OPPGAVE 1

a) i) $f(x) = (x^2+1)^3 + \sin(x)$

$$f'(x) = 3(x^2+1)(2x) + \cos(x)$$

$$= 6x(x^2+1) + \cos(x)$$

ii) $g(x) = \dfrac{\ln(x)}{x^2}$

$$g'(x) = \frac{\frac{1}{x} \cdot x^2 - \ln(x) \cdot 2x}{x^4}$$

$$= \frac{x - 2x \ln x}{x^4}$$

$$= \frac{1 - 2\ln x}{x^3}$$

b) i) $\int (2x^2 + \sqrt{2})\, dx = \frac{2}{3}x^3 + \sqrt{2}\, x + C$

ii) $\displaystyle\int \frac{\cos(\ln x)}{x}\, dx$

VARIABELSKIFTE: $u = \ln x$

$$\frac{du}{dx} = \frac{1}{x}$$

$$du = \frac{dx}{x}$$

$$dx = x\, du$$

$$= \int \frac{\cos(u)}{\cancel{x}} \cancel{x}\, du$$

$$= \int \cos u \, du$$

$$= \sin u + C$$

$$= \sin(\ln x) + C$$

c) $\int_0^{\pi/3} x \sin(3x)\,dx$

LØSER UBESTEMT INTEGRAL VED HJELP AV DELVIS INTEGRASJON

$\int u v' = uv - \int u' v$

$u = x \quad , \quad u' = 1 \quad , \quad v' = \sin(3x) \quad , \quad v = -\frac{1}{3}\cos(3x)$

$$\int x \sin(3x)\,dx = -\frac{1}{3}x\cos(3x) - \int 1\cdot\left(-\frac{1}{3}\cos(3x)\right)dx$$

$$= -\frac{1}{3}x\cos(3x) + \frac{1}{3}\int \cos(3x)\,dx$$

$$= -\frac{1}{3}x\cos(3x) + \frac{1}{3}\cdot\frac{1}{3}\sin(3x) + C$$

$$= \frac{1}{3}\left(\frac{1}{3}\sin(3x) - x\cos(3x)\right) + C$$

$$\int_0^{\pi/3} x\sin(3x)\,dx = \frac{1}{3}\left[\frac{1}{3}\sin(3x) - x\cos(3x)\right]_0^{\pi/3}$$

$$= \frac{1}{3}\left(\frac{1}{3}\sin(\pi) - \frac{\pi}{3}\cos(\pi) - \frac{1}{3}\sin(0) + 0\cdot\cos(0)\right)$$

$$= \frac{1}{3}\left(0 + \frac{\pi}{3} - 0 + 0\right) = \underline{\underline{\frac{\pi}{9}}}$$

d) $$\frac{e^{x^2}}{e^{3x+4}} = 1$$

$$e^{x^2} = e^{3x+4}$$

$$\ln(e^{x^2}) = \ln(e^{3x+4})$$

$$x^2 \ln e = (3x+4)\ln e$$

$$x^2 = 3x+4$$

$$x^2 - 3x - 4 = 0$$

$$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4 \cdot 1 \cdot (-4)}}{2}$$

$$= \frac{3 \pm \sqrt{25}}{2} = \frac{3 \pm 5}{2}$$

$$x_1 = \frac{3+5}{2} = \underline{\underline{4}} \quad \vee \quad x_2 = \frac{3-5}{2} = \underline{\underline{-1}}$$

$\Bigg($ EN ENKLERE METODE:

FOR AT $\frac{e^{x^2}}{e^{3x+4}}$ SKAL VÆRE LIK 1 MÅ

$e^{x^2} = e^{3x+4}$, SOM BARE ER MULIG

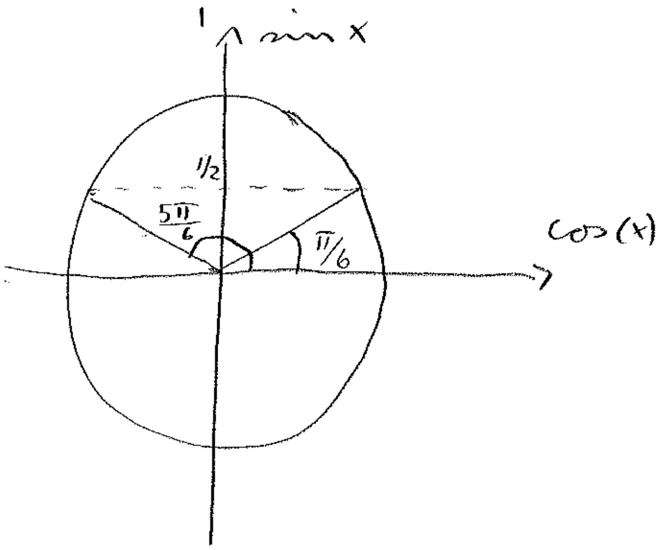
DERSOM $x^2 = 3x+4$ $\Bigg)$

e) $\sin(2x) - \frac{1}{2} = 0 \quad , \quad x \in [0, \pi\rangle$

$\sin(2x) = \frac{1}{2}$

$2x = \frac{\pi}{6} + 2\pi n$

eller $2x = \frac{5\pi}{6} + 2\pi n$



①:

$2x = \frac{\pi}{6} + 2\pi \cdot 0$

$\underline{\underline{x_1 = \frac{\pi}{12}}}$

② $2x = \frac{5\pi}{6} + 2\pi \cdot 0$

$\underline{\underline{x_2 = \frac{5\pi}{12}}}$

FOR $n \geq 1$ LIGGER LØSNINGENE UTENFOR D_f

OPPGAVE 2

a) $A - C = \begin{bmatrix} -2 & 1 \\ 0 & -3 \\ 2 & 2 \end{bmatrix} - \begin{bmatrix} 0 & 0 \\ 1 & 2 \\ -2 & -4 \end{bmatrix} = \underline{\underline{\begin{bmatrix} -2 & 1 \\ -1 & -5 \\ 4 & 6 \end{bmatrix}}}$

$$AB = \begin{bmatrix} -2 & 1 \\ 0 & -3 \\ 2 & 2 \end{bmatrix} \begin{bmatrix} 1 & -1 & 0 \\ 3 & -1 & -1 \end{bmatrix} = \begin{bmatrix} (-2)\cdot 1 + 1\cdot 3 & (-2)(-1)+1\cdot(-1) & -2\cdot 0 + 1\cdot(-1) \\ 0\cdot 1 + (-3)\cdot 3 & 0\cdot(-1)+(-3)(-1) & 0\cdot 0 + (-3)(-1) \\ 2\cdot 1 + 2\cdot 3 & 2\cdot(-1) + 2\cdot(-1) & 2\cdot 0 + 2(-1) \end{bmatrix}$$

$$= \underline{\underline{\begin{bmatrix} 1 & 1 & -1 \\ -9 & 3 & 3 \\ 8 & -4 & -2 \end{bmatrix}}}$$

$$BA = \begin{bmatrix} 1 & -1 & 0 \\ 3 & -1 & -1 \end{bmatrix} \begin{bmatrix} -2 & 1 \\ 0 & -3 \\ 2 & 2 \end{bmatrix} = \begin{bmatrix} 1\cdot(-2) + (-1)\cdot 0 + 0\cdot 2 & 1\cdot 1 + (-1)\cdot(-3) + 0\cdot 2 \\ 3\cdot(-2) + (-1)\cdot 0 + (-1)\cdot 2 & 3\cdot 1 + (-1)(-3) + (-1)\cdot 2 \end{bmatrix}$$

$$= \underline{\underline{\begin{bmatrix} -2 & 4 \\ -8 & 4 \end{bmatrix}}}$$

AC: <u>Ikke definert siden antall kolonner i A (2) ikke er det samme som antall rader i C (3).</u>

b) $A\vec{x} = \vec{b}$

Radreduserer den utvidede matrisa:

$$\begin{bmatrix} -2 & 1 & 0 \\ 0 & -3 & -3 \\ 2 & 2 & 6 \end{bmatrix} \cdot\left(-\tfrac{1}{3}\right) \sim \begin{bmatrix} -2 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 3 & 6 \end{bmatrix} \begin{matrix} \cdot\left(-\tfrac{1}{2}\right) \\ (-3) \\ \end{matrix}$$

$$\sim \begin{bmatrix} 1 & -\tfrac{1}{2} & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 3 \end{bmatrix} \leftarrow$$ Ligningssettet er inkonsistent. Ingen løsningen.

$C\vec{x} = \vec{b}$

$$\begin{bmatrix} 0 & 0 & 0 \\ 1 & 2 & -3 \\ -2 & -4 & 6 \end{bmatrix} \begin{matrix} \\ \\ \cdot\left(-\tfrac{1}{2}\right) \end{matrix} \sim \begin{bmatrix} 1 & 2 & -3 \\ 1 & 2 & -3 \\ 0 & 0 & 0 \end{bmatrix} \begin{matrix} \cdot(-1) \\ \\ \end{matrix} \sim \begin{bmatrix} 1 & 2 & -3 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$x_1 = \underline{\underline{-3 - 2x_2}}$

$\underline{\underline{x_2 \text{ er fri variabel.}}}$

$$x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -3 - 2x_2 \\ x_2 \end{bmatrix} = \underline{\underline{\begin{bmatrix} -3 \\ 0 \end{bmatrix} + x_2 \begin{bmatrix} -2 \\ 1 \end{bmatrix}}}$$

OPPGAVE 3

a) $\begin{bmatrix} \frac{1}{2} & 1 & 0 \\ 1 & -1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$ GJØR EN KOFAKTOREKSPANSJON OVER 3. KOLONNE:

$$\begin{vmatrix} \frac{1}{2} & 1 & 0 \\ 1 & -1 & 1 \\ 1 & 0 & 1 \end{vmatrix} = 0 \cdot \begin{vmatrix} 1 & -1 \\ 1 & 0 \end{vmatrix} + (-1) \begin{vmatrix} \frac{1}{2} & 1 \\ 1 & 0 \end{vmatrix} + 1 \cdot \begin{vmatrix} \frac{1}{2} & 1 \\ 1 & -1 \end{vmatrix}$$

$$= (-1)\left(\tfrac{1}{2} \cdot 0 - 1 \cdot 1\right) + \left(\tfrac{1}{2} \cdot (-1) - 1 \cdot 1\right)$$

$$= 1 - \tfrac{1}{2} - 1 = \underline{\underline{-\tfrac{1}{2}}}$$

<u>$\det(A) \neq 0$. Det betyr at A er inverterbar</u>

b) $\begin{bmatrix} 2 & 2 & -2 \\ 0 & -1 & 1 \\ -2 & -2 & 3 \end{bmatrix} \cdot \begin{bmatrix} \frac{1}{2} & 1 & 0 \\ 1 & -1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$

$$= \begin{bmatrix} 2 \cdot \frac{1}{2} + 2 \cdot 1 - 2 \cdot 1 & 2 \cdot 1 + 2 \cdot (-1) - 2 \cdot 0 & 2 \cdot 0 + 2 \cdot 1 - 2 \cdot 1 \\ 0 \cdot \frac{1}{2} - 1 \cdot 1 + 1 \cdot 1 & 0 \cdot 1 - 1 \cdot (-1) + 1 \cdot 0 & 0 \cdot 0 - 1 \cdot 1 + 1 \cdot 1 \\ -2 \cdot \frac{1}{2} - 2 \cdot 1 + 3 \cdot 1 & -2 \cdot 1 - 2 \cdot (-1) + 3 \cdot 0 & -2 \cdot 0 - 2 \cdot 1 + 3 \cdot 1 \end{bmatrix}$$

$$= \underline{\underline{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}}}$$

c) Ligningssettet kan skrives som

$A\vec{x} = \vec{b}$ der $\vec{b} = \begin{bmatrix} 1 \\ -6 \\ -4 \end{bmatrix}$ og $\vec{x} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$

Da har vi:

$$A^{-1}A\vec{x} = A^{-1}\vec{b}$$

$$\vec{x} = A^{-1}\vec{b}$$

$$= \begin{bmatrix} 2 & 2 & -2 \\ 0 & -1 & 1 \\ -2 & -2 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ -6 \\ -4 \end{bmatrix} = \begin{bmatrix} 2\cdot 1 + 2\cdot(-6) - 2\cdot(-4) \\ 0\cdot 1 - 1\cdot(-6) + 1\cdot(-4) \\ -2\cdot 1 - 2(-6) + 3(-4) \end{bmatrix}$$

$$= \underline{\begin{bmatrix} -2 \\ 2 \\ -2 \end{bmatrix}}$$

$\underline{\underline{x = -2, \quad y = 2, \quad z = -2}}$

d) <u>A er invertibel. Det betyr at $A\vec{x} = \vec{b}$ har løsninger for alle $\vec{b}$, og dermed også alle verdier av a.</u>

OPPGAVE 4

a) $\frac{dy}{dx} - \frac{2x}{e^y} = 0$

$$\frac{dy}{dx} = \frac{2x}{e^y}$$

$$\int e^y dy = \int 2x\, dx$$

$$e^y = 2x + C$$

$$\ln(e^y) = \ln(x^2+c)$$

$$y = \underline{\ln(x^2+C)}$$

$$y(0) = \ln(c) = 1 \Rightarrow C = 1$$

$$y = \underline{\ln(x^2+1)}$$

b) $3y' = y'' - 10$

$$y'' - 3y' = -10$$

Finner først homogen løsning:

kar. lign.: $r^2 - 3r = 0$

$$r(r-3) = 0 \Rightarrow r_1 = 3,\ r_2 = 0$$

$$y_h(x) = Ce^{3x} + De^{0x}$$

$$= Ce^{3x} + D$$

Partikulær løsn.:

gjetter formen $y_P(x) = ax$

$y_P'(x) = a \quad , \quad y_P''(x) = 0$

$3y_P' = y_P'' - 10$

$3a = 0 - 10$

$\underline{a = -\frac{10}{3}} \Rightarrow y_P(x) = -\frac{10}{3}x$

$y(x) = y_P(x) + y_h(x) = Ce^{3x} - \frac{10}{3}x + D$

$y(0) = C + D = 7 \Rightarrow D = 7 - C$

$y'(x) = 3Ce^{3x} - \frac{10}{3}$

$y'(0) = 3C - \frac{10}{3} = 0$

$3C = \frac{10}{3}$

$\underline{C = \frac{10}{9}} \qquad D = 7 - C = 7 - \frac{10}{9} = \underline{\frac{53}{9}}$

$$\underline{\underline{y(x) = \frac{10}{9}e^{3x} - \frac{10}{3}x + \frac{53}{9}}}$$

5 c)

$$-2y'+2y+4x=0$$

$$y'-y=+2x$$

$$e^{-x}(y'-y)=2e^{-x}\ x$$

$$(e^{-x}y)'=2e^{-x}x$$

$$e^{-x}y = 2\int x e^{-x}dx =$$

$$2\left(-xe^{-x}-\int 1\cdot(-e^{-x})dx\right)=$$

$$2\left(-xe^{-x}+\int e^{-x}dx\right)=$$

$$2\left(-xe^{-x}-e^{-x}\right)+C$$

$$\underline{y=-2x-2+Ce^{+x}}$$

Kontroll:

$$-2y'+2y+4x= -2\cdot(-2-0+Ce^{+x})+$$

$$2(-2x-2+Ce^{+x})+4x = 4-2Ce^{-x}-4x-4+$$

$$2Ce^{-x}+4x=0$$

Oppg. 5

a) $T\left(\begin{bmatrix}1\\1\\0\end{bmatrix}\right) = \begin{bmatrix}1\\3\\-1\end{bmatrix}$, $T\left(\begin{bmatrix}0\\0\\1\end{bmatrix}\right) = \begin{bmatrix}2\\0\\0\end{bmatrix}$

T er lineær:

$$T\left(\begin{bmatrix}1\\1\\1\end{bmatrix}\right) = T\left(\begin{bmatrix}1\\1\\0\end{bmatrix} + \begin{bmatrix}0\\0\\1\end{bmatrix}\right) =$$

$$T\left(\begin{bmatrix}1\\1\\0\end{bmatrix}\right) + T\left(\begin{bmatrix}0\\0\\1\end{bmatrix}\right) = \begin{bmatrix}1\\3\\-1\end{bmatrix} + \begin{bmatrix}2\\0\\0\end{bmatrix} = \underline{\begin{bmatrix}3\\3\\-1\end{bmatrix}}$$

$$T\left(\begin{bmatrix}2\\2\\0\end{bmatrix}\right) = T\left(2\begin{bmatrix}1\\1\\0\end{bmatrix}\right) = 2T\left(\begin{bmatrix}1\\1\\0\end{bmatrix}\right) = 2\begin{bmatrix}1\\3\\-1\end{bmatrix} = \underline{\begin{bmatrix}2\\6\\-2\end{bmatrix}}$$

b) Vis $B = PDP^{-1}$ med $B = \begin{bmatrix}0 & 1\\2 & 1\end{bmatrix}$,

$P = \begin{bmatrix}1 & -1\\2 & 1\end{bmatrix}$ og $D = \begin{bmatrix}2 & 0\\0 & -1\end{bmatrix}$

Vet: P skal ha egenvektorer til B som søyler, og diagonalmatrisa D skal ha de tilsvarende egenverdiene langs diagonalen

$$B\vec{x} = \lambda\vec{x}, \quad \vec{x} \neq \vec{0} \Rightarrow \det(B - \lambda I) = 0 \Leftrightarrow$$

$$0 = \begin{vmatrix}-\lambda & 1\\2 & 1-\lambda\end{vmatrix} = -\lambda(1-\lambda) - 2 = \lambda^2 - \lambda - 2 \Leftrightarrow$$

$$\lambda = \frac{-(-1) \pm \sqrt{(-1)^2 - 4 \cdot 1 \cdot (-2)}}{2 \cdot 1} = \frac{1 \pm 3}{2}$$

$$\lambda_1 = \frac{1+3}{2} = 2 \quad \wedge \quad \lambda_2 = \frac{1-3}{2} = -1$$

Egenvektor for $\lambda_1 = 2$:

$$(B - \lambda_2 I)\vec{x} = \vec{0}, \quad \vec{x} = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

$$B - \lambda_2 I = \begin{bmatrix} -2 & 1 \\ 2 & -1 \end{bmatrix} \begin{matrix} +1 \\ \swarrow \end{matrix} \sim \begin{bmatrix} -2 & 1 \\ 0 & 0 \end{bmatrix}$$

$$-2x_1 + x_2 = 0, \quad x_2 = 2x_1$$

$$\vec{x} = \begin{bmatrix} x_1 \\ 2x_1 \end{bmatrix} = x_1 \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \text{ egenvektor: } \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

Egenvektor for $\lambda_2 = -1$:

$$B - \lambda_1 I = \begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix} \begin{matrix} -2 \\ \swarrow \end{matrix} \sim \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$$

$$x_1 + x_2 = 0, \quad x_2 = -x_1$$

$$\vec{x} = \begin{bmatrix} x_1 \\ -x_1 \end{bmatrix} = -x_1 \begin{bmatrix} -1 \\ 1 \end{bmatrix}, \text{ egenvektor: } \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

Altså: $B = PDP^{-1}$ med $P = \begin{bmatrix} 1 & -1 \\ 2 & 1 \end{bmatrix}$ og $D = \begin{bmatrix} 2 & 0 \\ 0 & -1 \end{bmatrix}$

Kontroll: $B = PDP^{-1} \Leftrightarrow BP = PD$

$$BP = \begin{bmatrix} 0 & 1 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 4 & -1 \end{bmatrix}$$

$$PD = \begin{bmatrix} 1 & -1 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 4 & -1 \end{bmatrix} = BP$$

c) $B^5 = (PDP^{-1})^5 = PD^5P^{-1} =$

$$P\begin{bmatrix} 2^5 & 0 \\ 0 & (-1)^5 \end{bmatrix} P^{-1} = \begin{bmatrix} 1 & -1 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 32 & 0 \\ 0 & -1 \end{bmatrix} \frac{1}{1-(-2)} \begin{bmatrix} 1 & 1 \\ -2 & 1 \end{bmatrix}$$

$$= \frac{1}{3} \begin{bmatrix} 32 & 1 \\ 64 & -1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ -2 & 1 \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 30 & 33 \\ 66 & 63 \end{bmatrix} = \underline{\begin{bmatrix} 10 & 11 \\ 22 & 21 \end{bmatrix}}$$

Oppg. 6

a) De regner med å selge

$10\,000 - 10x$ billetter

Samle inntekt, når prisen er $(400+x)$ kr blir da

$$I(x) = (400+x)\cdot(10\,000-10x) =$$

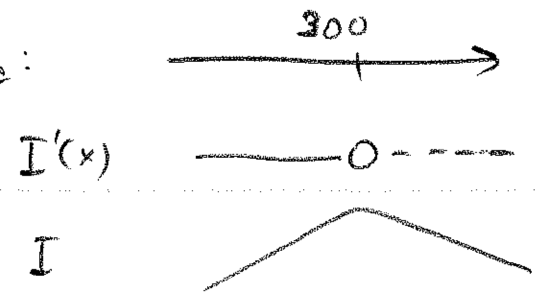
$$-10x^2 + (10\,000 - 400\cdot 10)\,x + 400\cdot 10\,000 =$$

$$\underline{10(-x^2 + 600x + 4\cdot 10^5)} \qquad (q.e.d.)$$

b) Deriverer:

$$I'(x) = 10(-2x + 600 + 0) = 20(300 - x)$$

Fortegnskjema:



Vi ser at I er maksimal når $x = 300$.

De bør altså sette prisen til

$(400 + 300)$ kr $=$ <u>700 kr</u>

Inntektene blir da (i kr):

$$I(300) = 10(-300^2 + 600\cdot 300 + 4\cdot 10^5) = 10\cdot(-9\cdot 10^4 + 1.8\cdot 10^5 + 4\cdot 10^5) = 10\cdot 10^4(18 + 40 - 9) = \underline{4.9\cdot 10^6}$$