} 2 \\ a \end{bmatrix}$

$$\underline{\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{3a + 2}\begin{bmatrix} a^2 + 2 \\ -2a + 6 \end{bmatrix}} \qquad a \neq \frac{-2}{3}$$

(For eksempel $a=0$: $\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 2 \\ 6 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$)

Radoperasjoner gir inversmatriser

$a=1$: $\left(\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{5}\begin{bmatrix} 3 \\ 4 \end{bmatrix} \right)$

$$[A \mid 1_n] \sim \underset{\text{(hvis mulig)}}{[1_n \mid A^{-1}]}$$

Vi viser hvorfor dette virker →

Radoperasjonen på et produkt $M \cdot N$ gir samme resultat som å utføre radoperasjonen på M og deretter gange med N.

2

$$\begin{bmatrix} m_1 \\ \vdots \\ m_k \end{bmatrix} [n_1, \ldots, n_\ell]$$

(overbevis deg selv om dette)

$$Ax = b = 1 \cdot b$$

(Anta A er inverterbar)

$$[A \mid 1] \sim [1 \mid C]$$

Da får vi

$$\begin{aligned} Ax &= 1\,b \\ 1 \cdot x &= C \cdot b \end{aligned}$$

,

$$Ax = b \quad \Updownarrow \quad x = Cb$$

Spesielt:

$$A \cdot A^{-1} = 1$$

$$A^{-1} = 1 \cdot A^{-1} = C$$

så $\underline{C = A^{-1}}$.

Eksempel

(3)

$$A = \begin{bmatrix} 2 & 0 & 4 \\ 3 & -1 & 5 \\ 1 & 2 & 2 \end{bmatrix}, \quad A^{-1} = \begin{bmatrix} -3 & 2 & 1 \\ -0.25 & 0 & 0.5 \\ 1.75 & -1 & -0.5 \end{bmatrix}$$

$$\left[\begin{array}{ccc|ccc} 2 & 0 & 4 & 1 & 0 & 0 \\ 3 & -1 & 5 & 0 & 1 & 0 \\ 1 & 2 & 2 & 0 & 0 & 1 \end{array}\right] \begin{array}{l} \cdot \\ -2 \\ -3 \end{array}$$

$$\sim \left[\begin{array}{ccc|ccc} 0 & -4 & 0 & 1 & 0 & -2 \\ 0 & -7 & -1 & 0 & 1 & -3 \\ 1 & 2 & 2 & 0 & 0 & 1 \end{array}\right] \begin{array}{l} \cdot -\frac{1}{4} \\ -2 \\ \end{array}$$

$$\sim \left[\begin{array}{ccc|ccc} 0 & 1 & 0 & -\frac{1}{4} & 0 & \frac{1}{2} \\ 0 & 1 & -1 & -2 & 1 & 1 \\ 1 & 2 & 2 & 0 & 0 & 1 \end{array}\right] \begin{array}{l} -1 \\ \\ -2 \end{array}$$

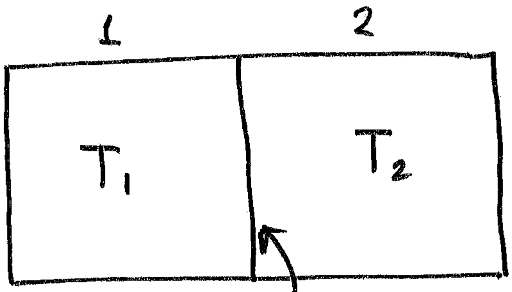
$$\sim \left[\begin{array}{ccc|ccc} 0 & 1 & 0 & -0{,}25 & 0 & 0.5 \\ 0 & 0 & -1 & -1{,}75 & 1 & 0.5 \\ 1 & 0 & 2 & 0.5 & 0 & 0 \end{array}\right] \begin{array}{l} \\ \cdot 2 \quad \cdot(-1) \\ \end{array}$$

$$\sim \left[\begin{array}{ccc|ccc} 0 & 1 & 0 & -0.25 & 0 & 0.5 \\ 0 & 0 & 1 & 1.75 & -1 & -0.5 \\ 1 & 0 & 0 & -3 & 2 & 1 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -3 & 2 & 1 \\ 0 & 1 & 0 & -0.25 & 0 & 0.5 \\ 0 & 0 & 1 & 1.75 & -1 & -0.5 \end{array}\right]$$

(4.)

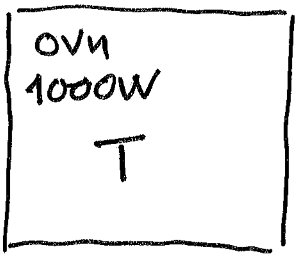
Eksempel



Varmeledningskoeffisient k

Varmeoverføring (energi / tid) fra system 2 til system 1 $= k(T_2 - T_1)$

$k = 50\,W/°C$ (Watt J/sek)



$T_0 = 10°C$

Hva er temperaturen i rommet når den har stabilisert seg?

$1000\,W = k(T - T_0)$

Varme: tilførsel = tap

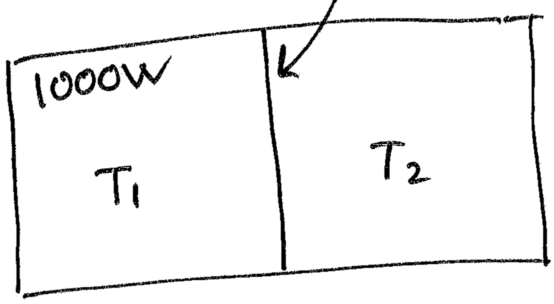
$$T = \frac{1000\,W}{50\,W/°C} + 10°C = 20°C + 10°C = \underline{30°C}$$

To rom:

$k_3 = 100\,W/°C$

temperatur $T_0 = 10°C$

$k_1 = 50\,W/°C$



k_2 ($= 70\,W/°C$) parameter

Hva er temperaturen i de to rommene når den har stabilisert seg?

$$\text{Rom 1}: \quad 1000\,W = k_1(T_1 - T_0) + k_3(T_1 - T_2)$$

$$\text{Rom 2}: \quad 0 = k_2(T_2 - T_0) + k_3(T_2 - T_{\cancel{3}\,1})$$

5 Lineært likningssystem i T_1, T_2.

$$(k_1 + k_3)T_1 - k_3 \cdot T_2 = 1000\,W + k_1 \cdot T_0$$

$$-k_3 T_1 + (k_2 + k_3)T_2 = k_2 \cdot T_0$$

setter inn for $T_0 = 10\,°C$, $k_1 = 50\,W/°C$

$k_3 = 100\,W/°C$

(unlater å skrive enheter)

$$150\,T_1 - 100\,T_2 = 1500 \qquad \cdot \tfrac{1}{100}$$

$$-100\,T_1 + (100 + k_2)T_2 = k_2 \cdot 10 \qquad \cdot \tfrac{1}{100}$$

$$\begin{bmatrix} 1.5 & -1 \\ -1 & 1 + \frac{k_2}{100} \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \end{bmatrix} = \begin{bmatrix} 15 \\ k_2/10 \end{bmatrix}$$

$$\begin{bmatrix} T_1 \\ T_2 \end{bmatrix} = \begin{bmatrix} 1.5 & -1 \\ -1 & 1 + k_2/100 \end{bmatrix}^{-1} \begin{bmatrix} 15 \\ k_2/10 \end{bmatrix}$$

$$= \frac{1}{1.5 \cdot (1 + k_2/100) - 1} \begin{bmatrix} 1 + k_2/100 & 1 \\ 1 & 1.5 \end{bmatrix} \begin{bmatrix} 15 \\ k_2/10 \end{bmatrix}$$

$$\begin{bmatrix} T_1 \\ T_2 \end{bmatrix} = \frac{1}{0.5 + k_2 \cdot 3/200} \begin{bmatrix} 15\left(1 + \frac{k_2}{100}\right) + k_2/10 \\ 15 + 1.5 \cdot k_2/10 \end{bmatrix}$$

Hvis $k_2 = 50 W/°c$, så får vi

(6) $$\begin{bmatrix} T_1 \\ T_2 \end{bmatrix} = \begin{bmatrix} 22°c \\ 18°c \end{bmatrix}$$

Hvis $k_2 = 70 W/°c$

$$\begin{bmatrix} T_1 \\ T_2 \end{bmatrix} = \begin{bmatrix} 20.96°c \\ 16.45°c \end{bmatrix}$$

Eksempel $A = \begin{bmatrix} 1 & 2 & -1 \\ 2 & 4 & 5 \\ 3 & 5 & 1 \end{bmatrix}$ $(\det A = 7 \neq 0)$.

La oss finne A^{-1}.

7

$$\begin{array}{cc} A & I_3 \\ \left[\begin{array}{ccc|ccc} 1 & 2 & -1 & 1 & 0 & 0 \\ 2 & 4 & 5 & 0 & 1 & 0 \\ 3 & 5 & 1 & 0 & 0 & 1 \end{array}\right] & -2 \quad -3 \end{array}$$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 2 & -1 & 1 & 0 & 0 \\ 0 & 0 & 7 & -2 & 1 & 0 \\ 0 & -1 & 4 & -3 & 0 & 1 \end{array}\right] \quad \frac{1}{7}$$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 2 & -1 & 1 & 0 & 0 \\ 0 & -1 & 4 & -3 & 0 & 1 \\ 0 & 0 & 1 & -\frac{2}{7} & \frac{1}{7} & 0 \end{array}\right] \quad -1$$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 2 & 0 & \frac{5}{7} & \frac{1}{7} & 0 \\ 0 & 1 & -4 & 3 & 0 & -1 \\ 0 & 0 & 1 & -\frac{2}{7} & \frac{1}{7} & 0 \end{array}\right] \quad 4 \quad -2$$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -3 & -1 & 2 \\ 0 & 1 & 0 & 13/7 & 4/7 & -1 \\ 0 & 0 & 1 & -\frac{2}{7} & \frac{1}{7} & 0 \end{array}\right]$$

$$A^{-1} = \begin{bmatrix} -3 & -1 & 2 \\ 13/7 & 4/7 & -1 \\ -2/7 & 1/7 & 0 \end{bmatrix}$$