Løysingsforslag aug. 2010

Oppg. 1

a) $f(x) = x^2(e^x + e^{-x})$

$f'(x) = (x^2)'(e^x + e^{-x}) + x^2(e^x + e^{-x})' =$

$2x(e^x + e^{-x}) + x^2(e^x + e^{-x} \cdot (-x)') =$

$2x e^x + 2x e^{-x} + x^2 e^x + x^2 e^{-x} \cdot (-1) =$

$\underline{\underline{e^x(2x + x^2) + e^{-x}(2x - x^2)}}$

$g(x) = \ln(\tan^2 x + 1)$

$g'(x) = \dfrac{1}{\tan^2 x + 1} \cdot (\tan^2 x + 1)' =$

$\dfrac{(2\tan x + 0) \cdot (\tan x)'}{\tan^2 x + 1} = \dfrac{2\tan x \cdot \frac{1}{\cos^2 x}}{\tan^2 x + 1} =$

$\dfrac{2\tan x}{\cos^2 x \left(\frac{\sin^2 x}{\cos^2 x} + 1\right)} = \dfrac{2\tan x}{\sin^2 x + \cos^2 x} =$

$\dfrac{2\tan x}{1} = \underline{\underline{2\tan x}}$

b) $4^y = x^2$

Implisitt derivasjon:

$4^y \cdot \ln 4 \cdot y' = 2x$

Med $x=4,\ y=2$:

$4^2 \ln 4 \cdot y' = 2\cdot 4, \quad y' = \dfrac{8}{16\ln 4} = \dfrac{1}{2\ln 4}$

Stigningstalet til tangenten blir altså $\dfrac{1}{2\ln 4}$.

Dermed blir likninga for tangenten

$$y - 2 = \frac{1}{2\ln 4}(x-4)$$

$$y = \frac{1}{2\ln 4}x - \frac{4}{2\ln 4} + 2$$

$$\underline{y = \frac{1}{2\ln 4}x - \frac{2}{\ln 4} + 2} \quad = \frac{1}{\ln 16}x + 2\left(1 - \frac{1}{\ln 4}\right)$$

Alternativt: $y = f(x) = 2\ln x / \ln 4$, $P_1 = f(4) + f'(4)(x-4)$

Oppg. 2

a) $A\vec{x} = \vec{b}$ med $a = -2$

Totalmatrise:

$$\begin{bmatrix} -2+1 & 1 & 1 & -2+3 \\ -1 & -2+3 & -(-2+2) & 1 \\ 1 & 1 & -2+1 & 3 \end{bmatrix} = \begin{array}{c}\begin{bmatrix} -1 & 1 & 1 & 1 \\ -1 & 1 & 0 & 1 \\ 1 & 1 & -1 & 3 \end{bmatrix}\end{array}\begin{array}{c} 1 \\ \\ 1 \end{array} \sim$$

$$\begin{bmatrix} -1 & 1 & 1 & 1 \\ 0 & 2 & -1 & 4 \\ 0 & 2 & 0 & 4 \end{bmatrix}\begin{array}{c} \\ -1 \\ \\ \end{array} \sim \begin{bmatrix} -1 & 1 & 1 & 1 \\ 0 & 2 & -1 & 4 \\ 0 & 0 & 1 & 0 \end{bmatrix}\begin{array}{c} \\ \\ 1 \quad -1 \end{array} \sim$$

$$\begin{bmatrix} -1 & 1 & 0 & 1 \\ 0 & 2 & 0 & 4 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{array}{l} \leftarrow (-1) \\ \leftarrow \frac{1}{2} \\ \\ \end{array} \sim \begin{bmatrix} 1 & -1 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{array}{l} \leftarrow \\ 1 \\ \\ \end{array} \sim$$

$$\begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

Løysing: $\underline{\underline{\vec{x} = \begin{bmatrix} 3 \\ 2 \\ 0 \end{bmatrix}}}$

b) Likningssystemet $A\vec{x} = \vec{b}$ har eintydig løysing

$$\Updownarrow$$

$\det A \neq 0$.

$$\det A = \begin{vmatrix} a+1 & 1 & 1 \\ -1 & a+3 & -(a+2) \\ 1 & 1 & a+1 \end{vmatrix} \begin{array}{l} \leftarrow \\ \leftarrow \\ 1 \quad -(a+1) \end{array} =$$

$$\begin{vmatrix} 0 & -a & -a^2-2a \\ 0 & a+4 & -1 \\ 1 & 1 & 1 \end{vmatrix} = +1 \cdot \begin{vmatrix} -a & -a^2-2a \\ a+4 & -1 \end{vmatrix} =$$

$$(-a)\cdot(-1) - (-a^2-2a)(a+4) = a + a^3 + 6a^2 + 8a =$$

$$a^3 + 6a^2 + 9a = a(a^2+6a+9) = a(a+3)^2 \neq 0 \Leftrightarrow$$

$$\underline{a \neq 0 \ \wedge \ a \neq -3 \Leftrightarrow a \notin \{-3, 0\}}$$

c) Totalmatrise med $a=0$:

$$\begin{bmatrix} 1 & 1 & 1 & 3 \\ -1 & 3 & -2 & 1 \\ 1 & 1 & 1 & 3 \end{bmatrix} \begin{matrix} 1 & -1 \\ & \\ & \end{matrix} \sim \begin{bmatrix} 1 & 1 & 1 & 3 \\ 0 & 4 & -1 & 4 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{matrix} \\ \leftarrow \frac{1}{4} \\ \\ \end{matrix} \sim$$

$$\begin{bmatrix} 1 & 1 & 1 & 3 \\ 0 & 1 & -\frac{1}{4} & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{matrix} \\ -1 \\ \\ \end{matrix} \sim \begin{bmatrix} 1 & 0 & \frac{5}{4} & 2 \\ 0 & 1 & \frac{1}{4} & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\left.\begin{matrix} x_1 + \frac{5}{4} x_3 = 2 \\ x_2 + \frac{1}{4} x_3 = 1 \end{matrix}\right\} \Leftrightarrow \begin{cases} x_1 = 2 - \frac{5}{4} x_3 \\ x_2 = 1 - \frac{1}{4} x_3 \end{cases}$$

x_3 er fri

$$\vec{x} = \begin{bmatrix} 2 - \frac{5}{4} x_3 \\ 1 - \frac{1}{4} x_3 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} + \frac{x_3}{4} \begin{bmatrix} -5 \\ -1 \\ 4 \end{bmatrix} = \underline{\underline{\begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} + t \begin{bmatrix} -5 \\ -1 \\ 4 \end{bmatrix}}}$$

$(t = x_3/4)$

Totalmatrise med $a=-3$:

$$\begin{bmatrix} -2 & 1 & 1 & 0 \\ -1 & 0 & 1 & 1 \\ 1 & 1 & -2 & 3 \end{bmatrix} \begin{matrix} \\ \\ 1 \quad 2 \end{matrix} \sim \begin{bmatrix} 0 & 3 & -3 & 6 \\ 0 & 1 & -1 & 4 \\ 1 & 1 & -2 & 3 \end{bmatrix} \begin{matrix} \\ -3 \\ \\ \end{matrix} \sim$$

$$\begin{bmatrix} 1 & 1 & -2 & 3 \\ 0 & 1 & -1 & 4 \\ 0 & 0 & 0 & -6 \end{bmatrix}$$

Av rekke 3 ser vi at likenings-systemet er inkonsisten.

Oppg. 3

a) $\int 8x^2 \ln(2x)\,dx$

Delvis integrasjon: $\int u v' dx = uv - \int u' v dx$

$u = \ln(2x) \Rightarrow u' = \frac{1}{2x} \cdot 2 = \frac{1}{x}$

$v' = 8x^2 \Leftarrow v = \frac{8}{3}x^3$

$\int 8x^2 \ln(2x)\,dx = \ln(2x) \cdot \frac{8}{3}x^3 - \int \frac{1}{x} \cdot \frac{8}{3}x^3 dx =$

$\frac{8}{3}x^3 \ln(2x) - \frac{8}{3}\int x^2 dx = \frac{8}{3}x^3 \ln(2x) - \frac{8}{3} \cdot \frac{1}{3}x^3 + C =$

$\underline{\frac{8}{3}x^3 \ln(2x) - \frac{8}{9}x^3 + C'}$

Kontroll: $\left(\frac{8}{3}x^3 \ln(2x) - \frac{8}{9}x^3 + C'\right)' =$

$\frac{8}{3} \cdot 3x^2 \ln(2x) + \frac{8}{3}x^3 \cdot \frac{1}{2x} \cdot 2 - \frac{8}{9} \cdot 3x^2 + 0 =$

$8x^2 \ln(2x) + \frac{8}{3}x^2 - \frac{8}{3}x^2 = 8x^2 \ln(2x)$ OK.

b) $\int \frac{\sqrt{x}}{\sqrt{x} - 1} dx$

Variabelbytte: $u = \sqrt{x} - 1$, $\frac{du}{dx} = \frac{1}{2\sqrt{x}}$, $dx = 2\sqrt{x}\,du$

$\int \frac{\sqrt{x}}{\sqrt{x} - 1} dx = \int \frac{\sqrt{x}}{u} \cdot 2\sqrt{x}\,du =$

$2 \int \frac{x}{u} du$

$u = \sqrt{x} - 1 \Leftrightarrow x = (u+1)^2$

$\int \frac{\sqrt{x}}{\sqrt{x} - 1} dx = 2\int \frac{(u+1)^2}{u} du = 2\int \frac{u^2 + 2u + 1}{u} du =$

$$2\int\left(u+2+\frac{1}{u}\right)du = 2\left(\tfrac{1}{2}u^2+2u+\ln|u|\right)+C'' =$$

$$(\sqrt{x}-1)^2 + 4(\sqrt{x}-1) + 2\ln|\sqrt{x}-1| + C'' =$$

$$x - 2\sqrt{x} + 4\sqrt{x} + \ln(\sqrt{x}-1)^2 + C' =$$

$$\underline{x + 2\sqrt{x} + \ln(\sqrt{x}-1)^2 + C} \qquad (C = C'-3)$$

Alternativt

$$\int \frac{\sqrt{x}}{\sqrt{x}-1}\,dx = \int \frac{\sqrt{x}-1+1}{\sqrt{x}-1}\,dx =$$

$$\int \frac{\sqrt{x}-1}{\sqrt{x}-1}\,dx + \int \frac{1}{\sqrt{x}-1}\,dx = \int dx + \int \frac{1}{\sqrt{x}-1}\,dx =$$

$$x + \int \frac{1}{\sqrt{x}-1}\,dx$$

Variabelbytte: $u = \sqrt{x}-1$, $dx = 2\sqrt{x}\,du = 2(u+1)\,du$

$$\int \frac{1}{\sqrt{x}-1}\,dx = \int \frac{1}{u}\,2(u+1)\,du =$$

$$2\int\left(1+\frac{1}{u}\right)du = 2\left(u+\ln|u|\right)+C' =$$

$$2(\sqrt{x}-1) + \ln(\sqrt{x}-1)^2 + C'$$

slik at

$$\int \frac{\sqrt{x}}{\sqrt{x}-1}\,dx = \underline{x + 2\sqrt{x} + \ln(\sqrt{x}-1)^2 + C} \qquad (C = C'-2)$$

Alternativt

$$u = \sqrt{x}, \quad \frac{du}{dx} = \frac{1}{2\sqrt{x}}, \quad dx = 2\sqrt{x}\,du$$

$$\int \frac{\sqrt{x}}{\sqrt{x}-1}\,dx = \int \frac{\sqrt{x}}{u-1}\cdot 2\sqrt{x}\,du = 2\int \frac{u^2}{u-1}\,du =$$

$$2\int \frac{u^2-1+1}{u-1}\,du = 2\left(\int \frac{u^2-1}{u-1}\,du + \int \frac{1}{u-1}\,du\right) =$$

$2\left(\int \frac{(u+1)(u-1)}{u-1}\,du + \ln|u-1|\right) =$

$2\int (u+1)\,du + \ln(u-1)^2 =$

$2\cdot\frac{1}{2}u^2 + 2\cdot u + \ln(u-1)^2 + C' =$

$\sqrt{x}^2 + 2\cdot\sqrt{x} + \ln(\sqrt{x}-1) + C' = \underline{\underline{x + 2\sqrt{x} + \ln(\sqrt{x}-1) + C'}}$

Oppg. 4

a) $y' + y = 4$

Integrerande faktor: e^F der F er ein antiderivert til 1. - Vel $F = x$.

$e^x(y'+y) = e^x 4$

$(e^x y)' = 4e^x$

$e^x y = \int 4e^x\,dx = 4e^x + C'$

$\underline{\underline{y = 4 + C e^{-x}}}$

Alternativt:

Homogen likning: $y' + y = 0$

Antar løysinga $y = e^{rx} \Rightarrow y' = re^{rx}$.

Det gir

$re^{rx} + e^{rx} = 0 \Leftrightarrow e^{rx}(r+1) = 0 \Leftrightarrow$

$r + 1 = 0 \Leftrightarrow r = -1.$

Generell løysing av homogen likning:

$y_h = Ae^{-x}$

Inhomogent ledd: 4

Antar at ein konstant vil vere ei partikulær løysing av den inhomogene likninga: $y_p = a$, $y_p' = 0$

$0 + a = 4$, $a = 4$

Generell løysing: $y = y_h + y_p = \underline{Ae^{-x} + 4}$

b) $y'' - 7y' + 12y = 3e^{2x}$

-Løyser homogen likning først:

$y'' - 7y' + 12y = 0$

Antar $y = e^{rx}$, $y' = re^{rx}$, $y'' = r^2 e^{rx}$

Set inn:

$r^2 e^{rx} - 7re^{rx} + 12e^{rx} = 0$

$e^{rx}(r^2 - 7r + 12) = 0$

$r^2 - 7r + 12 = 0$

$$r = \frac{-(-7) \pm \sqrt{(-7)^2 - 4 \cdot 1 \cdot 12}}{2 \cdot 1} = \frac{7 \pm 1}{2}$$

$$r = \frac{7-1}{2} = 3 \quad \vee \quad r = \frac{7+1}{2} = 4$$

Generell løysing av homogen likning:

$y_h = Ae^{3x} + Be^{4x}$

Vi antar partikulær løysing av den inhomogene likninga på forma

$y_p = ae^{2x}$, $y_p' = 2ae^{2x}$, $y_p'' = 4ae^{2x}$

Set inn:

$4ae^{2x} - 7 \cdot 2ae^{2x} + 12ae^{2x} = 3e^{2x}$

$4a - 14a + 12a = 3$, $2a = 3$, $a = \frac{3}{2}$

$y_p = \frac{3}{2}e^{2x}$

Generell løysing: $y = y_h + y_p = \underline{\underline{Ae^{3x} + Be^{4x} + \frac{3}{2}e^{2x}}}$

Oppg. 5

a) $A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$

Karakteristisk likning: $\det(A - \lambda I) = 0$

$0 = \begin{vmatrix} 1-\lambda & 2 \\ 2 & 1-\lambda \end{vmatrix} = (1-\lambda)^2 - 2^2 = \lambda^2 - 2\lambda + 1 - 4 =$

$\lambda^2 - 2\lambda - 3 \Leftrightarrow \lambda = \frac{-(-2) \pm \sqrt{(-2)^2 - 4 \cdot 1 \cdot (-3)}}{2 \cdot 1} = \frac{2 \pm 4}{2}$

$\lambda_1 = \frac{2-4}{2} = -1$ og $\lambda_2 = \frac{2+4}{2} = 3$

Eigenvektor for λ_1:

$$\begin{bmatrix} 1-(-1) & 2 \\ 2 & 1-(-1) \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$2x_1 + 2x_2 = 0$$

$$x_2 = -x_1$$

$$\vec{x} = \begin{bmatrix} x_1 \\ -x_1 \end{bmatrix} = x_1 \begin{bmatrix} 1 \\ -1 \end{bmatrix}, \quad \vec{v}_1 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

Eigenvektor for λ_2:

$$\begin{bmatrix} 1-3 & 2 \\ 2 & 1-3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \vec{0}$$

$$-2x_1 + 2x_2 = 0, \quad x_1 = x_2$$

$$\vec{x} = \begin{bmatrix} x_1 \\ x_1 \end{bmatrix} = x_1 \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \quad \vec{v}_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

Vi har altså funne desse eigenverdiane og -vektorane:

$$\underline{\lambda_1 = -1 \text{ med } \vec{v}_1 = \begin{bmatrix} 1 \\ -1 \end{bmatrix} \text{ og } \lambda_2 = 3 \text{ med } \vec{v}_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}}$$

b) Søylene i B er lineært uavhengige

$$\Updownarrow$$

$$\det B \neq 0$$

$$\det B = \begin{vmatrix} 1 & 2 & 1 & 4 \\ 2 & 1 & 1 & 7 \\ 0 & 0 & 2 & 1 \\ 0 & 0 & 1 & 2 \end{vmatrix} = \begin{vmatrix} 1 & 2 & 1 & 4 \\ 0 & -3 & -1 & -1 \\ 0 & 0 & 0 & -3 \\ 0 & 0 & 1 & 2 \end{vmatrix} =$$

$$- \begin{vmatrix} 1 & 2 & 1 & 4 \\ 0 & -3 & -1 & -1 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & -3 \end{vmatrix} = - 1 \cdot (-3) \cdot 1 \cdot (-3) = -9 \neq 0$$

Søylene i B er lineært uavhengige.

c) $B\vec{x} = \vec{x} \Leftrightarrow B\vec{x} = I\vec{x} \Leftrightarrow B\vec{x} - I\vec{x} = \vec{0} \Leftrightarrow$

$(B - I)\vec{x} = \vec{0}$

$$B - I = \begin{bmatrix} 1 & 2 & 1 & 4 \\ 2 & 1 & 1 & 7 \\ 0 & 0 & 2 & 1 \\ 0 & 0 & 1 & 2 \end{bmatrix} - \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} =$$

$$\begin{bmatrix} 0 & 2 & 1 & 4 \\ 2 & 0 & 1 & 7 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix} \begin{array}{l} \\ \\ -1 \\ \\ \end{array} \sim \begin{bmatrix} 2 & 0 & 1 & 7 \\ 0 & 2 & 1 & 4 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{array}{l} \\ \\ -1 \quad -1 \\ \\ \end{array} \sim$$

$$\begin{bmatrix} 2 & 0 & 0 & 6 \\ 0 & 2 & 0 & 3 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{array}{l} \leftarrow \frac{1}{2} \\ \leftarrow \frac{1}{2} \\ \\ \\ \end{array} \sim \begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & \frac{3}{2} \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\left.\begin{aligned} x_1 + 3x_4 &= 0 \\ x_2 + \tfrac{3}{2}x_4 &= 0 \\ x_3 + x_4 &= 0 \end{aligned}\right\} \Leftrightarrow \left\{\begin{aligned} x_1 &= -3x_4 \\ x_2 &= -\tfrac{3}{2}x_4 \\ x_3 &= -x_4 \end{aligned}\right.$$

x_4 er fri

$$\vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} -3x_4 \\ -\frac{3}{2}x_4 \\ -x_4 \\ x_4 \end{bmatrix} = -\frac{x_4}{2} \begin{bmatrix} 6 \\ 3 \\ 2 \\ -2 \end{bmatrix} = \underline{t \begin{bmatrix} 6 \\ 3 \\ 2 \\ -2 \end{bmatrix}}$$

$$(-x_4/2 = t \in \mathbb{R})$$

Sidan $B\vec{x} = 1\vec{x}$, er $\vec{x}$ ein eigenvektor og $\lambda_1 = 1$ er ein eigenverdi. Vi ser at <u>eigenrommet til λ_1 har dimension 1.</u>

c) Karakteristisk likning:

$$\det(B - \lambda I) = 0 \Leftrightarrow$$

$$0 = \begin{vmatrix} 1-\lambda & 2 & 1 & 4 \\ 2 & 1-\lambda & 1 & 7 \\ 0 & 0 & 2-\lambda & 1 \\ 0 & 0 & 1 & 2-\lambda \end{vmatrix} =$$

$$(1-\lambda) \begin{vmatrix} 1-\lambda & 1 & 7 \\ 0 & 2-\lambda & 1 \\ 0 & 1 & 2-\lambda \end{vmatrix} - 2 \begin{vmatrix} 2 & 1 & 4 \\ 0 & 2-\lambda & 1 \\ 0 & 1 & 2-\lambda \end{vmatrix} =$$

$$(1-\lambda)\cdot(1-\lambda) \begin{vmatrix} 2-\lambda & 1 \\ 1 & 2-\lambda \end{vmatrix} - 2\cdot 2 \begin{vmatrix} 2-\lambda & 1 \\ 1 & 2-\lambda \end{vmatrix} =$$

$$(1-\lambda)^2 \left((2-\lambda)^2 - 1 \right) - 4 \left((2-\lambda)^2 - 1 \right) =$$

$$\left((2-\lambda)^2 - 1 \right) \left((1-\lambda)^2 - 4 \right) = 0 \Leftrightarrow$$

$(2-\lambda)^2 - 1 = 0 \quad \vee \quad (1-\lambda)^2 - 4 = 0 \iff$

$(2-\lambda)^2 = 1 \quad \vee \quad (1-\lambda)^2 = 4 \iff$

$\lambda - 2 = \pm 1 \quad \vee \quad \lambda - 1 = \pm 2 \iff$

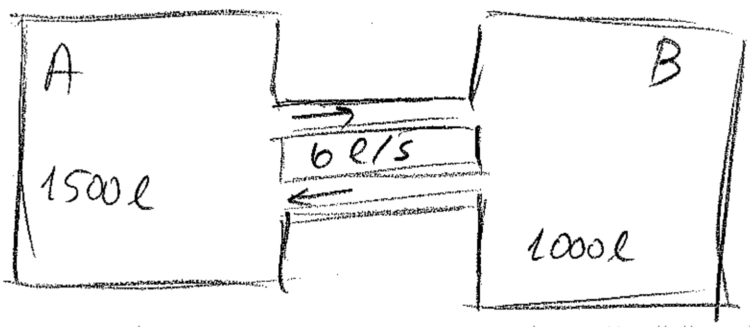
$\lambda = 2 \pm 1 \quad \vee \quad \lambda = 1 \pm 2 \iff$

$\lambda = 2-1 = 1 \quad \vee \quad \lambda = 2+1 = 3 \quad \vee \quad \lambda = 1-2 = -1 \quad \vee \quad \lambda = 1+2 = 3$

Eigenverdiar:

$\underline{\lambda_1 = 1, \quad \lambda_2 = -1 \quad \text{og} \quad \lambda_3 = 3}$

Oppg. 6



$S_A'(t)$: Giftmengde, målt i gram, i tank A etter t sekund.

$S_B'(t)$: Tilsvarande for tank B.

Tank B mottar gift frå A med farten

$\frac{6}{1500} \cdot S_A'(t)$ g/s og gir frå seg gift med farten $\frac{6}{1000} S_B'(t)$ g/s til A.

Farten gifta i B aukar med er $S_B'(t)$.

Vi får:

$$S_B'(t) = +\frac{6}{1500} S_A(t) - \frac{6}{1000} S_B(t).$$

Giftmengda er konstant lik 100g:

$$S_A(t) + S_B(t) = 20 \Leftrightarrow S_A = 100 - S_B(t)$$

Altså:

$$S_B'(t) = \frac{6}{1500}(100 - S_B) - \frac{6}{1000} S_B =$$

$$\frac{600}{1500} - \left(\frac{6}{1500} + \frac{6}{1000}\right) S_B =$$

$$\frac{2}{5} - \frac{1}{100} S_B \Leftrightarrow$$

$$S_B'(t) + \frac{1}{100} S_B = \frac{2}{5}$$

Integrerande faktor: $e^{\frac{t}{100}}$

$$e^{t/100}\left(S_B' + \frac{1}{100} S_B\right) = \frac{2}{5} e^{t/100}$$

$$\left(e^{t/100} S_B\right)' = \frac{2}{5} e^{t/100}$$

$$e^{t/100} S_B = \int \frac{2}{5} e^{t/100}\, dt = \frac{2}{5}\cdot 100\, e^{t/100} + C$$

$$S_B(t) = e^{-t/100}\left(\frac{200}{5} e^{t/100} + C\right) = 40 + C e^{-t/100}$$

Initialkrav: $S_B(0) = 0$ (Inga gift i B i starten)

$40 + C \cdot e^0 = 0$, $C = -40$

$S_B(t) = 40 - 40\,e^{-t/100} = 40(1 - e^{-t/100})$

Skal finne tida t då S_B er 20:

$S_B(t) = 20$

$40(1 - e^{-t/100}) = 20$

$1 - e^{-t/100} = \frac{20}{4} = \frac{1}{2}$

$e^{-t/100} = \frac{1}{2}$

$-\frac{t}{100} = \ln\frac{1}{2} = -\ln 2$

$t = 100 \ln 2 \approx 100 \cdot 0.693 = 69.3$

Det vil altså ta $\underline{\underline{69.3 \text{ sekund}}}$