oblig 3 LF

$\sqrt{a \cdot b} = \sqrt{a} \cdot \sqrt{b}$

$a, b \geq 0$

1.1 $f(x) = \dfrac{\sqrt[3]{8x}}{\sqrt{9x}}$

$= \dfrac{\sqrt[3]{8}\ \sqrt[3]{x}}{\sqrt{9}\ \sqrt{x}} = \dfrac{2}{3} \dfrac{x^{1/3}}{x^{1/2}}$

$= \dfrac{2}{3} x^{1/3} \underbrace{\left(x^{1/2}\right)^{-1}}_{x^{-1/2}} = \dfrac{2}{3} x^{1/3 - 1/2}$

$= \dfrac{2}{3} x^{\frac{2}{6} - \frac{3}{6}} = \underline{\dfrac{2}{3} x^{-1/6}}$

$f'(x) = \left(\dfrac{2}{3} x^{-1/6}\right)' = \dfrac{2}{3} \left(\dfrac{-1}{6}\right) x^{-1/6 - 1}$

$= \underline{\dfrac{-1}{9} x^{-7/6}} = \underline{\dfrac{-1}{9 \sqrt[6]{x^7}}}$

1.2 $f(x) = \sqrt{x^3} + \sqrt[3]{x^2} = (x^3)^{1/2} + (x^2)^{1/3}$

$= x^{3/2} + x^{2/3}$

$f'(x) = \dfrac{3}{2} x^{\frac{3}{2} - 1} + \dfrac{2}{3} x^{\frac{2}{3} - 1}$

$= \underline{\dfrac{3}{2} x^{1/2} + \dfrac{2}{3} x^{-1/3}}$

$= \underline{\dfrac{3}{2} \sqrt{x} + \dfrac{2}{3 \sqrt[3]{x}}}$

$$1.3 \quad g(x) = x^2\sqrt{x^4+4}$$

$$g'(x) = \underbrace{(x^2)'}_{2x}\left(\sqrt{x^4+4}\right) + (x^2)\underbrace{\left(\left(x^4+4\right)^{1/2}\right)'}_{\frac{1}{2\sqrt{x^4+4}}\underbrace{(x^4+4)'}_{4x^3}}$$

$$\underline{g'(x) = 2x\sqrt{x^4+4} + \frac{x^2 \cdot 4x^3}{2\sqrt{x^4+4}}}$$

Felles "nevner"

$$g'(x) = \frac{2x \cdot 2(x^4+4) + 4x^5}{2\sqrt{x^4+4}}$$

$$= \underline{\frac{8x^5 + 16x}{2\sqrt{x^4+4}}}$$

1.4 $h(x) \quad \overbrace{x^3(2x+3)^4}\overbrace{(3x+4)^5}$

$h'(x) = \left(x^3(2x+3)^4\right)'(3x+4)^5 + \left(x^3(2x+3)^4\right)\underbrace{\left((3x+4)^5\right)'}_{5(3x+4)^4\underbrace{(3x+4)'}_{3}}$

$= \underbrace{(x^3)'}_{3x^2}(2x+3)^4(3x+4)^5 + x^3\underbrace{\left((2x+3)^4\right)'}_{4(2x+3)^3\underbrace{(2x+3)'}_{2}}(3x+4)^5 + \text{—"—}$

$= \underline{3x^2(2x+3)^4(3x+4)^5 + 8x^3(2x+3)^3(3x+4)^5 + 15x^3(2x+3)^4 \cdot (3x+4)^4}$

(kan forenkles...)
se oppg 2.

1.5 $f(x) = \left(3+(x^3-4)^5\right)^7$

$f'(x) = 7\left(3+(x^3-4)^5\right)^6\left(3+\underbrace{(x^3-4)^5}_{0+5(x^3-4)^4\underbrace{(x^3-4)'}_{3x^2}}\right)'$

$f'(x) = 3\cdot5\cdot7\left(3+(x^3-4)^5\right)^6(x^3-4)^4\cdot x^2$

$= \underline{105\,x^2(x^3-4)^4\left(3+(x^3-4)^5\right)^6}$

2 $\quad P(x) = (2x+1)^3(3-x)^4$

$$P'(x) = \left((2x+1)^3\right)'(3-x)^4 + (2x+1)^3\left((3-x)^4\right)'$$

$$= 3(2x+1)^2\underbrace{(2x+1)'}_{2}(3-x)^4 + (2x+1)^3\cdot 4(3-x)^3\underbrace{(3-x)'}_{-1}$$

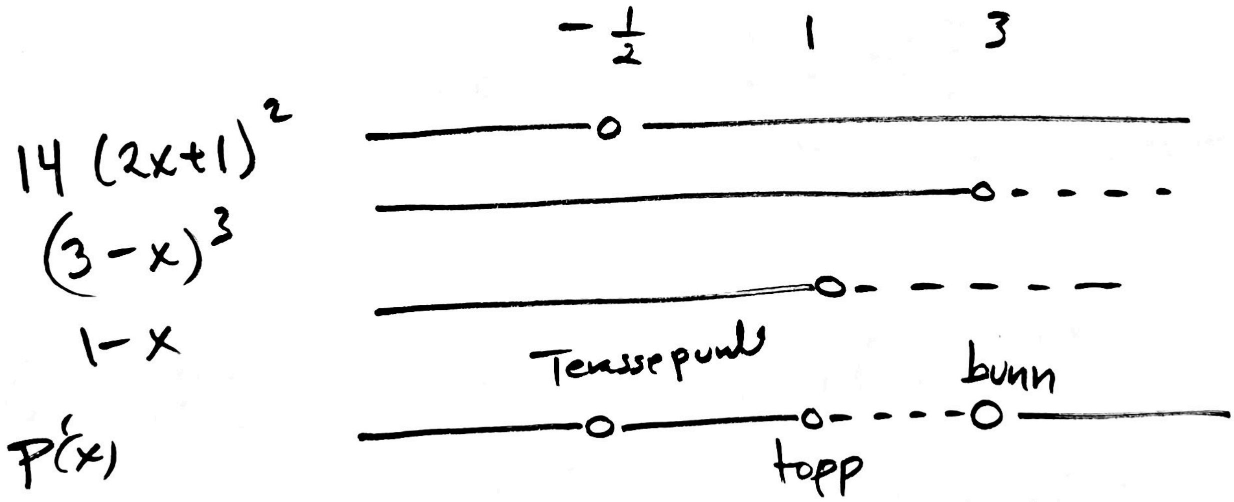
$$P'(x) = 2\cdot 3(2x+1)^2(3-x)^4 + (-1)\cdot 4(2x+1)^3(3-x)^3$$

$$= (2x+1)^2(3-x)^3\left[6(3-x) - 4(2x+1)\right]$$

$$= (2x+1)^2(3-x)^3\left[\underbrace{18-4}_{14}\underbrace{-6x-8x}_{-14x}\right]$$

$$= 14(2x+1)^2(3-x)^3(1-x)$$

Vi bestemmer fortegnet til $P'(x)$ for ulike verdier av x.



(benytter at $(3-x)^3$ har samme fortegn som $3-x$.)

Funksjonen har toppunkt i $(1, 3^3\cdot 2^4)$

$\underline{(1, 432)}$

Funksjonen har bunnpunkt i $\underline{(3, 0)}$

3 $f(x) = \begin{cases} 2(x+3)^2 & -3 \le x \le -2 \\ 2 - \sqrt{3x+6} & -2 \le x \le 1 \\ x^3 - 2x^2 & 1 \le x \le 2 \end{cases}$

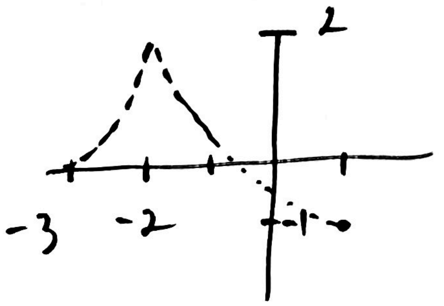
$f(x)$ er kont på (def. mengde) $[-3, 2]$

Ekstremalverdisetningen gir at det finnes globalt max og min punkt (siden $f(x)$ er kont. på en lukket begrenset mengde)

$$\left(2(x+3)^2\right)' = 2 \cdot 2(x+3)^1 (x+3)'$$
$$= 4(x+3) \quad i \ [-3,-2)$$
$$\geq 0 \quad i \ [-3,-2]$$

bare lik 0 i $x = -3$

$$\left(2 - \sqrt{3x+6}\right)' = 0 - \frac{1}{2\sqrt{3x+6}} (3x+6)'$$
$$= \frac{-3}{2\sqrt{3x+6}} \quad < 0 \quad i \ (-2, 1)$$



x^3-2x^2 $\qquad\qquad x \in [1,2]$

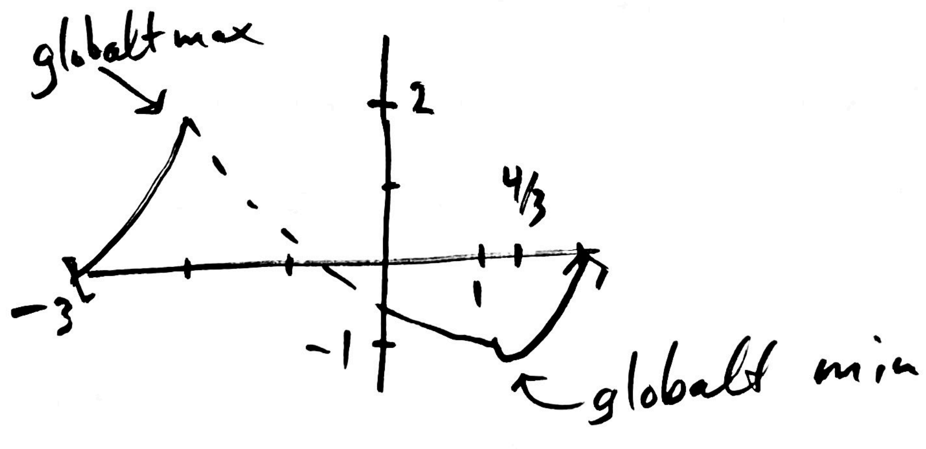
$$(x^3-2x^2)' = 3x^2-4x$$
$$= x(3x-4)$$
$$= 3x\left(x-\frac{4}{3}\right)$$

synke i $\left[1, \frac{4}{3}\right]$

stige i $\left[\frac{4}{3}, 2\right]$

$$f(2) = 2^3 - 2\cdot 2^2 = 0$$

$$f\left(\frac{4}{3}\right) = \left(\frac{4}{3}\right)^3 - 2\cdot\left(\frac{4}{3}\right)^2$$
$$= \left(\frac{4}{3}\right)^2\left(\frac{4}{3}-2\right)$$
$$= \frac{4\cdot 4\,(-2)}{3^3} = \underline{\frac{-32}{27}} = -1-\frac{5}{27}$$



Globalt maksimumspunkt $\underline{(-2, 2)}$

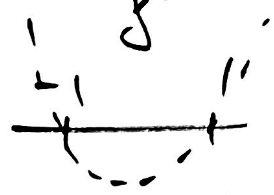
Globalt minimumspunkt $\underline{\left(\frac{4}{3}, \frac{-32}{27}\right)}$

4.1 $f(x) = x^3 - 3x + 1$

$f'(x) = 3x^2 - 3 = 3(x^2 - 1)$

$f''(x) = 6x$

c) $f(x)$ har ingen asymptoter.



a) $f'(x) = 3(x-1)(x+1)$

f stigende fra $-\infty$ til -1

f avtagende — -1 til 1

f stigende fra $x > 1$.

Vi har derfor toppunkt i $(-1, f(-1)) = (-1, 3)$

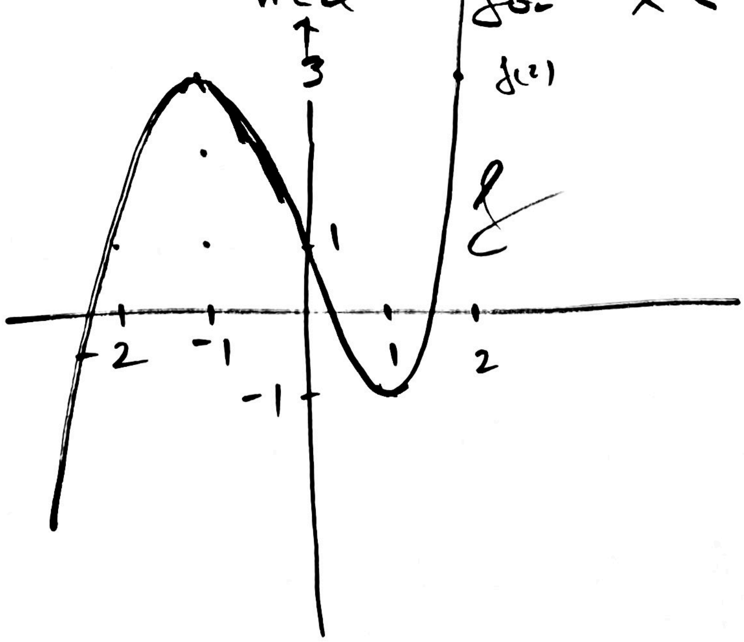
og bunnpunkt i $(1, f(1)) = (1, -1)$

b)

$f''(x) > 0$ konkav opp for $x > 0$

$f''(x) < 0$ — ned for $x < 0$.

skisse:



Vendepunkt i $(0, 1)$.

4.2 $r(x) = \frac{x^2+3}{x+3}$

Vi utfører polynomdivisjon for å få $r(x)$ på en mer egnet form.

$$r(x) = \frac{(x+3)(x-3)+9+3}{x+3}$$

$$= (x-3) + \frac{12}{x+3}.$$

a) $$r'(x) = (x-3)' + \frac{-12}{(x+3)^2}(x+3)'$$

$$= 1 - \frac{12}{(x+3)^2}$$

$$= \frac{(x+3)^2-12}{(x+3)^2} \quad \left(= \frac{x^2+6x-3}{(x+3)^2}\right)$$

$r'(x)=0$ når $(x+3)^2=12$ så

$$x = -3 \pm \sqrt{12} = \underline{-3 \pm 2\sqrt{3}}$$

nevnern >0 for $x \neq -3$.

telleren er en parabel med positiv ledende koeffisient. Derfor er

$r'(x) > 0$ for $x < -3-2\sqrt{3}$
stigende

$r'(x) < 0$ for $-3-2\sqrt{3} < x < 3+2\sqrt{3}$.
avtagende

$r'(x) > 0$ for $x > 3+2\sqrt{3}$

toppunkt: $\left(\overset{\approx -6.464}{-3-2\sqrt{3}},\ \overset{\approx -12.928}{r(-3-2\sqrt{3})}\right)$

bunnpunkt: $\left(\underset{\approx 0.464}{-3+2\sqrt{3}},\ \underset{0.928}{r(-3+2\sqrt{3})}\right)$

b) $r''(x) = (1)' - 12\left(\frac{1}{(x+3)^2}\right)'$

$= -12 \cdot (-2)\frac{1}{(x+3)^3}\underbrace{(x+3)'}_{1}$

$= \frac{24}{(x+3)^3}$.

$r''(x) > 0$ for $x > -3$ konkav opp

$r''(x) < 0$ for $x < -3$ konkav ned

ingen vendepunkt

($r(x)$ ikke definert i $x = -3$)

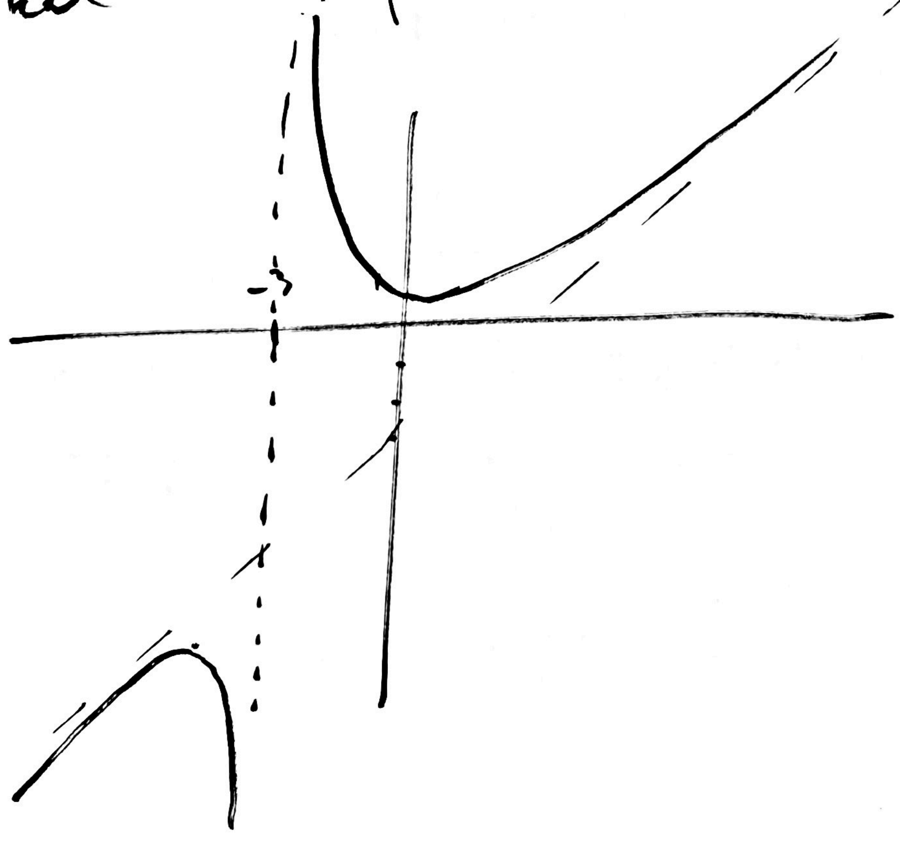
c) $y = x - 3$ skrå asymptote

siden $\lim_{x\to\infty}(r(x) - (x-3))$

$= \lim_{x\to\infty}\left(\frac{12}{x+3}\right) = 0$

Vertikal asymptote: $x = -3$.

d)



4.3

$$s(x) = e^{-x^2}$$

$$s'(x) = e^{-x^2}(-x^2)'$$

$$= -2x\,e^{-x^2}$$

$$s''(x) = (-2x)'\,e^{-x^2} + (-2x)(e^{-x^2})'$$

$$= -2\,e^{-x^2} + (-2x)^2\,e^{-x^2}$$

$$= \underline{2(2x^2-1)\,e^{-x^2}}$$

a) $s'(x) > 0$ når $x < 0$

$s'(x) < 0$ når $x > 0$

$s(x)$ vokser fra $-\infty$ til 0 og deretter avtar den.

Toppunkt i $\underline{(0, 1)}$

b) $2x^2 - 1 = 0$ når $x = \pm\frac{1}{\sqrt{2}}$

$\sim \pm 0{,}707$

konkav opp $x < \frac{-1}{\sqrt{2}}$

og $x > \frac{1}{\sqrt{2}}$

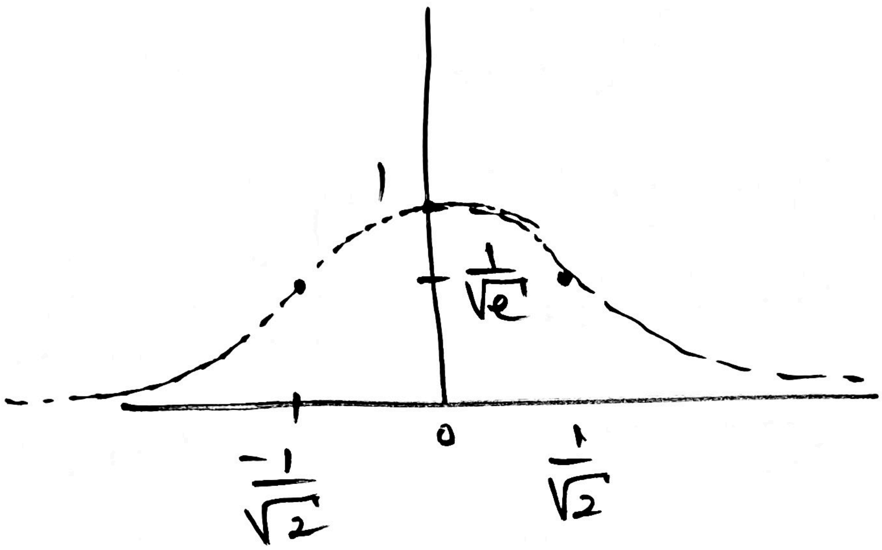
konkav ned for $\frac{-1}{\sqrt{2}} < x < \frac{1}{\sqrt{2}}$

Vendepunkt i $\left(\frac{-1}{\sqrt{2}}, e^{-1/2}\right)$

og i $\left(\frac{1}{\sqrt{2}}, e^{-1/2}\right)$.

c) $s(x)$ har horisontal asymptote $y = 0$, siden $\lim\limits_{x \to \infty} s(x) = 0$.
(og $-\infty$)

d)



(sjekk gjerne wikipedia...)

Til orientering. Grafer av denne formen

$$\frac{1}{\sqrt{2\pi}\,\sigma} e^{-(x-\mu)^2/2\sigma^2}$$

kalles normalfordeling med gjennomsnitt μ og varians σ^2.
sentralgrenseteoremet i statistikk sier at gjennomsnitt av mange, stokastiske variabler med samme fordeling tilnærmes av normalfordelingen.

4.4. $g(x) = \dfrac{(\log x)^2}{x}$

vi velger å uttrykke $\log x$ ved hjelp av $\ln x$: $\log x = \dfrac{\ln x}{\ln 10}$

Så $g(x) = \dfrac{1}{(\ln 10)^2} \cdot \dfrac{(\ln x)^2}{x}$

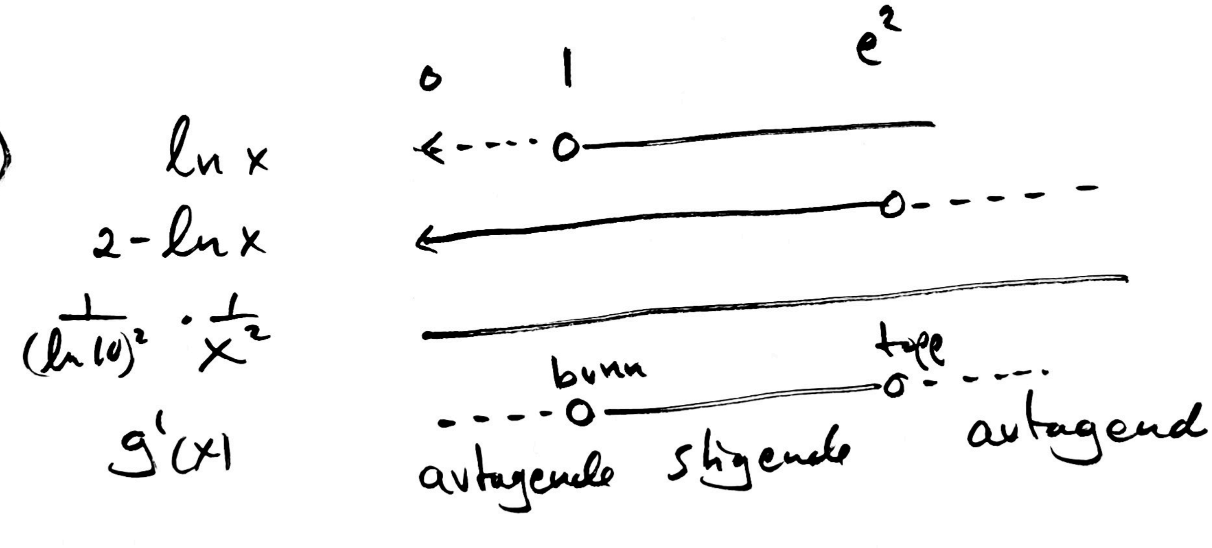
$$g'(x) = \frac{1}{(\ln 10)^2}\left[\left((\ln x)^2\right)' \cdot \frac{1}{x} + (\ln x)^2 \left(\frac{1}{x}\right)' \right]$$

$$= \frac{1}{(\ln 10)^2}\left[2\ln x \cdot (\ln x)' \cdot \frac{1}{x} + (\ln x)^2 \cdot \frac{-1}{x^2} \right]$$

$$= \frac{1}{(\ln 10)^2}\left(\frac{2\ln x}{x^2} - \frac{(\ln x)^2}{x^2} \right)$$

$$= \underline{\frac{1}{(\ln 10)^2} \cdot \frac{\ln x(2-\ln x)}{x^2}}.$$

a)



bunnpunkt : $(1, 0)$

glemt faktor $\frac{1}{(\ln 10)^2}$

toppunkt $\cancel{\left(e^2, \frac{4}{e^2}\right) \approx (7.39, 0.541)}$

$\left(e^2, \frac{4}{(\ln 10)^2 e^2}\right) \approx \underline{(7.39, 0.102)}$

b) $g''(x) = \frac{1}{(\ln 10)^2}\left[\left(2\ln x - (\ln x)^2\right)' \frac{1}{x^2} + \left(2\ln x - (\ln x)^2\right)\left(x^{-2}\right)'\right]$

$= \frac{1}{(\ln 10)^2}\left[\left(\frac{2}{x} - 2\ln x\,\underbrace{(\ln x)'}_{1/x}\right)\frac{1}{x^2} + \left(2\ln x - (\ln x)^2\right)\frac{-2}{x^3}\right]$

$g''(x) = \frac{1}{(\ln 10)^2}\left[\frac{2 - 2\ln x + (-2)(2\ln x - (\ln x)^2)}{x^3}\right]$

$= \frac{1}{(\ln 10)^2} \cdot \frac{1}{x^3}\left[2 - 6\ln x + 2(\ln x)^2\right]$

$= \frac{1}{(\ln 10)^2} \cdot \frac{2}{x^3}\left[\underbrace{(\ln x)^2 - 3\ln x + 1}_{(\ln x - \frac{3}{2})^2 \underbrace{- (\frac{3}{2})^2 + 1}_{-\frac{5}{4}}}\right]$

$= \frac{1}{(\ln 10)^2} \cdot \frac{2}{x^3}\left(\ln x - \frac{3}{2} - \frac{\sqrt{5}}{2}\right) \cdot \left(\ln x - \frac{3}{2} + \frac{\sqrt{5}}{2}\right)$

$g''(x) = 0$ $\qquad \ln x = \frac{3}{2} \pm \frac{\sqrt{5}}{2}$

$x = e^{(3 \pm \sqrt{5})/2}$

$x_1 = e^{(3-\sqrt{5})/2} \sim 1.465$

$x_2 = e^{(3+\sqrt{5})/2} \sim 13.709$

$g(x)$ konkav opp $\langle 0, x_1\rangle$

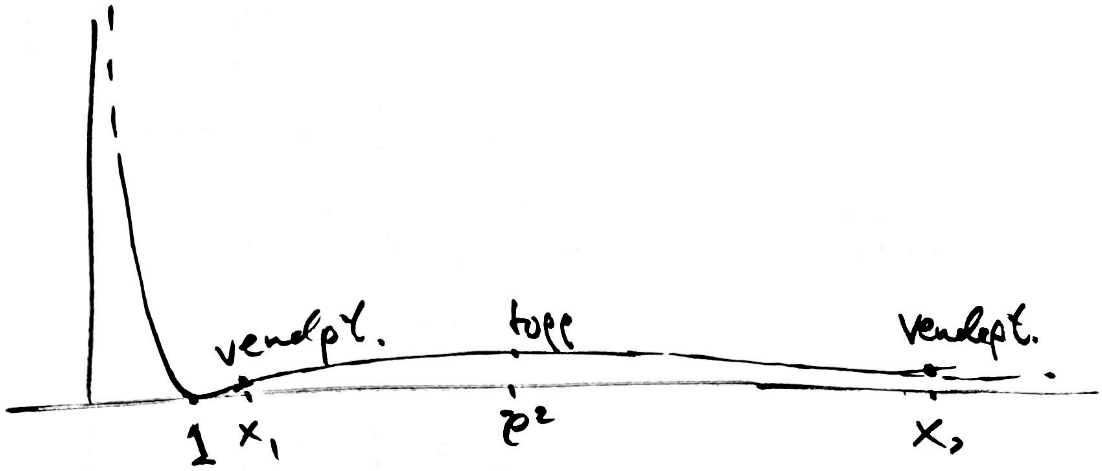
og i $\langle x_2, \infty\rangle$

$g(x)$ konkav ned $\langle x_1, x_2\rangle$

Vendepunkt $(x_1, g(x_1))$

$(x_2, g(x_2))$

4.4



c) Vertikal asymptote : y-aksen

Horisontal asymptote x-aksen.

5.1 $\quad 10^x = 5$

$$\text{Log}(10^x) = \text{Log}\,5$$

$$\underline{x = \text{Log}\,5 = 0.69897\ldots}$$

5.2 $\quad \text{Log}\,|x+2| = 1$

$$|x+2| = 10^{\text{Log}\,|x+2|} = 10^1 = 10 \qquad \left(10^{\text{Log}\,u} = u\right)$$

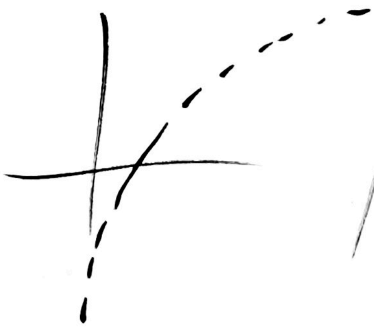
så $\quad x+2 = \pm 10$

$$x = -2 \pm 10$$

Løsningene er $\quad \underline{x = 8}$ og $\underline{x = -12}$

(Alternativt: $\quad \text{Log}\,|x+2| = 1 = \text{Log}\,10$

så da må $\quad |x+2| = 10$)



5.3 $\quad 3 \cdot 2^x = 10^x$

$$\log(3 \cdot 2^x) = \log 10^x$$

$$\log 3 + x \log 2 = x \qquad \text{lin. likning i } x$$

$$\log 3 = x - x\log 2 = x(1 - \log 2)$$

$$\underline{x = \frac{\log 3}{1 - \log 2}} = \frac{\log 3}{\log(10) - \log 2} = \underline{\frac{\log 3}{\log 5}}.$$

(Alternativt: $3 = \frac{10^x}{2^x} = \left(\frac{10}{2}\right)^x = 5^x$

$$\log 3 = \log 5^x = x \log 5$$

$$\underline{x = \frac{\log 3}{\log 5}}$$
)

5.4 $\quad 32^x = 10^x$

$$(3.2)^x = \frac{32^x}{10^x} = 1 \quad \text{så } x = 0$$

Alternativt $\quad \log 32^x = \log 10^x$

$$x \log(32) = x$$

$$x\underbrace{(\log(32) - 1)}_{\neq 0} = 0$$

$$\text{så } x = 0.$$

5.5 $\quad \log|\ln|x|| = 1 = \log 10$

$$|\ln|x|| = 10$$

så $\quad \ln|x| = \pm 10$

$$|x| = e^{\ln|x|} = e^{\pm 10}$$

så $\quad x = \underline{-e^{10}, -e^{-10}, e^{-10}, e^{10}}$

5.6 $e^{2x} + e^x - 6 = 0$

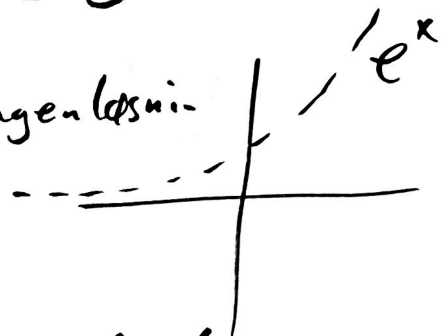
$(e^x)^2 + e^x - 6 = 0$

2.gradslikning i e^x

$\Leftrightarrow (e^x + 3)(e^x - 2) = 0$

løsningen til 2.grads-pol. :

$e^x = -3$ ingen løsn.



$e^x = 2$

Løsningen er $\underline{x = \ln(2)}$

5.7 $1000^x = 100^x + 10^x$

$(10^3)^x = (10^2)^x + 10^x$

$(10^x)^3 = (10^x)^2 + 10^x$

3.gradslikning i $u = 10^x$

$u^3 - u^2 - u = 0$

$u(u^2 - u - 1) = 0$

$u = 0$, abc-formel

$u = \dfrac{1 \pm \sqrt{1 - 4 \cdot 1(-1)}}{2 \cdot 1}$

$u = \frac{1}{2}(1 \pm \sqrt{5})$

$10^x = u = 0$ ingen løsning

$10^x = u = \frac{1}{2}(1-\sqrt{5}) < 0$ ingen løsning

$10^x = u = \frac{1}{2}(1+\sqrt{5})$

$x = \log\left(\frac{1}{2}(1+\sqrt{5})\right)$

Løsningen er $x = \underline{\log(1+\sqrt{5}) - \log(2)}$

$\sim \underline{0.20}898\ldots$

5.8 $\log|x| + \log|x+2| = 3$

$\log|x(x+2)| = 3 = \log(10^3)$

$|x(x+2)| = 10^3 = 1000$

$x(x+2) = \pm 1000$

$x^2 + 2x \mp 1000 = 0$

$a=1 \quad b=2 \quad c = \mp 1000$

abc-formel:

$$x = \frac{-2 \pm \sqrt{4 - 4(\mp 1000)}}{2}$$

$$= -1 \pm \sqrt{1 \pm 1000}.$$

Løsningene er $x = \underline{-1+\sqrt{1001}} \sim 30.638$

$= \underline{-1-\sqrt{1001}} \sim -32.638$

5.9

$$(\ln x)^2 = \ln x + 6$$

$$(\ln x)^2 - \ln x - 6 = 0$$

$$(\ln x - 3)(\ln x + 2) = 0$$

$$e^{\ln x} = e^{3} \quad \text{eller} \quad e^{\ln x} = e^{-2}$$

Løsningene er $\underline{x = e^{-2} \text{ og } e^{3}}$

5.10

$$e^{\ln\left(\frac{e^x}{e^x+1} + 2\right)} = e^{1}$$

$$\frac{e^x}{e^x+1} + 2 = e$$

$$\frac{e^x}{e^x+1} = e - 2$$

$$e^x = (e-2)(e^x+1)$$

$$e^x(\underbrace{1-(e-2)}_{3-e}) = e-2$$

$$e^x = \frac{e-2}{3-e}$$

Løsningen er $\underline{\underline{x = \ln\left(\frac{e-2}{3-e}\right)}}$

6 toppunkt i (1.2) $\Rightarrow$

$\underline{p(1)=2}$ og $\underline{p'(1)=0}$

nullpunkt i $(-1,0)$: $\underline{p(-1)=0}$

1. $p(x) = a + bx + cx^2$ grad $p \le 2$

$p(1)=2$ $a+b+c=2$

$p(-1)=0$ $a-b+c=0$

$p'(1)=0$ $b+2c=0$

L1+L2 gir $2a+2c=2$

L1−L2 — $2b=2$

så $\underline{b=1}$ $a+c=1$

L3 gir $c = -\frac{1}{2}b = -\frac{1}{2}$

$a = 1-c = 1+\frac{1}{2}b = \frac{3}{2}$

$p(x) = \frac{3}{2} + (-\frac{1}{2})x^2 + x$

$= \underline{-\frac{1}{2}x^2 + x + \frac{3}{2}}$

6.2

$$P(x) = a + bx + cx^2 + dx^3$$
$$p'(x) = b + 2cx + 3dx^2$$

$p(1) = 2$: $\quad a + b + c + d = 2$

$p(-1) = 0 \quad a - b + c - d = 0$

$p'(1) = 0 \quad b + 2c + 3d = 0$

L1+L2 gir $\quad a + c = 1$

L1−L2 $\quad b + d = 1$

$\underline{b = 1 - d}$

$2c = -b - 3d = d - 1 - 3d = -1 - 2d$

$\underline{c = \frac{-1}{2}(1+2d)}$

$a = 1 - c = 1 + \frac{1}{2}(1+2d) = \underline{\frac{3+2d}{2}}$

$$\underline{p(x) = dx^3 - \frac{1}{2}(1+2d)x^2 + (1-d)x + \frac{3+2d}{2}}$$

når har vi toppunkt : $(1,2)$?

$p''(1) = 2c + 6d = -(1+2d) + 6d$
$= \underline{-1 + 4d}$

konkav ned : $-1 + 4d < 0$
$d < \frac{1}{4}$

konkav opp $\quad -1 + 4d > 0 \quad : \quad d > \frac{1}{4}$

terrassepunkt for $d = \frac{1}{4}$

3 polynome er derfor

$$p(x) = dx^3 - \frac{1+2d}{2}x^2 + (1-d)x + \frac{3+2d}{2}$$

for $d < \frac{1}{4}$

(test $d=0$ gir 2. grads pol: $-\frac{x^2}{2} + x + \frac{3}{2}$
fra 6.1)

6.3 $\quad p(x) = a + bx + cx^2 + dx^3 + ex^4$

$p(1) = 2 \qquad a+b+c+d+e = 2$

$p(-1) = 0 \qquad a-b+c-d+e = 0$

$p'(-1) = \qquad b + 2c + 3d + 4e = 0$

3 Likningen, 5 ukjente
Det blir $5-3=2$ "frie parametre".

$L1+L2: \qquad a+c+e = 1$

$L1-L2: \qquad b+d = 1$

$b = 1-d$

$2c = -b-3d-4e$

$= d-1-3d-4e = -1-2d-4e$

$c = \frac{-1}{2}(1 + 2d + 4e)$

$a = 1 - c - e = 1 + \frac{1}{2} + d + 2e - e$

$= \underline{\frac{3}{2} + d + e}$

$p(x) = \left(\frac{3}{2} + d + e\right) + (1-d)x - \frac{(1+2d+4e)}{2}x^2 + dx^3 + ex^4$

$p''(1) = 2c + 6d + 12e$

$= -(1+2d+4e) + 6d + 12e$

$= -1 + 4d + 8e$

toppunkt når $p''(1) < 0$: $4d + 8e < 1$

bunnpunkt når $p''(1) > 0$ $4d + 8e > 1$.

Vi undersøker hva som skjer

når $\underline{4d + 8e = 1}$

$p'''(1) = 6d + 24e$ visetter inn $8e = 1 - 4d$

$= 6d + 3(1 - 4d)$

$= 3 - 6d = 6\left(\frac{1}{2} - d\right).$

Når $p'''(1) \neq 0$ så ser grafen lokalt rundt $(1,2)$ ut som et 3.gradspolynom m. terrassepunkt i $(1,2)$.

Hvis $p'''(1) = 0$ da er $d = \frac{1}{2}$

$e = \frac{1}{8}(1-4d) = \frac{-1}{8} < 0.$

$p^{(4)}(1) = 24e < 0.$

grafen ser lokalt rundt $(1,2)$ ut som

$\frac{-1}{8}(x-1)^4 + 2.$

Vi har da et toppunkt.

Løsningene er:

$$P(x) = ex^4 + dx^3 - \frac{1+2d+4e}{2}x^2 + (1-d)x + \left(\frac{3}{2} + d + e\right)$$

for $d + 2e < \frac{1}{4}$

samt $d = \frac{1}{2}$ og $e = \frac{-1}{8}$

7 C-14 metoden

t er variabelen (tid)

a) $r(t) = r_0 e^{-kt}$

$r(0) = r_0 e^0 = r_0$.

$r'(t) = r_0 (e^{-kt})'$

$= r_0 (e^{-kt}) \cdot \underbrace{(-kt)'}_{-k}$

$= -k r_0 e^{-kt}$

$\underline{r'(t) = -k\, r(t)}$

b) $r(t_{1/2}) = \frac{1}{2} r_0$ halvparten igjen.

$r_0 e^{-kt_{1/2}} = \frac{1}{2} r_0$

så $\ln(e^{-kt_{1/2}}) = \ln(1/2)$

$-k t_{1/2} = -\ln 2$

så $\boxed{k \cdot t_{1/2} = \ln 2}$

c) $\frac{r(t)}{r_0} = e^{-kt} = 0.7$ gir

$-kt = \ln(0.7)$ så

$t = \frac{-\ln 0.7}{k} = t_{1/2} \frac{-\ln(0.7)}{\ln 2}$

$= 5700$ år $\cdot 0.51457$

$\underline{\sim 2933 \text{ år}}$

d) år 0 ~ $t = 2022$.

$$\frac{r(t)}{r_0} = e^{-k\cdot(2022)} = e^{-\ln 2 \cdot \frac{2022}{5700}}$$

$$= \left(\frac{1}{2}\right)^{\frac{2022}{5700}} = \underline{0.7820}$$

Vi kan forvente et forhold $\frac{r(t)}{r_0}$ på ca $\underline{0.78}$

8

diameter b

Lengde på gjerde er

$$L(a,b) = a + \pi b$$

Areal avgrenset av gjerde og vegg:

$$A = ab + \pi\left(\frac{b}{2}\right)^2 = 10 \text{ m}^2$$

(utelatte enheter)

Dette lar oss uttrykke a ved hjelp av b:

$$a = \frac{A - \pi b^2/4}{b} = \underline{\frac{A}{b} - \frac{\pi}{4}\cdot b}$$

Setter dette inn i uttrykket for lengden til gjerdet, når $A = 10$

$$L(b) = a + \pi b = \frac{A}{b} - \frac{\pi}{4} b + \pi b$$

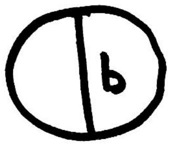
$$= \frac{A}{b} + b\left(-\frac{\pi}{4} + \pi\right)$$

$$L(b) = \frac{A}{b} + \frac{3\pi}{4} b$$

når $b \to 0^+$ vil $L(b) \to \infty$.

Det største b kan være er slik at

$a = 0$



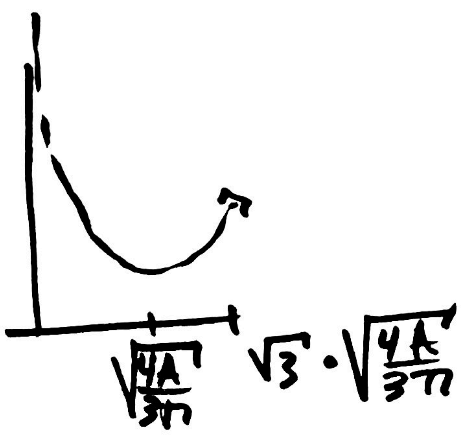
$$\pi\left(\frac{b}{2}\right)^2 = A$$

$$b_{max} = \sqrt{\frac{4A}{\pi}}.$$

$$L'(b) = \frac{-A}{b^2} + \frac{3\pi}{4} \cdot 1$$

$L'(b) = 0$ når $\frac{A}{b^2} = \frac{3\pi}{4}$

så $\underline{b = \sqrt{\frac{4A}{3\pi}}}$ $(< b_{max})$



Gjerdet er kortest når arealet er
A for

$b = \sqrt{\frac{4A}{3\pi}}$ og $a = \frac{A}{\sqrt{\frac{4A}{3\pi}}} - \frac{\pi}{4}\sqrt{\frac{4A}{3\pi}}$

$$a = \frac{\sqrt{A}\cdot\sqrt{3\pi}}{2} - \frac{\pi}{4}\frac{\cdot 2\sqrt{A}}{\sqrt{3}\sqrt{\pi}}$$

$$= \sqrt{A\pi}\left(\frac{\sqrt{3}}{2} - \frac{1}{2\sqrt{3}}\right) = \sqrt{A\pi}\left(\frac{3-1}{2\sqrt{3}}\right)$$

$$\underline{\underline{a = \sqrt{A\pi/3}}}$$

Forholdet mellom a og b er da:

$$\frac{a}{b} = \frac{\sqrt{\cancel{x}A\pi/3}}{\sqrt{4A}}\cdot\sqrt{3\pi} = \underline{\underline{\frac{\pi}{2}}} \sim \underline{1.57}$$

Når A = 10 får vi

$$b = 2\sqrt{\frac{10}{3\pi}} \sim \underline{\underline{2.0601}}$$

$$a = \sqrt{\frac{10\cdot\pi}{3}} \sim \cancel{9.7087}\ \underline{\underline{3,236}}$$

9.1 $x^e + e^x$

potensfunksjon ↑ ↖ eksponentfunksjon

$$(x^e + e^x)' = \underline{e x^{e-1} + e^x}$$

9.2 $$2^x \cdot 5^{x+1} = 2^x \cdot 5^x \cdot 5^1$$
$$= 10^x \cdot 5$$
(benytter $10 = e^{\ln 10}$)

Så $(2^x 5^{x+1})' = 5(10^x)'$
$$= 5\left(e^{\ln 10 \cdot x}\right)' = 5\ln(10)\, e^{\ln 10 \cdot x}$$
kjerneregelen

$$= \underline{5\ln(10)\ 10^x}$$

9.3 $$\frac{3}{e^{2x}} = 3 \cdot e^{-2x}$$

så den deriverte er lik
$$3\, e^{-2x} \cdot (-2x)' = \underline{-6\, e^{-2x}}$$

9.4 $$\frac{x^2}{(2^x)^3} = x^2 \cdot 2^{-3x} = x^2 e^{-(\ln 2)\cdot 3 \cdot x}$$
Benytter produkt og kjerneregelen
$$\left(\frac{x^2}{(2^x)^3}\right)' = (x^2)' \cdot 2^{-3x} + x^2\left(e^{-(\ln 2)3x}\right)'$$
$$= 2x \cdot 2^{-3x} + x^2\, 2^{-3x} \cdot (-(\ln 2)\cdot 3)$$
$$= \underline{(2x - 3\ln(2)\, x^2)\cdot 2^{-3x}}$$

9.5 $r(x) = \dfrac{e^{2x^3}}{3x+2} + 2e^3$

konstant

$$r'(x) = \frac{(e^{2x^3})'(3x+2) - e^{2x^3}(3x+2)'}{(3x+2)^2}$$

(kvotientregel)

$$r'(x) = \frac{6x^2(3x+2)\,e^{2x^3} - 3e^{2x^3}}{(3x+2)^2}$$

$$= \underline{\frac{(18x^3+12x^2-3)\,e^{2x^3}}{(3x+2)^2}}$$

9.6 $\ell(x) = \log\left|\dfrac{2}{3x+1}\right| = \underbrace{\log 2}_{\text{konstant}} - \log|3x+1|$

$$\ell'(x) = 0 - \frac{1}{(3x+1)\cdot\ln 10}\,\underbrace{(3x+1)'}_{3}$$

$$= \underline{\frac{-3}{\ln(10)(3x+1)}}$$

9.7 $k(x) = \dfrac{10^x - 3}{2^x} = \dfrac{10^x}{2^x} - \dfrac{3}{2^x}$

$$= 5^x - 3\cdot 2^{-x}$$

$$k'(x) = \left(e^{\ln 5\cdot x} - 3\cdot e^{-x\ln 2}\right)'$$

$$= \underline{(\ln 5)\,5^x + 3\ln(2)\cdot 2^{-x}}$$

9.8 $m(x) = \log \sqrt{1+x^2}$

$= \log \left((1+x^2)^{1/2} \right)$

$= \frac{1}{2} \log (1+x^2)$

$m'(x) = \frac{1}{2} \frac{1}{\ln 10} \cdot \frac{1}{1+x^2} (1+x^2)'$

$= \underline{\dfrac{x}{(\ln 10)(1+x^2)}}$

9.9. $V(x) = (e^x)^4 \log |3-x|$

$= e^{4x} \log |3-x|$

$V'(x) = (e^{4x})' \log |3-x| + e^{4x} (\log|3-x|)'$

$= 4e^{4x} \log |3-x| + e^{4x} \cdot \frac{1}{\ln 10 (3-x)} \underbrace{(3-x)'}_{-1}$

$V'(x) = \underline{4e^{4x} \log |3-x| - \frac{1}{(\ln 10)(3-x)} \cdot e^{4x}}$

10 Vi benytter at

$$e = \lim_{x\to\infty} \left(1+\frac{1}{x}\right)^x = \lim_{x\to-\infty} \left(1+\frac{1}{x}\right)^x.$$

1. $$\lim_{x\to\infty} \left(1+\frac{2}{x}\right)^x$$

reduserer til tilfellet ovenfor.

$y = \frac{x}{2}$, $x = 2 \cdot y$

$x\to\infty \Leftrightarrow y\to\infty$

$$\lim_{y\to\infty} \left(1+\frac{1}{y}\right)^{2y}$$

$$= \left(\lim_{y\to\infty} \left(1+\frac{1}{y}\right)^y\right)^2$$

(siden $(-)^2$ er kontinuerlig)

$$= \underline{\underline{e^2}}$$

(vi tester $\left(1+\frac{2}{10\,000}\right)^{10\,000} \approx 7.3875\ldots$)

2. $$\lim_{x\to\infty} \left(\frac{x-1}{x}\right)^x = \lim_{x\to\infty} \left(1-\frac{1}{x}\right)^x$$

La $-x = y$

$x\to\infty \Leftrightarrow y\to-\infty$

grensen er lik:

$$\lim_{y\to-\infty} \left(1+\frac{1}{y}\right)^{-y}$$

$$= \left(\lim_{y\to\infty} \left(1+\frac{1}{y}\right)^y\right)^{-1} = \underline{\underline{\frac{1}{e}}}$$

3. $$\lim_{x\to 0} \frac{\log(1+x)}{x}$$

$$= \lim_{x\to 0} \frac{\log(1+x) - \overbrace{\log 1}^{0}}{x}$$

= Den deriverte av $\log(z)$ i $z=1$.

$$= \frac{1}{\ln 10} \cdot \frac{1}{z} \Big|_{z=1} = \underline{\underline{\frac{1}{\ln 10}}}$$

10.4. $$\lim_{x\to\infty} \left(1 - \frac{1}{x^2}\right)^x$$

$x^2 = -y$

$x \to \infty$

$\Leftrightarrow y \to -\infty$

$$= \lim_{y\to -\infty} \left(1 + \frac{1}{y}\right)^{\sqrt{-y}}$$

$$= \lim_{y\to -\infty} \left(1 + \frac{1}{y}\right)^{-y/\sqrt{-y}}$$

$$\leq \sqrt[n]{\lim_{y\to -\infty} \left(1 + \frac{1}{y}\right)^{-y}} = \sqrt[n]{\frac{1}{e}}$$

for alle naturlige tall n.

(la $-y$ være så stor at $\sqrt{-y} > n$ ...)

$\sqrt[n]{\frac{1}{e}} \to 1$ når n blir stor.

Derfor er grensen $\lim_{x\to\infty} \left(1 - \frac{1}{x^2}\right)^x = \underline{\underline{1}}$